Lecture (02)
Number Systems (2)

By:

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Number base conversion

- Convert decimal 41 to binary
- First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of 1/2.
- Then the quotient is again divided by 2 to give a new quotient and remainder.
- The process is continued until the integer quotient becomes 0.
- The *coefficients* of the desired binary number are obtained from the *remainders* as follows

	Integer Quotient		Remainder	Coefficien
$41/2 =$	20	+	1	$a_0 = 1$
$20/2 =$	10	+	0	$a_1 = 0$
$10/2 =$	5	+	0	$a_2 = 0$
$5/2 =$	2	+	1	$a_3 = 1$
$2/2 =$	1	+	0	$a_4 = 0$
$1/2 =$	0	+	1	$a_5 = 1$

Therefore, the answer is $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$.

Example

- Convert decimal 153 to octal.
- The required base *r* is 8.
- First, 153 is divided by 8 to give an integer quotient of 19 and a remainder of 1.
- Then 19 is divided by 8 to give an integer quotient of 2 and a remainder of 3.
- Finally, 2 is divided by 8 to give a quotient of 0 and
- a remainder of 2.

	quotient		reminder	Coefficient
153/8	19	+	1	a0=1
19/8	2	+	3	a1=3
2/8	0	+	2	a2=2

$(153)_{10} = (231)_8$

Example

- Convert $(0.6875)_{10}$ to binary
- First, 0.6875 is multiplied by 2 to give an integer and a fraction.
- Then the new fraction is multiplied by 2 to give a new integer and a new fraction.
- The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

	Integer		Fraction	Coefficient
0.6875x2	1	+	0.3750	$a_{-1}=1$
0.3750x2	0	+	0.7500	$a_{-2}=0$
0.7500 x2	1	+	0.5000	$a_{-3}=1$
0.5000 x2	1	+	0.0000	$a_{-4}=1$

the answer is $(0.6875)_{10} = (0.\ a_{-1}\ a_{-2}\ a_{-3}\ a_{-4})_2 = (0.1011)_2$.

Example

$$
\begin{aligned}
0.513 \times 8 &= 4.104 \\
0.104 \times 8 &= 0.832 \\
0.832 \times 8 &= 6.656 \\
0.656 \times 8 &= 5.248 \\
0.248 \times 8 &= 1.984 \\
0.984 \times 8 &= 7.872
\end{aligned}
$$

- Convert (0.513)10 to octal.

	Integer		Fraction	Coefficient
0.513x8	4	+	0.104	$a_{-1}=4$
0.104x8	0	+	0.832	$a_{-2}=0$
0.832 x8	6	+	0.656	$a_{-3}=6$
0.656 x8	5	+	0.248	$a_{-4}=5$
0.248x8	1	+	0.984	$a_{-5}=1$
0.984	7		0.872	$a_{-6}=7$

- $(0.513)_{10} = (0.406517\ \ldots.)_8$

- By combining previous examples:

$$(41.6875)_{10} = (101001.1011)_2$$

$$(153.513)_{10} = (231.406517)_8$$

Octal and Hexadecimal numbers

- The conversion from and to binary, octal, and hexadecimal plays an important role in digital computers, because shorter patterns of hex characters are easier to recognize than long patterns
- $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

The conversion from binary to octal

- partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.
- The corresponding octal digit is then assigned to each group.

$$\begin{matrix}(10 & 110 & 001 & 101 & 011 & \cdot & 111 & 100 & 000 & 110)_2 = (26153.7406)_8 \\ 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6\end{matrix}$$

Conversion from binary to hexadecimal

- similar, except that the binary number is divided into groups of *four* digits:

$$
\begin{array}{cccccccc}
(10 & 1100 & 0110 & 1011 & \cdot & 1111 & 0010)_2 = (2C6B.F2)_{16} \\
2 & C & 6 & B & & F & 2
\end{array}
$$

Conversion from octal to binary

- is done by reversing the preceding procedure.
- Each octal digit is converted to its three-digit binary equivalent.

$$
\begin{array}{lcccccc}
(673.124)_8 = (110 & 111 & 011 & \cdot & 001 & 010 & 100)_2 \\
\quad\quad\quad\quad\quad\quad 6 & 7 & 3 & & 1 & 2 & 4
\end{array}
$$

Conversion from hexadecimal to binary

- is done by reversing the preceding procedure.
- each hexadecimal digit is converted to its four-digit binary equivalent

$$(306.D)_{16} = (0011 \quad 0000 \quad 0110 \quad \cdot \quad 1101)_2$$
$$\qquad\qquad\quad 3 \qquad\quad 0 \qquad\quad 6 \qquad\qquad\quad D$$

- that's why we need to use hex and octal,

Easy conversion between binary ←→ octal , and binary ←→hexadecimal,

- Binary numbers are difficult to work with because they require three or four times as many digits as their decimal equivalents.
- For example, the binary number 111111111111 is equivalent to decimal 4095.

Arithmetic operations

$$\begin{array}{r} 101101 \\ +100111 \\ \hline 1010100 \end{array}$$

```
 1010001
+1000101
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10010110
```


•

$$\begin{array}{r} 101101 \\ -100111 \\ \hline 000110 \end{array}$$

```
 1010001
- 1000101
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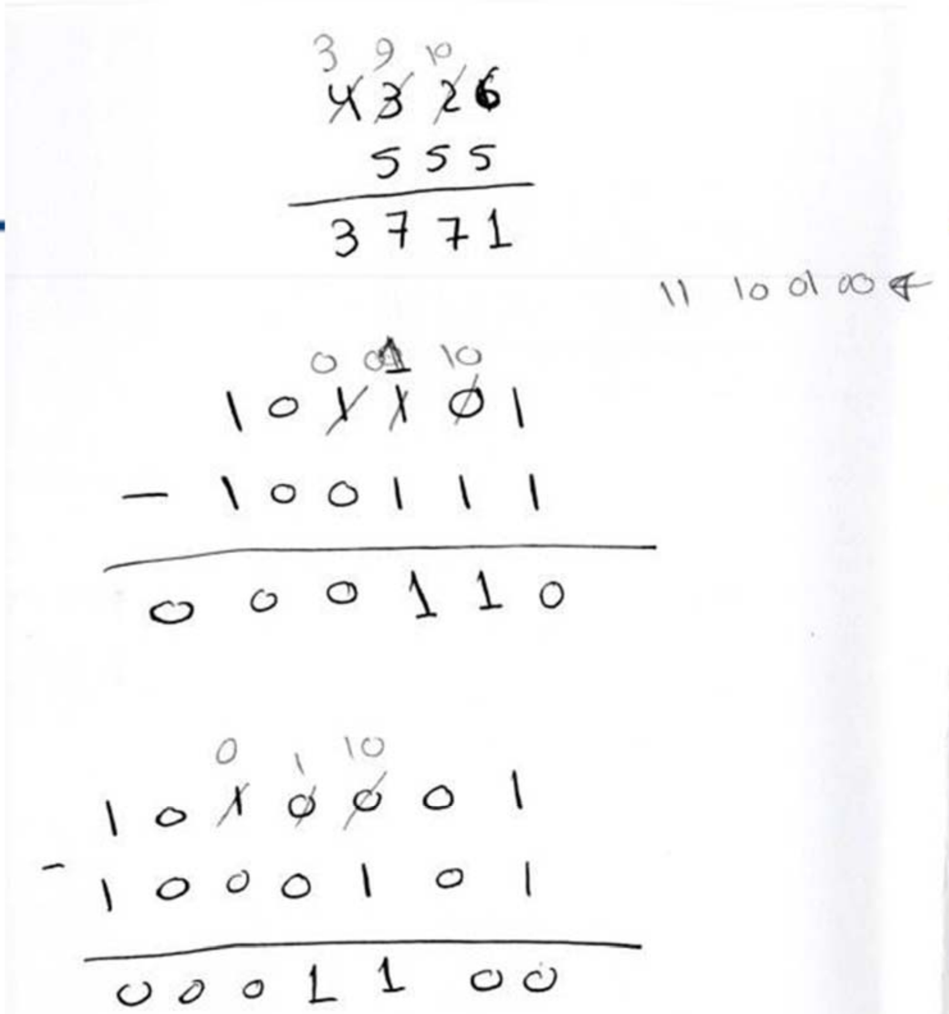
  0001100
```



d:

```
  1011
 × 101
  ----
  1011
 0000
1011
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110111
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Thanks,..
See you next week isA,..
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