



# Lecture (03) Boolean Algebra I

By:

**Dr. Ahmed ElShafee**

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Design I

## objectives

- Security system installed in a house; consists of
  - Door sensor (D) : 0 = closed; 1= opened
  - Light sensor (L): 0 = night; 1 = day
  - Alarm (A) : 0 = no alarm; 1 = Alarm
- System is designed to protect home at night, that if door is opened during day light; alarm will not activated, if door is open during night; alarm will be activated.

- Doorsensor (D) : 0 = closed; 1= opened
- Light sensor (L): 0 = night; 1 = day
- Alarm (A) : 0 = no alarm; 1 = Alarm

```

If((D==0))
{
    A=0;
}
Else if((D==1)&&(L==1))
{
    A=0;
}
Else if((D==1)&&(L==0))
{
    A=1;
}

```

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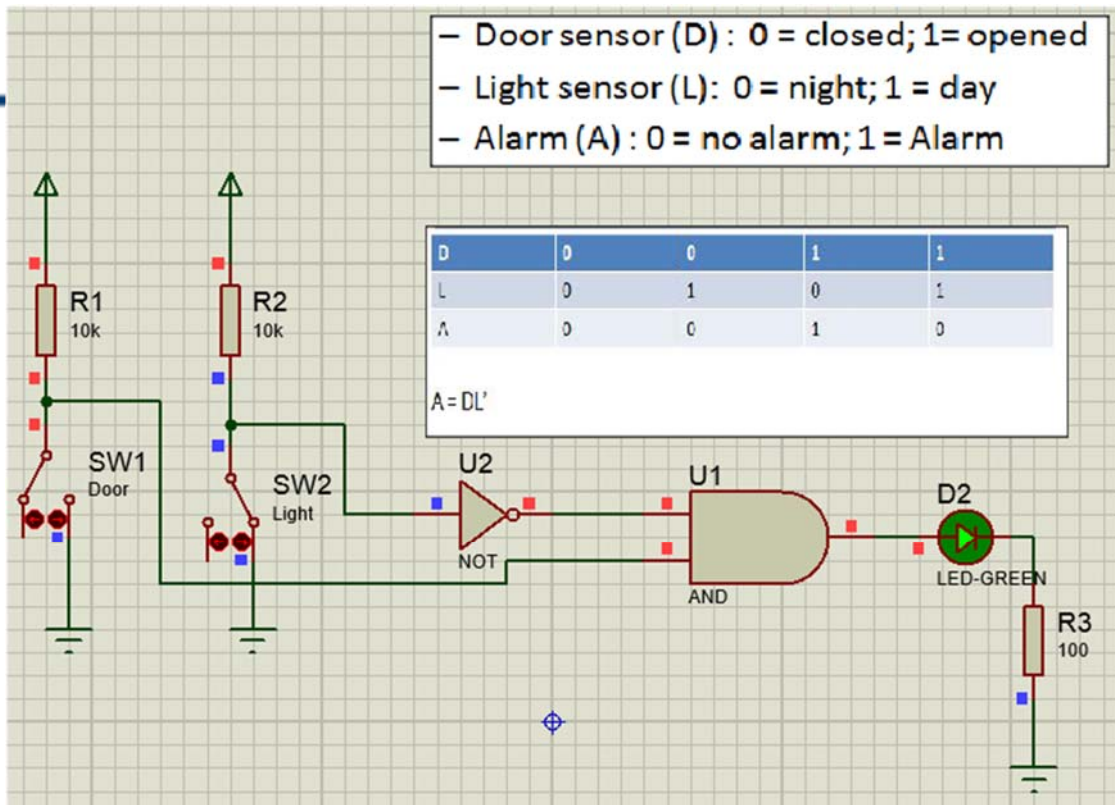
- Doorsensor (D) : 0 = closed; 1= opened
- Light sensor (L): 0 = night; 1 = day
- Alarm (A) : 0 = no alarm; 1 = Alarm

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D	L	A
0	0	0
0	1	0
1	0	1
1	1	0

$$A = DL'$$

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## objectives

- Security system installed in a house; consists of
  - Door sensor (D) : 0 = closed; 1= opened
  - Light sensor (L): 0 = night; 1 = day
  - Windows (w) : 0: closed; 1 = opened
  - Alarm (A) : 0 = no alarm; 1 = Alarm
- System is designed to protect home at day and night, that if
  - door is opened during day light; alarm will not be activated, if door is open during night; alarm will be activated.
  - If window is opened during day light; alarm will not be activated, if door is open during night; alarm will be activated.

- Door sensor (D) : 0 = closed; 1= opened
- Light sensor (L): 0 = night; 1 = day
- Windows (w) : 0: closed; 1 = opened
- Alarm (A) : 0 = no alarm; 1 = Alarm

<pre> If((D==0) &amp;&amp; (W==0)) {     A=0; } Else if(L==1) {     A=0; } Else if((L==0)&amp;&amp;(D==1)) {     A=1; } </pre>	<pre> Else if((L==0)&amp;&amp;(W==1)) {     A=1; } Else if((L==0)&amp;&amp;(W==1)&amp;&amp;(D==1)) {     A=1; } Else {     A=0; } </pre>
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- Door sensor (D) : 0 = closed; 1= opened
- Light sensor (L): 0 = night; 1 = day
- Windows (w) : 0: closed; 1 = opened
- Alarm (A) : 0 = no alarm; 1 = Alarm

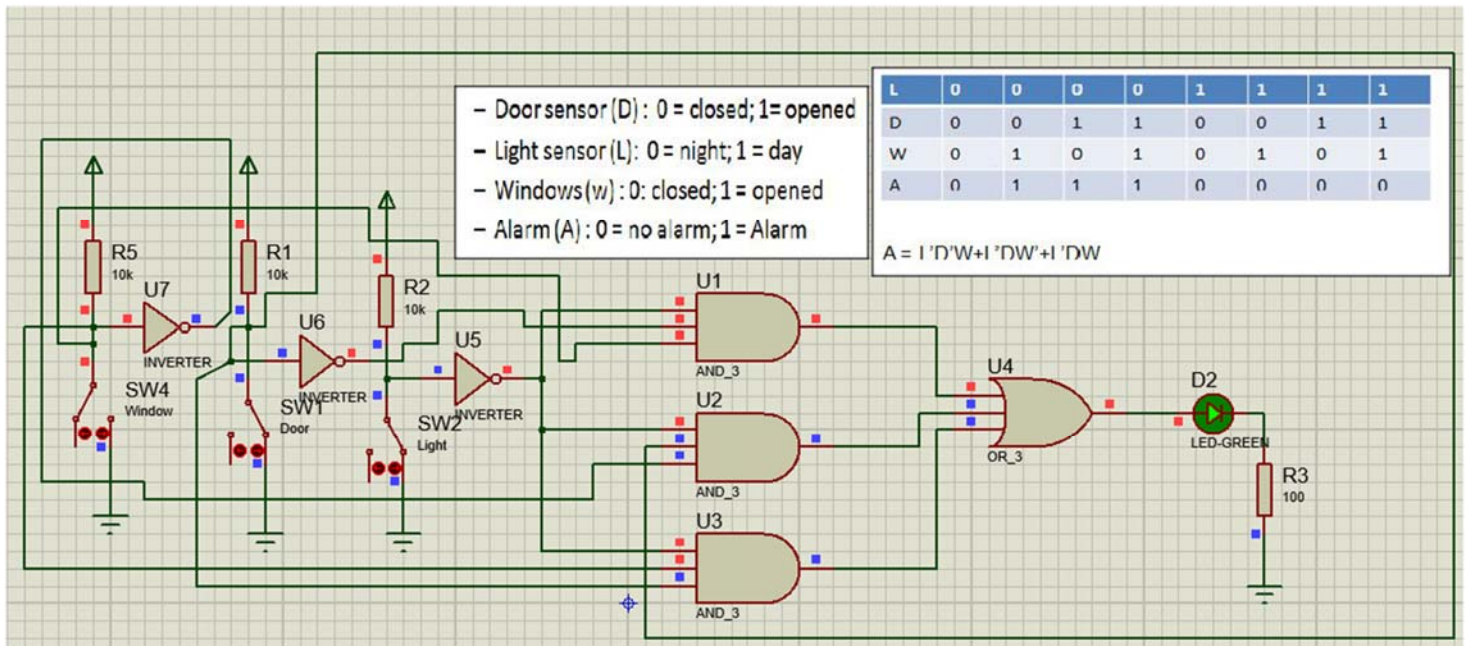
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L	D	W	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$$A = L'D'W + L'DW' + L'DW$$

A





# Introduction

- George Boole developed Boolean algebra in 1847 and used it to solve problems in mathematical logic.

## Boolean Algebra

- is an *algebra*  $(\mathbf{B}; ., +, ' ; \mathbf{0}, \mathbf{1})$  consisting of a set  $\mathbf{B}$  (which contains at least two elements  $\mathbf{0}$  and  $\mathbf{1}$ )
- together with *three* operations,
- the **AND** (Boolean *product*) operation  $.$ ,
- the **OR** (Boolean *sum*) operation  $+$ , and
- the **NOT** (*complement*) operation  $'$ , defined on the set, such that

- 
- An element of a Boolean algebra  $B$  is called a *constant* on  $B$ .
  - e.g. 0,a,b,1 in  $B_4$ .
  
  - A symbol that may represent any one of element of  $B$  is called a (Boolean) *variable* on  $B$ .
  - e.g. x,y,z,...

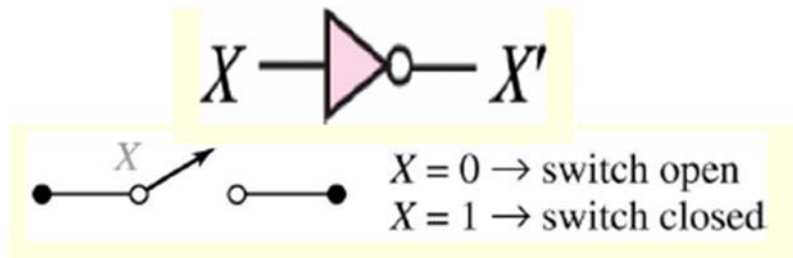
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- A *Boolean expression* over an algebra system  $(B; ., +, ' ; 0, 1)$  is defined as follows:
    1. Any element of  $B$  ( **constant** ) is a Boolean **expression**.
    2. Any variable name is a Boolean expression.
    3. If  $e_1$  and  $e_2$  are Boolean expression, then  $e_1'$ ,  $e_2'$ ,  $e_1+e_2$ ,  $e_1 \cdot e_2$  are Boolean expressions.
    4. Any expression that can be constructed by a finite number of applications of the above rules, and only such a expression is a Boolean expression.

# Boolean Operations

- The basic operations of Boolean algebra are AND, OR, and NOT (complement, or inverse).
- NOT (Complement)

$$0' = 1 \quad 1' = 0$$
$$X' = 1 \text{ if } X = 0 \text{ and } X' = 0 \text{ if } X = 1$$

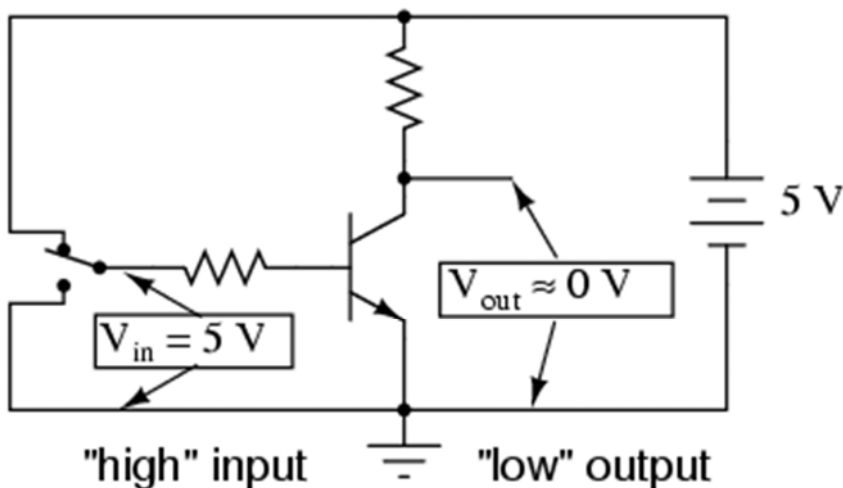
- Inverter



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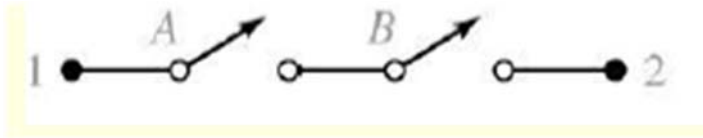
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0 V = "low" logic level (0)

5 V = "high" logic level (1)

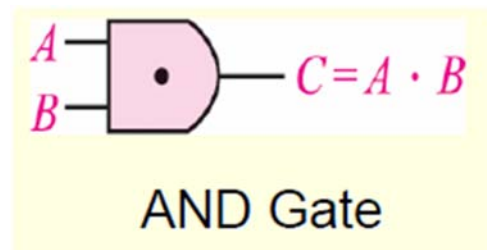
- AND Operation
- Omit the symbol “.”,  $A \cdot B = AB$



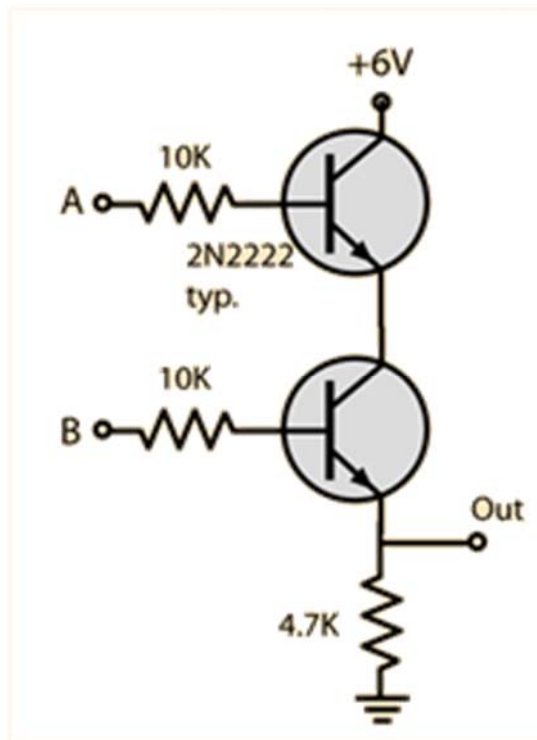
$T = 0 \rightarrow$  open circuit between terminals 1 and 2  
 $T = 1 \rightarrow$  closed circuit between terminals 1 and 2

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$A$	$B$	$C = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1



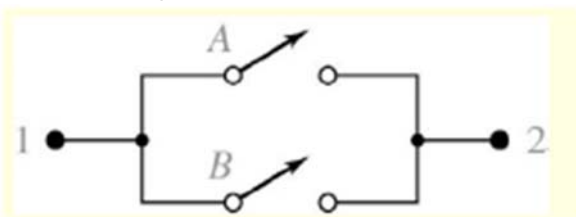




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• OR operation

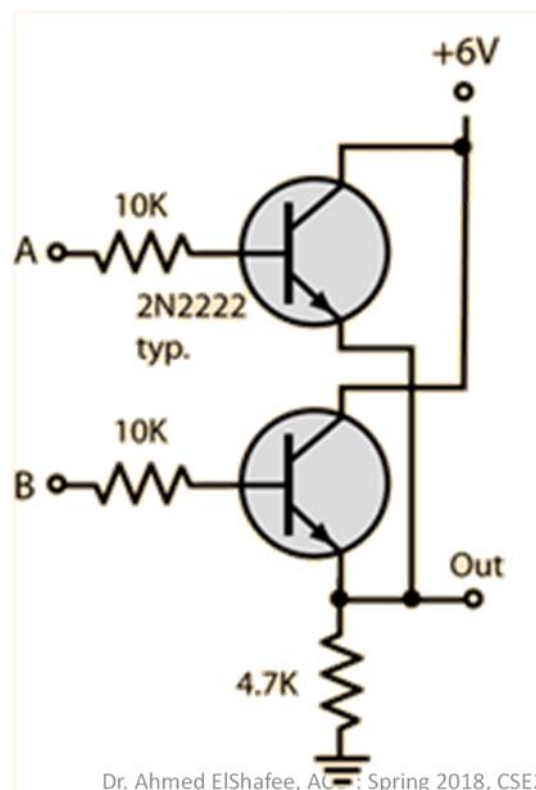
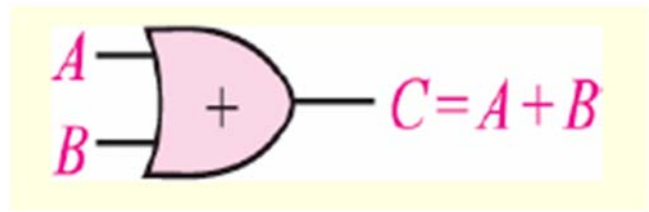


$$0+0=0 \quad 0+1=1 \quad 1+0=1 \quad 1+1=1$$

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$A$	$B$	$C=A+B$
0	0	0
0	1	1
1	0	1
1	1	1



# Boolean functions/expressions

- A function  $f(x_1, x_2, \dots, x_n), f : B^n \rightarrow B$

is called a Boolean function if it can be specified by a Boolean expression of  $n$  variables  $x_1, x_2, \dots, x_n$ .

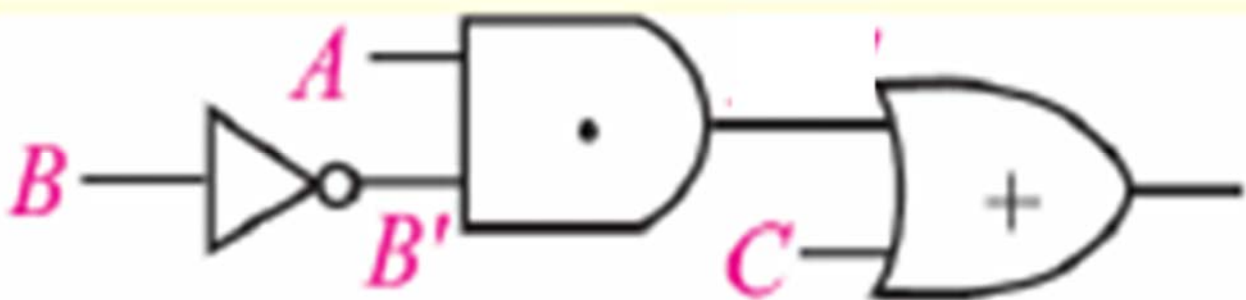
$$f(a,b,c)=ab'c+a'b+b'c'$$

- Each appearance of a variable or its complement in an expression is referred to as a literal.

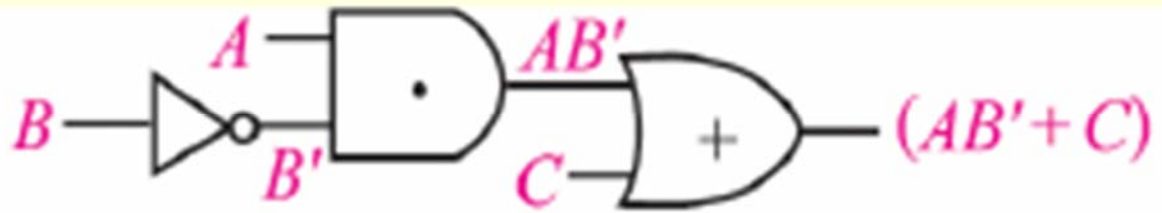
$$f(a,b,c)=ab'c+a'b+b'c'$$

- has 3 variables,  $a, b$ , and  $c$ , 7 literals( $a, b', c, a', b, b', c'$ ).

$$f(a,b,c)=C + (AB')$$

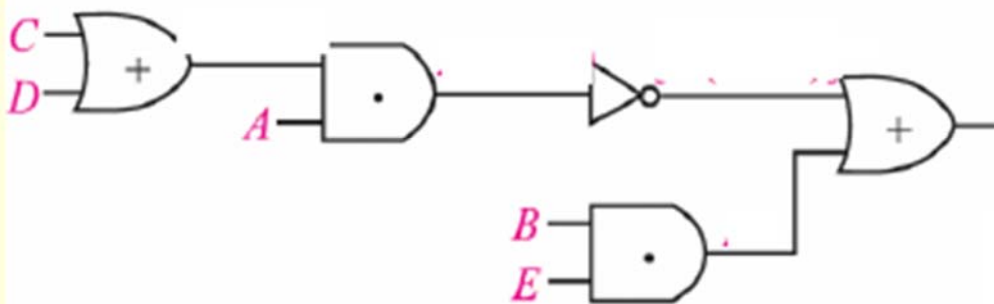


$$f(a,b,c) = C + (AB')$$



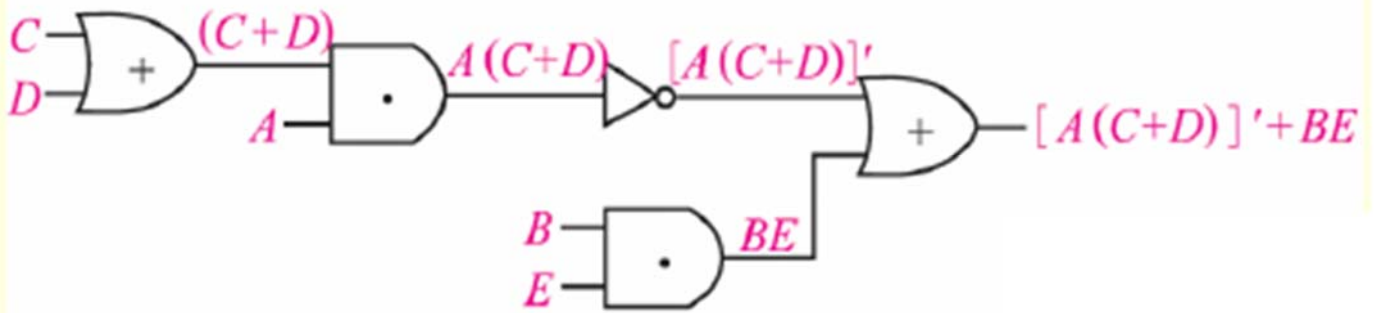
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Thanks,..