Lecture (01) Introduction Number Systems and Conversion

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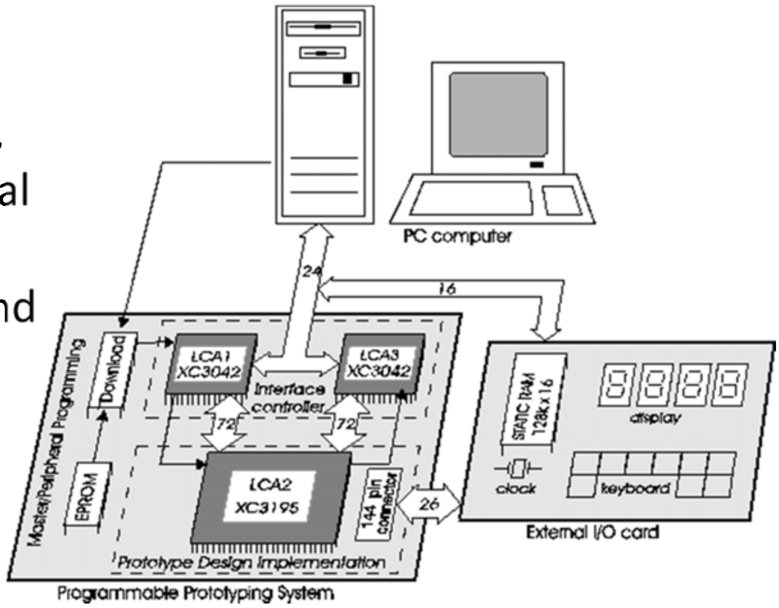
Digital systems

- Digital systems are used in communication, business transactions, traffic control, spacecraft guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises
- Most, if not all, of these devices have a special-purpose digital computer embedded within them.

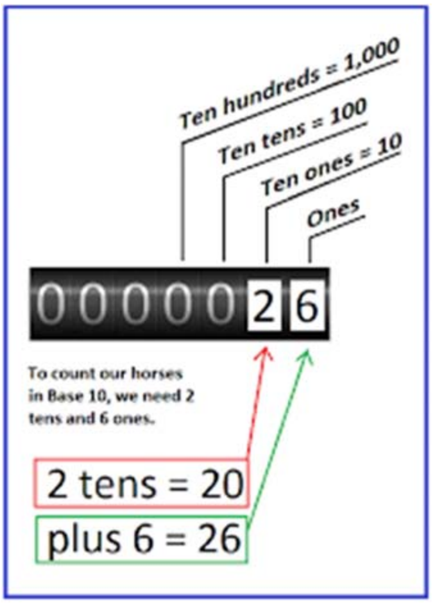


- These devices follow a sequence of instructions, called a program, that operates on given data.
- One characteristic of digital systems is their ability to represent and manipulate discrete elements of information.
- Any set that is restricted to a finite number of elements contains discrete information.

- Examples of discrete sets are the 10 decimal digits, the 26 letters of the alphabet, the 52 playing cards, and the 64 squares of a chessboard
- Discrete elements of information are represented in a digital system by physical quantities called signals.
- Electrical signals such as voltages and currents are the most common.

- electronic digital systems use just two discrete values and are therefore said to be *binary*.
- A binary digit, called a *bit,* has two values: 0 and 1.
- Discrete elements of information are represented with groups of bits called *binary codes*.
- For example, the decimal digits 0 through 9 are represented in a digital system with a code of four bits (e.g., the number 7 is represented by 0111).

- we could write $(0111)_2$ to indicate that the pattern 0111 is to be interpreted in a binary system, and $(0111)_{10}$ to indicate that the reference system is decimal.
- Then $0111_2 = 7_{10}$, which is not the same as 0111_{10}, or one hundred eleven.



Numbering systems

- A decimal number 7323 is a shorthand notation for what should be written as

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

- In general, a number with a decimal point is represented by a series of coefficients:

$$a_5a_4a_3a_2a_1a_0.\ a_{-1}a_{-2}a_{-3}$$

- The coeff nicients *aj* are any of the 10 digits (0, 1, 2, …, 9), subscript value *j* gives the place value and, hence, the power of 10 by which the coefficient must be multiplied.

$$10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

- $a_3 = 7$, $a_2 = 3$, $a_1 = 9$, and $a_0 = 2$.

- The decimal number system is said to be of *base,* or *radix,* 10 because it uses 10 digits and the coefficients are multiplied by powers of 10.

The *binary* system

- The coefficients have only two possible values: 0 and 1.
- coefficient *aj* is multiplied by a power of the radix, e.g., 2^j, and the results are added to obtain the decimal equivalent of the number
- The radix (float) point distinguishes positive powers of 2 from negative powers of 2.
- 11010.11_2 =

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

- There are many different number system, number expressed in a base-*r* system has coefficients multiplied by powers of *r*: stems,

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

- The coefficients *aj* range in value from 0 to *r* - 1

- the conversion from binary to decimal can be obtained by adding only the numbers with powers of two corresponding to the bits that are equal to 1. For example,

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

Other numbering systems

- An example of a base-5 number is

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

- The coefficient values for base 5 can be only 0, 1, 2, 3, and 4.

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

- An example of base-8 number (octal system)

hexadecimal (base-16) number system

- The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10
- The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15, respectively

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46{,}687)_{10}$$

- **Why the hexadecimal is important in computer systems?**
- The hexadecimal system is used commonly by designers to represent long strings of bits in the addresses, instructions, and data in digital systems.
- For example, B65F is used to represent 1011011001010000

Binary system prefixes:

- In computer work, 2^{10} is referred to as K (kilo), 2^{20} as M (mega), 2^{30} as G (giga), and 2^{40} as T (tera).
- 4K = 2^{12} = 4,096 and 16M = 2^{24} = 16,777,216.

- Computer capacity is usually given in bytes.
- A *byte* is equal to eight bits which presents a one keyboard character

- A computer hard disk with four gigabytes of storage has a capacity of 4G = 2^{32} bytes
- A terabyte is 1024 gigabytes, = 2^{40} bytes.

Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Number base conversion

- Convert decimal 41 to binary
- First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of 1/2.
- Then the quotient is again divided by 2 to give a new quotient and remainder.
- The process is continued until the integer quotient becomes 0.
- The *coefficients* of the desired binary number are obtained from the *remainders* as follows

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	Integer Quotient		Remainder	Coefficien
$41/2 =$	20	+	1	$a_0 = 1$
$20/2 =$	10	+	0	$a_1 = 0$
$10/2 =$	5	+	0	$a_2 = 0$
$5/2 =$	2	+	1	$a_3 = 1$
$2/2 =$	1	+	0	$a_4 = 0$
$1/2 =$	0	+	1	$a_5 = 1$

Therefore, the answer is $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$.

Example

- Convert decimal 153 to octal.
- The required base *r* is 8.
- First, 153 is divided by 8 to give an integer quotient of 19 and a remainder of 1.
- Then 19 is divided by 8 to give an integer quotient of 2 and a remainder of 3.
- Finally, 2 is divided by 8 to give a quotient of 0 and
- a remainder of 2.

	quotient		reminder	Coefficient
153/8	19	+	1	a0=1
19/8	2	+	3	a1=3
2/8	0	+	2	a2=2

$(153)_{10} = (231)_8$

Example

- Convert $(0.6875)_{10}$ to binary
- First, 0.6875 is multiplied by 2 to give an integer and a fraction.
- Then the new fraction is multiplied by 2 to give a new integer and a new fraction.
- The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

	Integer		Fraction	Coefficient
0.6875x2	1	+	0.3750	$a_{-1}=1$
0.3750x2	0	+	0.7500	$a_{-2}=0$
0.7500 x2	1	+	0.5000	$a_{-3}=1$
0.5000 x2	1	+	0.0000	$a_{-4}=1$

the answer is $(0.6875)_{10} = (0.\ a_{-1}\ a_{-2}\ a_{-3}\ a_{-4})_2 = (0.1011)_2$.

Example

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5.248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

- Convert (0.513)10 to octal.

	Integer		Fraction	Coefficient
0.513x8	4	+	0.104	$a_{-1}=4$
0.104x8	0	+	0.832	$a_{-2}=0$
0.832 x8	6	+	0.656	$a_{-3}=6$
0.656 x8	5	+	0.248	$a_{-4}=5$
0.248x8	1	+	0.984	$a_{-5}=1$
0.984	7		0.872	$a_{-6}=7$

- $(0.513)_{10} = (0.406517\ ....)_8$

- By combining previous examples:

$$(41.6875)_{10} = (101001.1011)_2$$

$$(153.513)_{10} = (231.406517)_8$$

Octal and Hexadecimal numbers

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- The conversion from and to binary, octal, and hexadecimal plays an important role in digital computers, because shorter patterns of hex characters are easier to recognize than long patterns
- $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

The conversion from binary to octal

- partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.
- The corresponding octal digit is then assigned to each group.

(10	110	001	101	011	·	111	100	000	$110)_2$	$= (26153.7406)_8$
2	6	1	5	3		7	4	0	6	

Conversion from binary to hexadecimal

- similar, except that the binary number is divided into groups of *four* digits:

(10	1100	0110	1011	·	1111	$0010)_2$	$= (2C6B.F2)_{16}$
2	C	6	B		F	2	

Conversion from octal to binary

- is done by reversing the preceding procedure.
- Each octal digit is converted to its three-digit binary equivalent.

$$(673.124)_8 = (\underset{6}{110} \quad \underset{7}{111} \quad \underset{3}{011} \quad \cdot \quad \underset{1}{001} \quad \underset{2}{010} \quad \underset{4}{100})_2$$

Conversion from hexadecimal to binary

- is done by reversing the preceding procedure.
- each hexadecimal digit is converted to its four-digit binary equivalent

$$(306.D)_{16} = (\underset{3}{0011} \quad \underset{0}{0000} \quad \underset{6}{0110} \quad \cdot \quad \underset{D}{1101})_2$$

- that's why we need to use hex and octal,

Easy conversion between binary ←→ octal , and binary ←→hexadecimal,

- Binary numbers are difficult to work with because they require three or four times as many digits as their decimal equivalents.
- For example, the binary number 111111111111 is equivalent to decimal 4095.

Arithmetic operations

```
  101101
 +100111
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 1010100
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```
  1010001
 +1000101
 --------------
 10010110
```

```
 101101
-100111
-------
 000110
```

```
  1010001
- 1000101
---------------
  0001100
```



```
   1011
  × 101
 ------
   1011
  0000
 1011
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 110111
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Thanks,..
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Dr. Ahmed ElShafee, ACU : Spring 2018, CSE202 Logic Design I