

Logic Design – Tutorial

07

#	Student ID	Student Name	Grade (10)
-			



Faculty of Engineering

Q1	For each of the following functions, find all of the prime implicants, using the Quine-McCluskey method. (a) $f(a, b, c, d) = \sum m(1, 5, 7, 9, 11, 12, 14, 15)$																																								
Sol 1	<table><tr><td><u>1</u></td><td>0001✓</td><td>1, 5</td><td>0-01</td><td>a'c'd</td></tr><tr><td>5</td><td>0101✓</td><td><u>1, 9</u></td><td>-001</td><td>b'c'd</td></tr><tr><td>9</td><td>1001✓</td><td>5, 7</td><td>01-1</td><td>a'bd</td></tr><tr><td><u>12</u></td><td><u>1100</u>✓</td><td>9, 11</td><td>10-1</td><td>ab'd</td></tr><tr><td>7</td><td>0111✓</td><td><u>12, 14</u></td><td>11-0</td><td>abd'</td></tr><tr><td>11</td><td>1011✓</td><td>7, 15</td><td>-111</td><td>bcd</td></tr><tr><td><u>14</u></td><td><u>1110</u>✓</td><td>11, 15</td><td>1-11</td><td>acd</td></tr><tr><td>15</td><td>1111✓</td><td>14, 15</td><td>111-</td><td>abc</td></tr></table> <p>Prime implicants: $a'c'd, b'c'd, a'bd, ab'd, abd', bcd, acd, abc$</p>	<u>1</u>	0001✓	1, 5	0-01	a'c'd	5	0101✓	<u>1, 9</u>	-001	b'c'd	9	1001✓	5, 7	01-1	a'bd	<u>12</u>	<u>1100</u> ✓	9, 11	10-1	ab'd	7	0111✓	<u>12, 14</u>	11-0	abd'	11	1011✓	7, 15	-111	bcd	<u>14</u>	<u>1110</u> ✓	11, 15	1-11	acd	15	1111✓	14, 15	111-	abc
<u>1</u>	0001✓	1, 5	0-01	a'c'd																																					
5	0101✓	<u>1, 9</u>	-001	b'c'd																																					
9	1001✓	5, 7	01-1	a'bd																																					
<u>12</u>	<u>1100</u> ✓	9, 11	10-1	ab'd																																					
7	0111✓	<u>12, 14</u>	11-0	abd'																																					
11	1011✓	7, 15	-111	bcd																																					
<u>14</u>	<u>1110</u> ✓	11, 15	1-11	acd																																					
15	1111✓	14, 15	111-	abc																																					



Faculty of Engineering

Q2	<p>For this function, find a minimum sum-of-products solution, using the Quine-McCluskey method.</p> $f(a, b, c, d) = \sum m(1, 3, 4, 5, 6, 7, 10, 12, 13) + \sum d(2, 9, 15)$																																																								
Sol 2	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">1 0001✓</td> <td style="border-right: 1px solid black; padding: 5px;">1, 3 00-1✓</td> <td style="padding: 5px;">1, 3, 5, 7 0--1 a'd</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">2 0010✓</td> <td style="border-right: 1px solid black; padding: 5px;">1, 5 0-01✓</td> <td style="padding: 5px;">1, 5, 3, 7 0--1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">4 0100✓</td> <td style="border-right: 1px solid black; padding: 5px;">1, 9 -001✓</td> <td style="padding: 5px;">1, 5, 9, 13 --01 c'd</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">3 0011✓</td> <td style="border-right: 1px solid black; padding: 5px;">2, 3 001-✓</td> <td style="padding: 5px;">1, 9, 5, 13 --01</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">5 0101✓</td> <td style="border-right: 1px solid black; padding: 5px;">2, 6 0-10✓</td> <td style="padding: 5px;">2, 3, 6, 7 0-1- a'c</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">6 0110✓</td> <td style="border-right: 1px solid black; padding: 5px;">2, 10 -010 b'cd'</td> <td style="padding: 5px;">2, 6, 3, 7 0-1-</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">9 1001✓</td> <td style="border-right: 1px solid black; padding: 5px;">4, 5 010-✓</td> <td style="padding: 5px;">4, 5, 6, 7 01-- a'b</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">10 1010✓</td> <td style="border-right: 1px solid black; padding: 5px;">4, 6 01-0✓</td> <td style="padding: 5px;">4, 5, 12, 13 -10- bc'</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">12 1100✓</td> <td style="border-right: 1px solid black; padding: 5px;">4, 12 -100✓</td> <td style="padding: 5px;">4, 6, 5, 7 01--</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">7 0111✓</td> <td style="border-right: 1px solid black; padding: 5px;">3, 7 0-11✓</td> <td style="padding: 5px;">4, 12, 5, 13 -10-</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">13 1101✓</td> <td style="border-right: 1px solid black; padding: 5px;">5, 7 01-1✓</td> <td style="padding: 5px;">5, 7, 13, 15 -1-1 bd</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">15 1111✓</td> <td style="border-right: 1px solid black; padding: 5px;">5, 13 -101✓</td> <td style="padding: 5px;">5, 13, 7, 15 -1-1</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">6, 7 011-✓</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">9, 13 1-01✓</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">12, 13 110-✓</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">7, 15 -111✓</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">13, 15 11-1✓</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td style="border-right: 1px solid black; padding: 5px;">13, 15 11-1✓</td> <td></td> </tr> </table> <p>Prime implicants: $b'cd', a'd, c'd, a'c, a'b, bc', bd$</p>			1 0001✓	1, 3 00-1✓	1, 3, 5, 7 0--1 a'd	2 0010✓	1, 5 0-01✓	1, 5, 3, 7 0--1	4 0100✓	1, 9 -001✓	1, 5, 9, 13 --01 c'd	3 0011✓	2, 3 001-✓	1, 9, 5, 13 --01	5 0101✓	2, 6 0-10✓	2, 3, 6, 7 0-1- a'c	6 0110✓	2, 10 -010 b'cd'	2, 6, 3, 7 0-1-	9 1001✓	4, 5 010-✓	4, 5, 6, 7 01-- a'b	10 1010✓	4, 6 01-0✓	4, 5, 12, 13 -10- bc'	12 1100✓	4, 12 -100✓	4, 6, 5, 7 01--	7 0111✓	3, 7 0-11✓	4, 12, 5, 13 -10-	13 1101✓	5, 7 01-1✓	5, 7, 13, 15 -1-1 bd	15 1111✓	5, 13 -101✓	5, 13, 7, 15 -1-1		6, 7 011-✓			9, 13 1-01✓			12, 13 110-✓			7, 15 -111✓			13, 15 11-1✓			13, 15 11-1✓	
1 0001✓	1, 3 00-1✓	1, 3, 5, 7 0--1 a'd																																																							
2 0010✓	1, 5 0-01✓	1, 5, 3, 7 0--1																																																							
4 0100✓	1, 9 -001✓	1, 5, 9, 13 --01 c'd																																																							
3 0011✓	2, 3 001-✓	1, 9, 5, 13 --01																																																							
5 0101✓	2, 6 0-10✓	2, 3, 6, 7 0-1- a'c																																																							
6 0110✓	2, 10 -010 b'cd'	2, 6, 3, 7 0-1-																																																							
9 1001✓	4, 5 010-✓	4, 5, 6, 7 01-- a'b																																																							
10 1010✓	4, 6 01-0✓	4, 5, 12, 13 -10- bc'																																																							
12 1100✓	4, 12 -100✓	4, 6, 5, 7 01--																																																							
7 0111✓	3, 7 0-11✓	4, 12, 5, 13 -10-																																																							
13 1101✓	5, 7 01-1✓	5, 7, 13, 15 -1-1 bd																																																							
15 1111✓	5, 13 -101✓	5, 13, 7, 15 -1-1																																																							
	6, 7 011-✓																																																								
	9, 13 1-01✓																																																								
	12, 13 110-✓																																																								
	7, 15 -111✓																																																								
	13, 15 11-1✓																																																								
	13, 15 11-1✓																																																								



		1	3	4	5	6	7	10	12	13
1, 3, 5, 7	a'd	x	x		x		x			
1, 5, 9, 13	c'd	x			x					x
2, 3, 6, 7	a'c		x			x	x			
4, 5, 6, 7	a'b			x	x	x	x			
4, 5, 12, 13	bc'			x	x			x	x	
5, 7, 13, 15	bd				x		x			x
2, 10	b'cd'							x		

$$f = \underline{bc'} + \underline{b'cd'} + a'd + a'b$$

$$f = \underline{bc'} + \underline{b'cd'} + c'd + a'c$$

$$f = \underline{bc'} + \underline{b'cd'} + a'c + a'd$$

Q3	<p>For each of the following functions, find all of the prime implicants using the Quine-McCluskey method.</p> <p>Using a prime implicant chart, find <i>all</i> minimum sum-of-products solutions for each of the functions given in</p> <p>(a) $f(a, b, c, d) = \sum m(0, 3, 4, 5, 7, 9, 11, 13)$</p>																																								
Sol 3	<table border="1"> <tbody> <tr> <td>0</td> <td>0000✓</td> <td>0, 4</td> <td>0-00</td> <td>a'c'd'</td> </tr> <tr> <td>4</td> <td>0100✓</td> <td>4, 5</td> <td>010-</td> <td>a'bc'</td> </tr> <tr> <td>3</td> <td>0011✓</td> <td>3, 7</td> <td>0-11</td> <td>a'cd</td> </tr> <tr> <td>5</td> <td>0101✓</td> <td>3, 11</td> <td>-011</td> <td>b'cd</td> </tr> <tr> <td>9</td> <td>1001✓</td> <td>5, 7</td> <td>01-1</td> <td>a'bd</td> </tr> <tr> <td>7</td> <td>0111✓</td> <td>5, 13</td> <td>-101</td> <td>bc'd</td> </tr> <tr> <td>11</td> <td>1011✓</td> <td>9, 11</td> <td>10-1</td> <td>ab'd</td> </tr> <tr> <td>13</td> <td>1101✓</td> <td>9, 13</td> <td>1-01</td> <td>ac'd</td> </tr> </tbody> </table> <p>Prime implicants: $a'c'd', a'bc', a'cd, b'cd, a'bd, bc'd, ab'd, ac'd$</p>	0	0000✓	0, 4	0-00	a'c'd'	4	0100✓	4, 5	010-	a'bc'	3	0011✓	3, 7	0-11	a'cd	5	0101✓	3, 11	-011	b'cd	9	1001✓	5, 7	01-1	a'bd	7	0111✓	5, 13	-101	bc'd	11	1011✓	9, 11	10-1	ab'd	13	1101✓	9, 13	1-01	ac'd
0	0000✓	0, 4	0-00	a'c'd'																																					
4	0100✓	4, 5	010-	a'bc'																																					
3	0011✓	3, 7	0-11	a'cd																																					
5	0101✓	3, 11	-011	b'cd																																					
9	1001✓	5, 7	01-1	a'bd																																					
7	0111✓	5, 13	-101	bc'd																																					
11	1011✓	9, 11	10-1	ab'd																																					
13	1101✓	9, 13	1-01	ac'd																																					



		0	3	4	5	7	9	11	13
0, 4	a'c'd'	⊗		×					
4, 5	a'bc'			×	×				
3, 7	a'cd		×		×	×			
3, 11	b'cd		×					×	
5, 7	a'bd				×	×			
5, 13	bc'd				×				×
9, 11	ab'd						×	×	
9, 13	ac'd							×	×

$f = a'c'd' + a'cd + ab'd + bc'd$
 $f = a'c'd' + ac'd + a'bd + b'cd$

Q4 (a) Using the Quine-McCluskey, method find all prime implicants of $f(A, B, C, D) = \sum m(1, 3, 5, 6, 8, 9, 12, 14, 15) + \sum d(4, 10, 13)$. Identify all essential prime implicants and find all minimum sum-of-products expressions.
 (b) Repeat Part (a) for f' .

Sol 4

1	0001√	1, 3	00-1*	1, 5, 9, 13	--01*
4	0100√	1, 5	0-01√	4, 5, 12, 13	-10-*
8	1000√	1, 9	-001√	4, 6, 12, 14	-1-0*
3	0011√	4, 5	010-√	8, 9, 12, 13	1-0-*
5	0101√	4, 6	01-0√	8, 10, 12, 14	1--0*
6	0110√	4, 12	-100√	12, 13, 14, 15	11--*
9	1001√	8, 9	100-√		
10	1010√	8, 10	10-0√		
12	1100√	8, 12	1-00√		
13	1101√	5, 13	-101√		
14	1110√	6, 14	-110√		
15	1111√	9, 13	1-01√		
		10, 14	1-10√		
		12, 13	110-√		
		12, 14	11-0√		
		13, 15	11-1√		
		14, 15	111-√		

Prime implicants: $A'B'D, AB, A'C', C'D, AD', B'D', B C'$

(1, 3)	A'B'D	⊗	×						
(1, 5, 9, 13)	C'D	×					×		
(4, 5, 12, 13)	BC'		×					×	
(4, 6, 12, 14)	BD'					⊗		×	×
(8, 9, 12, 13)	AC'						×	×	×
(8, 10, 12, 14)	AD'						×	×	×
(12, 13, 14, 15)	AB							×	⊗

Essential Prime Implicants: $AB, BD', A'B'D$

$f = AB + BD' + A'B'D + C'D + AD'$
 $f = AB + BD' + A'B'D + AC' + C'D$
 $f = AB + BD' + A'B'D + AC' + B C'$

0	0000√	0, 2	00-0*		
2	0010√	0, 4	0-00*		
4	0100√	2, 10	-010*		
10	1010√	10, 11	101-*		
7	0111*				
11	1011√				
13	1101*				

Prime Implicants of f' : $A'BCD, A'B'D', ABC'D, AB'C, B'CD', A'C'D'$

(7)	A'BCD				⊗
(13)	ABC'D				
(0, 2)	A'B'D'	×	×		
(0, 4)	A'C'D'	×			
(2, 10)	B'CD'		×		
(10, 11)	AB'C				⊗

Essential Prime Implicants: $AB'C, A'BCD$

$f' = AB'C + A'BCD + A'B'D'$



Q5	<p>For each of the following functions, find all of the prime implicants, using the Quine-McCluskey method.</p> <p>(b) $f(a, b, c, d) = \sum m(0, 1, 3, 5, 6, 7, 8, 10, 14, 15)$</p>																																																																																				
Sol 5	<p>.....</p> <table border="0" style="width: 100%; border-collapse: collapse;"> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>0</u></td> <td style="border-bottom: 1px solid black;"><u>0000</u>✓</td> <td style="border-bottom: 1px solid black;"><u>0, 1</u></td> <td style="border-bottom: 1px solid black;"><u>000- a'b'c'</u></td> <td style="border-bottom: 1px solid black;"><u>1, 3, 5, 7</u></td> <td style="border-bottom: 1px solid black;"><u>0--1 a'd</u></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>1</u></td> <td style="border-bottom: 1px solid black;"><u>0001</u>✓</td> <td style="border-bottom: 1px solid black;"><u>0, 8</u></td> <td style="border-bottom: 1px solid black;"><u>-000 b'c'd'</u></td> <td style="border-bottom: 1px solid black;"><u>1, 5, 3, 7</u></td> <td style="border-bottom: 1px solid black;"><u>0--1</u></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>8</u></td> <td style="border-bottom: 1px solid black;"><u>1000</u>✓</td> <td style="border-bottom: 1px solid black;"><u>1, 3</u></td> <td style="border-bottom: 1px solid black;"><u>00-1</u>✓</td> <td style="border-bottom: 1px solid black;"><u>6, 7, 14, 15</u></td> <td style="border-bottom: 1px solid black;"><u>-11- bc</u></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>3</u></td> <td style="border-bottom: 1px solid black;"><u>0011</u>✓</td> <td style="border-bottom: 1px solid black;"><u>1, 5</u></td> <td style="border-bottom: 1px solid black;"><u>0-01</u>✓</td> <td style="border-bottom: 1px solid black;"><u>6, 14, 7, 15</u></td> <td style="border-bottom: 1px solid black;"><u>=11=</u></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>5</u></td> <td style="border-bottom: 1px solid black;"><u>0101</u>✓</td> <td style="border-bottom: 1px solid black;"><u>8, 10</u></td> <td style="border-bottom: 1px solid black;"><u>10-0 ab'd'</u></td> <td></td> <td></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>6</u></td> <td style="border-bottom: 1px solid black;"><u>0110</u>✓</td> <td style="border-bottom: 1px solid black;"><u>3, 7</u></td> <td style="border-bottom: 1px solid black;"><u>0-11</u>✓</td> <td></td> <td></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>10</u></td> <td style="border-bottom: 1px solid black;"><u>1010</u>✓</td> <td style="border-bottom: 1px solid black;"><u>5, 7</u></td> <td style="border-bottom: 1px solid black;"><u>01-1</u>✓</td> <td></td> <td></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>7</u></td> <td style="border-bottom: 1px solid black;"><u>0111</u>✓</td> <td style="border-bottom: 1px solid black;"><u>6, 7</u></td> <td style="border-bottom: 1px solid black;"><u>011-</u>✓</td> <td></td> <td></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>14</u></td> <td style="border-bottom: 1px solid black;"><u>1110</u>✓</td> <td style="border-bottom: 1px solid black;"><u>6, 14</u></td> <td style="border-bottom: 1px solid black;"><u>-110</u>✓</td> <td></td> <td></td> </tr> <tr> <td>...</td> <td style="border-bottom: 1px solid black;"><u>15</u></td> <td style="border-bottom: 1px solid black;"><u>1111</u>✓</td> <td style="border-bottom: 1px solid black;"><u>10, 14</u></td> <td style="border-bottom: 1px solid black;"><u>1-10 acd'</u></td> <td></td> <td></td> </tr> <tr> <td>...</td> <td></td> <td></td> <td style="border-bottom: 1px solid black;"><u>7, 15</u></td> <td style="border-bottom: 1px solid black;"><u>-111</u>✓</td> <td></td> <td></td> </tr> <tr> <td>...</td> <td></td> <td></td> <td style="border-bottom: 1px solid black;"><u>14, 15</u></td> <td style="border-bottom: 1px solid black;"><u>111-</u>✓</td> <td></td> <td></td> </tr> </table> <p>... Prime implicants: $a'b'c', b'c'd', ab'd', acd', a'd, bc$</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p>	...	<u>0</u>	<u>0000</u> ✓	<u>0, 1</u>	<u>000- a'b'c'</u>	<u>1, 3, 5, 7</u>	<u>0--1 a'd</u>	...	<u>1</u>	<u>0001</u> ✓	<u>0, 8</u>	<u>-000 b'c'd'</u>	<u>1, 5, 3, 7</u>	<u>0--1</u>	...	<u>8</u>	<u>1000</u> ✓	<u>1, 3</u>	<u>00-1</u> ✓	<u>6, 7, 14, 15</u>	<u>-11- bc</u>	...	<u>3</u>	<u>0011</u> ✓	<u>1, 5</u>	<u>0-01</u> ✓	<u>6, 14, 7, 15</u>	<u>=11=</u>	...	<u>5</u>	<u>0101</u> ✓	<u>8, 10</u>	<u>10-0 ab'd'</u>			...	<u>6</u>	<u>0110</u> ✓	<u>3, 7</u>	<u>0-11</u> ✓			...	<u>10</u>	<u>1010</u> ✓	<u>5, 7</u>	<u>01-1</u> ✓			...	<u>7</u>	<u>0111</u> ✓	<u>6, 7</u>	<u>011-</u> ✓			...	<u>14</u>	<u>1110</u> ✓	<u>6, 14</u>	<u>-110</u> ✓			...	<u>15</u>	<u>1111</u> ✓	<u>10, 14</u>	<u>1-10 acd'</u>			...			<u>7, 15</u>	<u>-111</u> ✓			...			<u>14, 15</u>	<u>111-</u> ✓		
...	<u>0</u>	<u>0000</u> ✓	<u>0, 1</u>	<u>000- a'b'c'</u>	<u>1, 3, 5, 7</u>	<u>0--1 a'd</u>																																																																															
...	<u>1</u>	<u>0001</u> ✓	<u>0, 8</u>	<u>-000 b'c'd'</u>	<u>1, 5, 3, 7</u>	<u>0--1</u>																																																																															
...	<u>8</u>	<u>1000</u> ✓	<u>1, 3</u>	<u>00-1</u> ✓	<u>6, 7, 14, 15</u>	<u>-11- bc</u>																																																																															
...	<u>3</u>	<u>0011</u> ✓	<u>1, 5</u>	<u>0-01</u> ✓	<u>6, 14, 7, 15</u>	<u>=11=</u>																																																																															
...	<u>5</u>	<u>0101</u> ✓	<u>8, 10</u>	<u>10-0 ab'd'</u>																																																																																	
...	<u>6</u>	<u>0110</u> ✓	<u>3, 7</u>	<u>0-11</u> ✓																																																																																	
...	<u>10</u>	<u>1010</u> ✓	<u>5, 7</u>	<u>01-1</u> ✓																																																																																	
...	<u>7</u>	<u>0111</u> ✓	<u>6, 7</u>	<u>011-</u> ✓																																																																																	
...	<u>14</u>	<u>1110</u> ✓	<u>6, 14</u>	<u>-110</u> ✓																																																																																	
...	<u>15</u>	<u>1111</u> ✓	<u>10, 14</u>	<u>1-10 acd'</u>																																																																																	
...			<u>7, 15</u>	<u>-111</u> ✓																																																																																	
...			<u>14, 15</u>	<u>111-</u> ✓																																																																																	

Q6 Find all prime implicants of the following function and then find all minimum solutions using Petrick's method:

$$F(A, B, C, D) = \Sigma m(9, 12, 13, 15) + \Sigma d(1, 4, 5, 7, 8, 11, 14)$$

Sol6

....	1	0001✓	1, 5	0-01✓	1, 5, 9, 13	--01	$C'D$
....	4	0100✓	1, 9	-001✓	1, 9, 5, 13	--01	
....	8	1000✓	4, 5	010-✓	4, 5, 12, 13	-10-	BC'
....	5	0101✓	4, 12	-100✓	4, 12, 5, 13	-10-	
....	9	1001✓	8, 9	100-✓	5, 7, 13, 15	-1-1	BD
....	12	1100✓	8, 12	1-00✓	5, 13, 7, 15	-1-1	
....	7	0111✓	5, 7	01-1✓	8, 9, 12, 13	1-0-	AC'
....	11	1011✓	5, 13	-101✓	8, 12, 9, 13	1-0-	
....	13	1101✓	9, 11	10-1✓	9, 11, 13, 15	1--1	AD
....	14	1110✓	9, 13	1-01✓	9, 13, 11, 15	1--1	
....	15	1111✓	12, 13	110-✓	12, 13, 14, 15	11--	AB
....			12, 14	11-0✓	12, 14, 13, 15	11--	
....			7, 15	-111✓			
....			11, 15	1-11✓			
....			13, 15	11-1✓			
....			14, 15	111-✓			

Prime implicants: $C'D, BC', BD, AC', AD, AB$

		9	12	13	15
P1 (1, 5, 9, 13)	$C'D$	×		×	
P2 (4, 5, 12, 13)	BC'		×	×	
P3 (5, 7, 13, 15)	BD			×	×
P4 (8, 9, 12, 13)	AC'	×	×	×	
P5 (9, 11, 13, 15)	AD	×		×	×
P6 (12, 13, 14, 15)	AB		×	×	×

$$\begin{aligned}
 &(P1 + P4 + P5) (P2 + P4 + P6) (\cancel{P1} + \cancel{P2} + P3 + P4 + P5 + P6) (P3 + P5 + P6) \\
 &= (P4 + P1P2 + P1P6 + P2P5 + P5P6) (P3 + P5 + P6) \\
 &= P3P4 + P4P5 + P4P6 + P1P2P3 + P1P2P5 + P1P2P6 + P1P3P6 \\
 &+ P1P5P6 + P1P6 + P2P3P5 + P2P5 + P2P5P6 + P3P5P6 + P5P6 = 1
 \end{aligned}$$

$$F = \underset{P4}{(AC')} + \underset{P3}{BD} \text{ or } \underset{P5}{AD} + \underset{P2}{BC'} \text{ or } \underset{P5}{AD} + \underset{P4}{AC'} \text{ or } \underset{P6}{AB} + \underset{P5}{AD} \text{ or } \underset{P6}{AB} + \underset{P4}{AC'} \text{ or } \underset{P6}{AB} + \underset{P1}{C'D}$$



Q7

For each of the following functions, find all of the prime implicants using the Quine-McCluskey method.

Using a prime implicant chart. find *all* minimum sum-of-products solutions for each of the functions given in

(b) $f(a, b, c, d) = \sum m(2, 4, 5, 6, 9, 10, 11, 12, 13, 15)$

Sol 7

2	0010✓	2, 6	0-10 a'cd'	4, 5, 12, 13	-10- bc'
4	0100✓	2, 10	-010 b'cd'	4, 12, 5, 13	-10-
5	0101✓	4, 5	010-✓	9, 11, 13, 15	1--1 ad
6	0110✓	4, 6	01-0 a'bd'	9, 13, 11, 15	1--1
9	1001✓	4, 12	-100✓		
10	1010✓	5, 13	-101✓		
12	1100✓	9, 11	10-1✓		
11	1011✓	9, 13	1-01✓		
13	1101✓	10, 11	101- ab'c		
15	1111✓	12, 13	110-✓		
		11, 15	1-11✓		
		13, 15	11-1✓		

Prime implicants: $ad, bc', a'cd', b'cd', a'bd', ab'c$

		2	4	5	6	9	10	11	12	13	15
2, 6	a'cd'	x			x						
2, 10	b'cd'	x					x				
4, 6	a'bd'		x		x						
10, 11	ab'c							x		x	
4, 5, 12, 13	bc'		x	x					x	x	
9, 11, 13, 15	ad					x		x			x

$f = bc' + ad + a'cd' + b'cd'$

$f = bc' + ad + a'cd' + ab'c$

$f = bc' + ad + a'bd' + b'cd'$



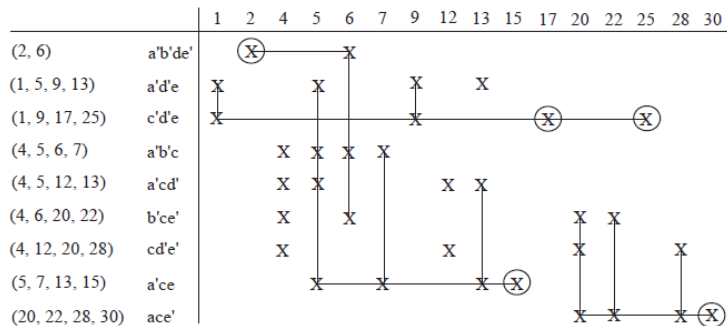
Q8

(a) Use the Quine-McCluskey method to find all prime implicants of $f(a, b, c, d, e) = \Sigma m(1, 2, 4, 5, 6, 7, 9, 12, 13, 15, 17, 20, 22, 25, 28, 30)$. Find all essential prime implicants, and find all minimum sum-of-products expressions.
(b) Repeat Part (a) for f' .

Sol 8

1	00001√	1, 5	00-01√	1, 5, 9, 13	0--01*
2	00010√	1, 9	0-001√	1, 9, 17, 25	--001*
4	00100√	1, 17	-0001√	4, 5, 6, 7	001--*
5	00101√	2, 6	00-10*	4, 5, 12, 13	0-10-*
6	00110√	4, 5	0010-√	4, 6, 20, 22	-01-0*
9	01001√	4, 6	001-0√	4, 12, 20, 28	--100*
12	01100√	4, 12	0-100√	5, 7, 13, 15	0-1-1*
17	10001√	4, 20	-0100√	20, 22, 28, 30	1-1-0*
20	10100√	5, 7	001-1√		
7	00111√	5, 13	0-101√		
13	01101√	6, 7	0011-√		
22	10110√	6, 22	-0110√		
25	11001√	9, 13	01-01√		
28	11100√	9, 25	-1001√		
15	01111√	12, 13	0110-√		
30	11110√	12, 28	-1100√		
		17, 25	1-001√		
		20, 22	101-0√		
		20, 28	1-100√		
		7, 15	0-111√		
		13, 15	011-1√		
		22, 30	1-110√		
		28, 30	111-0√		

Prime Implicants: $a c e'$, $a' c e$, $c d' e'$, $a' c d'$, $a' b' c$, $b' c e'$, $a' b' d' e'$, $c' d' e$, $a' d' e$



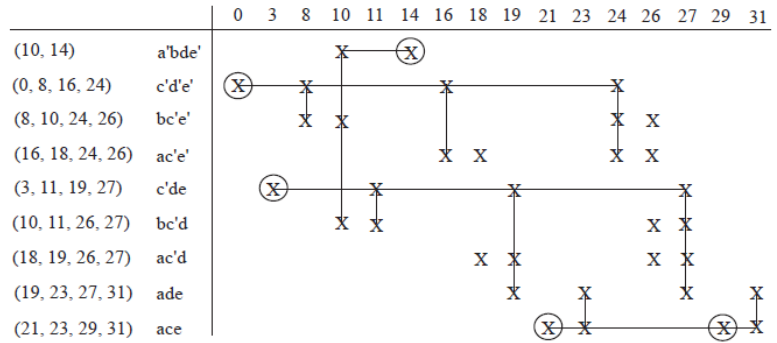
Essential Prime Implicants: $a c e'$, $c' d' e$, $a' c e$, $a' b' d' e'$

$$f = a c e' + c' d' e + a' c e + a' b' d' e' + a' c d'$$



0	00000√	0, 8	0-000√	0, 8, 16, 24	--000*
8	01000√	0,16	-0000√	8, 10, 24, 26	-10-0*
16	10000√	8, 10	010-0√	16, 18, 24, 26	1-0-0*
3	00011√	8, 24	-1000√	3, 11, 19, 27	--011*
10	01010√	16, 18	100-0√	10, 11, 26, 27	-101-*
18	10010√	16, 24	1-000√	18, 19, 26, 27	1-01-*
24	11000√	3, 11	0-011√	19, 23, 27, 31	1--11*
11	01011√	3, 19	-0011√	21, 23, 29, 31	1-1-1*
14	01110√	10, 11	0101-√		
19	10011√	10, 14	01-10*		
21	10101√	10, 26	-1010√		
26	11010√	18, 19	1001-√		
23	10111√	18, 26	1-010√		
27	11011	24, 26	110-0√		
29	11101√	11, 27	-1011√		
31	11111√	19, 23	10-11√		
		19, 27	1-011√		
		21, 23	101-1√		
		21, 29	1-101√		
		26, 27	1101-√		
		23, 31	1-111√		
		27, 31	11-11√		
		29, 31	111-1√		

Prime Implicants of f' : $ace, ade, ac'd, ac'e', bc'd, a'bde', bc'e', c'de, c'd'e'$



Essential Prime Implicants: $ace, a'bde', c'de, c'd'e'$
 $f' = ace + a'bde' + c'de + c'd'e' + ac'e'$
 $f' = ace + a'bde' + c'de + c'd'e' + ac'd$