Logic Design – Tutorial 06

#	Student ID	Student Name	Grade (10)
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Q1

Find the minimum sum-of-products expression for each function. Underline the essential prime implicants in your answer and tell which minterm makes each one essential.

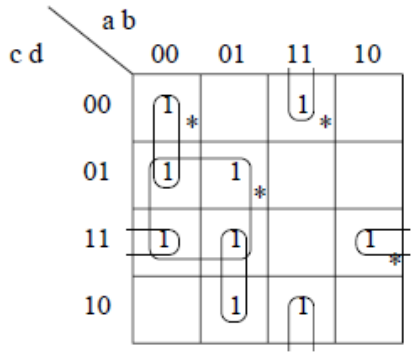
(a) $f(a, b, c, d) = \Sigma\, m(0, 1, 3, 5, 6, 7, 11, 12, 14)$

(b) $f(a, b, c, d) = \Pi\, M(1, 9, 11, 12, 14)$

(c) $f(a, b, c, d) = \Pi\, M(5, 7, 13, 14, 15) \bullet \Pi\, D(1, 2, 3, 9)$

Sol 1

(a)



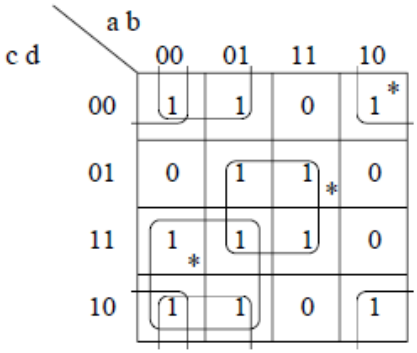
f = a'b'c' + a'd + b'cd + abd' + bcd'

Alt: f = a'b'c' + a'd + b'cd + abd' + a'bc

(*) Indicates a minterm that makes the corresponding prime implicant essential.

a'd $\rightarrow m_5$; a'b'c' $\rightarrow m_0$; b'cd $\rightarrow m_{11}$; abd' $\rightarrow m_{12}$

(b)



F = a'c + b'd' + bd + a'd'

Alt: F = a'c + b'd' + bd + a'b

(*) Indicates a minterm that makes the corresponding prime implicant essential.

bd $\rightarrow m_{13}$ or m_{15}; a'c $\rightarrow m_3$; b'd' $\rightarrow m_8$ or m_{10}

Q2

Find the minimum sum-of-products expression for each function.

(a) $f(a, b, c, d) = \Sigma\, m(0, 2, 3, 4, 7, 8, 14)$

(b) $f(a, b, c, d) = \Sigma\, m(1, 2, 4, 15) + \Sigma\, d(0, 3, 14)$

(c) $f(a, b, c, d) = \Pi\, M(1, 2, 3, 4, 9, 15)$

(d) $f(a, b, c, d) = \Pi\, M(0, 2, 4, 6, 8) \bullet \Pi\, D(1, 12, 9, 15)$

Sol 2

(a)

c d \ a b	00	01	11	10
00	1	1		1
01				
11	1	1		
10	1		1	

f = a'c'd' + a'cd + b'c'd' + abcd' + a'b'd'

Alt: f = a'c'd' + a'cd + b'c'd' + abcd' + a'b'c

(b)

c d \ a b	00	01	11	10
00	X	1		
01	1			
11	X		1	
10	1		X	

f = a'b' + a'c'd' + abc

(c)

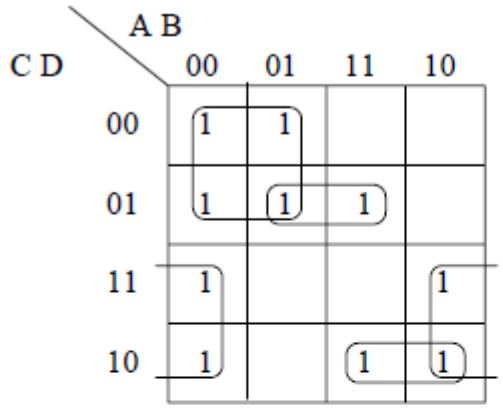
c d \ a b	00	01	11	10
00	1	0	1	1
01	0	1	1	0
11	0	1	0	1
10	0	1	1	1

f = b'c'd' + ab'c + a'bc + bc'd + ad'

(d)

C D \ A B	00	01	11	10
00	0	0	X	0
01	X	1	1	X
11	1	1	X	1
10	0	0	1	1

F = D + A C

Q3	Find the minimum sum of products for the given expression. Then, make minterm 5 a don't-care term and verify that the minimum sum of products is unchanged. Now, start again with the original expression and find each minterm which could *individually* be made a don't-care without changing the minimum sum of products. $F(A, B, C, D) = A'C' + B'C + ACD' + BC'D$
Sol 3	 F = A C D' + B C'D + B'C + A'C' Minterms m_0, m_1, m_2, m_3, m_4, m_{10}, and m_{11} can be made don't cares, individually, without changing the given expression. However, if m_{13} or m_{14} is made a don't care, the term BC'D or the term ACD' (respectively) is not needed in the expression.

Q4

Find the minimum sum of products for each of these functions.

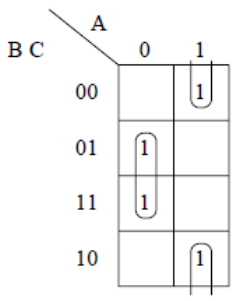
(a) $f_1(A, B, C) = m_1 + m_3 + m_4 + m_6$ (b) $f_2(d, e, f) = \Sigma\, m(1, 4, 5, 7)$

(c) $f_3(r, s, t) = r't' + rs' + rs$ (d) $f_1(a, b, c) = m_3 + m_4 + m_6 + m_7$

(e) $f_2(n, p, q) = \Sigma\, m(2, 3, 5, 7)$ (f) $f_4(x, y, z) = M_3M_6$

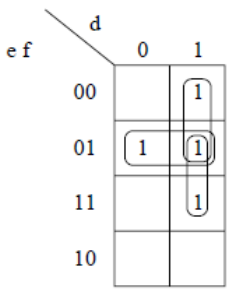
Sol 4

(a)



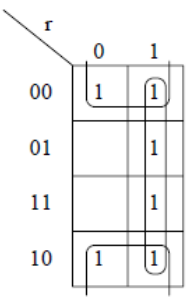
$f_1 = A'C + AC'$

(b)



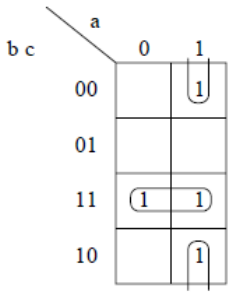
$f_2 = e'f + de' + df$

(c)



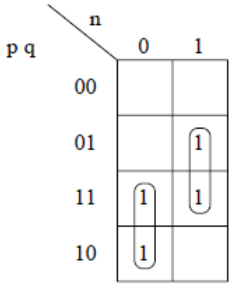
$f_3 = t' + r$

(d)



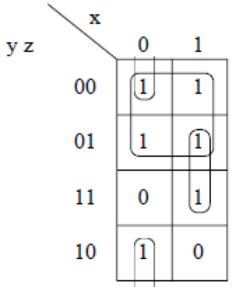
$f_1 = bc + ac'$

(e)



$f_2 = n'p + nq$

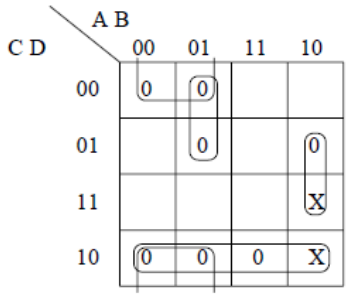
(f)



$f_4 = y' + x'z' + xz$

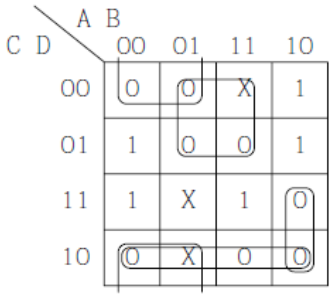
Q5	Find the minimum product of sums for the following. Underline the essential prime implicants in your answer. (a) $\Pi\, M(0, 2, 4, 5, 6, 9, 14) \bullet \Pi\, D(10, 11)$ (b) $\Sigma\, m(1, 3, 8, 9, 15) + \Sigma\, d(6, 7, 12)$
Sol 5	

(a)



F = (C'+ D) (A'+ B + D') (A + B'+ C) (A + D)

(b)



F = (B'+ C) (A'+ B + C') (A + D) (C'+ D)

Alt: F = (B'+ C) (A'+ B + C') (A + D) (B'+ D)

Q 6

Given $F = AB'D' + A'B + A'C + CD$.

(a) Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebric notation).

(b) Use a Karnaugh map to find the minimum sum-of-products form for F'.

(c) Find the minimum product of sums for F.

Sol 6

(a)

C D \ A B	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

F = A B'D' + A'B + A'C + C D

F = ∏ M(0, 1, 9, 12, 13, 14)
= (A + B + C + D)(A + B + C + D')
(A' + B + C + D')(A' + B' + C + D)
(A' + B' + C + D')(A' + B' + C' + D)

(b)

C D \ A B	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	0	0	0	0
10	0	0	1	0

F'= ABD' + A'B'C' + AC'D

(c)

C D \ A B	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

F = (A'+ B'+ D)(A + B + C)(A'+ C + D')

Q7	(a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.) $F(A,B,C,D) = BD' + B'CD + ABC + ABC'D + B'D'$ (b) Find the minimum sum of products. (c) Find the minimum product of sums.

Sol 7

(a)

C D \ A B	00	01	11	10
00	1	1	1	1
01			1	
11	1		1	1
10	1	1	1	1

$F = BD' + B'CD + ABC + ABC'D + B'$

(b)

C D \ A B	00	01	11	10
00	1	1	1	1
01			1	
11	1		1	1
10	1	1	1	1

$F = D' + B'C + A B$

(c)

C D \ A B	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	1	0	1	1
10	1	1	1	1

$F = (A + B' + D')(B + C + D')$

Q8

Find the minimum sum of products and the minimum product of sums for each function:

(a) $f(a, b, c, d) = \Pi M(0, 1, 6, 8, 11, 12) \bullet \Pi D(3, 7, 14, 15)$

(b) $f(a, b, c, d) = \Sigma m(1, 3, 4, 11) + \Sigma d(2, 7, 8, 12, 14, 15)$

Sol 8

(a)

c d \ a b	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	X	X	X	0
10	1	0	X	1

f = (c'+ d')(b'+ c')(a + b + c)(a'+ c + d)

c d \ a b	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	X	X	X	0
10	1	0	X	1

f = a'bc' + ac'd + b'cd'

(b)

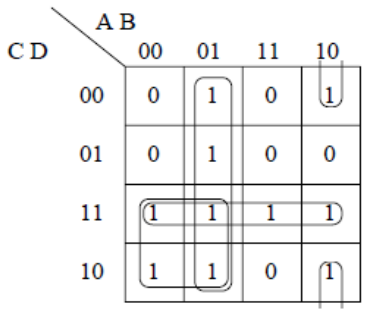
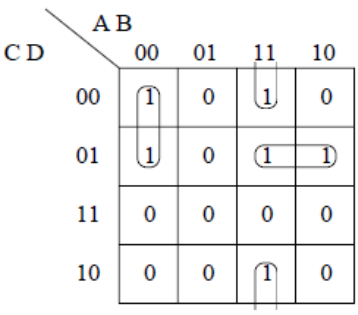
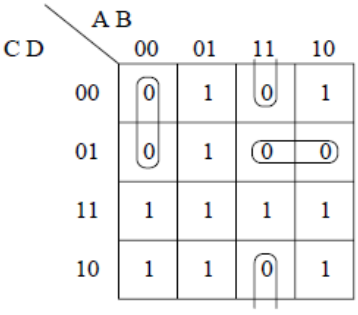
c d \ a b	00	01	11	10
00	0	1	X	X
01	1	0	0	0
11	1	X	X	1
10	X	0	X	0

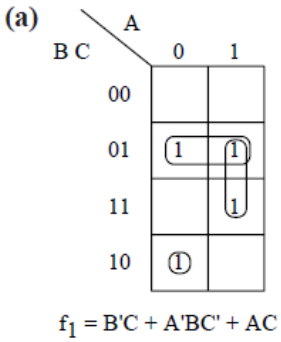
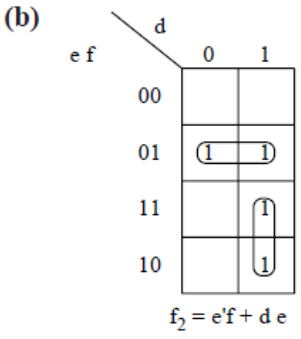
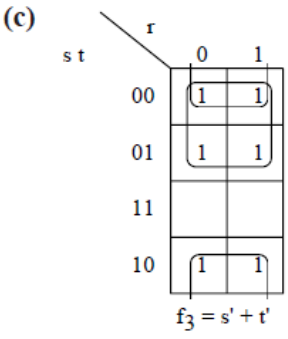
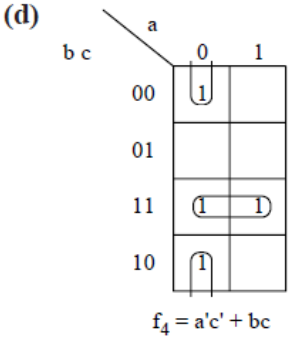
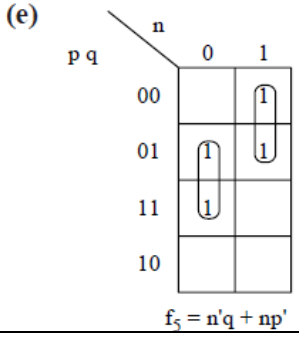
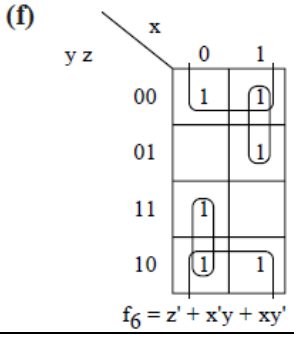
f = (a'+ c)(b'+ d')(b + d)(c'+ d)

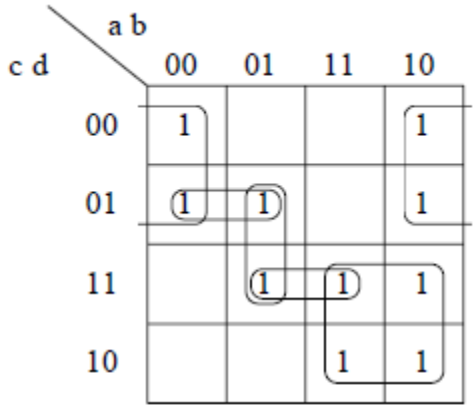
Alt: f = (a'+ c)(b'+ d')(b + d)(b'+ c')

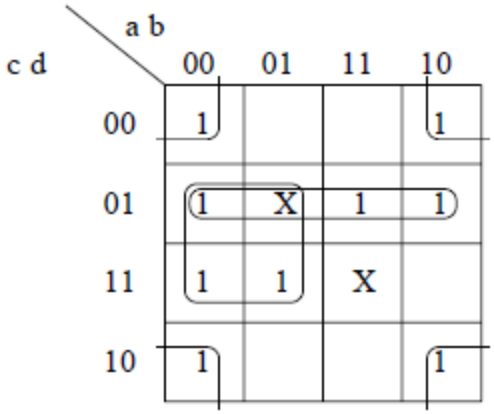
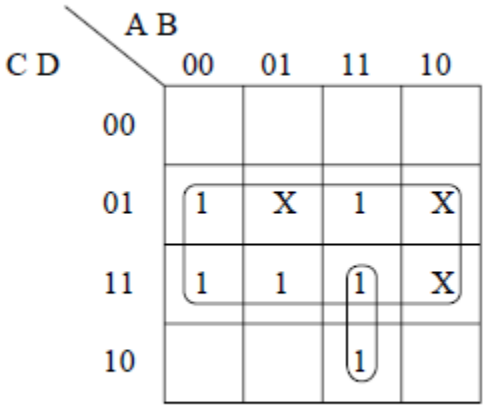
c d \ a b	00	01	11	10
00	0	1	X	X
01	1	0	0	0
11	1	X	X	1
10	X	0	X	0

f = a'b'd + bc'd' + cd

Q9	Given $F = AB'D' + A'B + A'C + CD$. (a) Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebric notation). (b) Use a Karnaugh map to find the minimum sum-of-products form for F'. (c) Find the minimum product of sums for F.
Sol 9	(a)  F = A B'D' + A'B + A'C + C D F = ∏ M(0, 1, 9, 12, 13, 14) = (A + B + C + D) (A + B + C + D') (A' + B' + C + D) (A' + B' + C + D') (A' + B' + C' + D) (A' + B + C + D') (b)  F' = A'B'C' + A B D' + A C'D (c)  F = (A'+ B'+ D)(A + B + C)(A'+ C + D')

Q10	Find the minimum sum-of-products expressions for each of these functions. (a) $f_1(A, B, C) = m_1 + m_2 + m_5 + m_7$ (b) $f_2(d, e, f) = \Sigma\, m(1, 5, 6, 7)$ (c) $f_3(r, s, t) = rs' + r's' + st'$ (d) $f_4(a, b, c) = m_0 + m_2 + m_3 + m_7$ (e) $f_5(n, p, q) = \Sigma\, m(1, 3, 4, 5)$ (f) $f_6(x, y, z) = M_1 M_7$
Sol 10	 $f_1 = B'C + A'BC' + AC$  $f_2 = e'f + d\,e$  $f_3 = s' + t'$  $f_4 = a'c' + bc$  $f_5 = n'q + np'$  $f_6 = z' + x'y + xy'$

Q11	Simplify the following expression first by using a map and then by using Boolean algebra. Use the map as a guide to determine which theorems to apply to which terms for the algebraic simplification. $F = a'b'c' + a'c'd + bcd + abc + ab'$
Sol 11	 F = a'b'c' + a'c'd + bcd +abc + a b' = (a'b'c' + ab') + a'c'd + bcd + (abc + a b') = (a'c' + a)b' + (a'c'd + bcd) + a(bc + b') = (c' + a)b' + (a'c'd + bcd +a'bd) + a(c + b') = (b'c' + a'bd + a'c'd) + (bcd + a'bd + ac) + ab' = (b'c' + ac + ab') + a'bd = b'c' + ac + a'bd

Q12	A logic *circuit* realizes the function $F(a, b, c, d) = a'b' + a'cd + ac'd + ab'd'$. Assuming that $a = c$ never occurs when $b = d = 1$, find a simplified expression for F.
Sol 12	 F = b'd' + a'd + c'd Notice that $abcd = 0101$ and 1111 never occur, so minterms 5 and 15 are don't cares.
Q13	Assuming that the inputs $ABCD = 0101$, $ABCD = 1001$, $ABCD = 1011$ never occur, find a simplified expression for $F = A'BC'D + A'B'D + A'CD + ABD + ABC$
Sol 13	 F = D + AB C

Q14

A logic circuit realizing the function f has four inputs a, b, c, d. The three inputs a, b, and c are the binary representation of the digits 0 through 7 with a being the most significant bit. The input d is an odd-parity bit; that is, the value of d is such that a, b, c, and d always contains an odd number of 1's. (For example, the digit 1 is represented by $abc = 001$ and $d = 0$, and the digit 3 is represented by $abcd = 0111$.) The function f has value 1 if the input digit is a prime number. (A number is prime if it is divisible only by itself and 1; 1 is considered to be prime, and 0 is not.)

(a) Draw a Karnaugh map for f.
(b) Find all prime implicants of f.
(c) Find all minimum sum of products for f.
(d) Find all prime implicants of f'.
(e) Find all minimum product of sums for f.

Sol 14

(a)

c d \ a b	00	01	11	10
00	X	1	X	
01		X		X
11	X	1	X	1
10	1	X	1	X

(b) & (c)

c d \ a b	00	01	11	10
00	X	1	X	
01		X		X
11	X	1	X	1
10	1	X	1	X

PIs: bd', a'b, a'd', c, ab'd

f = b d' + c or
= a'b + c or
= a'd' + c

(d) & (e)

c d \ a b	00	01	11	10
00	X		X	0
01	0	X	0	X
11	X		X	
10		X		X

PIs: (c + d'), (a'+ c), (b + c), (a + b +d'), (a + b'+c'+d), (a'+b'+d'), (a'+b+d)

f = (c + d')(a'+ c) or
= (b + c)(c + d') or
= (b + c)(a'+ c)