Logic Design – Tutorial
04 – Boolean Algebra

#	Student ID	Student Name	Grade (10)
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Q1	Prove the following theorems algebraically: (a) $X(X' - Y) = XY$ (b) $X + XY = X$ (c) $XY + XY' = X$ (d) $(A + B)(A + B') = A$
Sol 1	(a) $X(X' + Y) = XX' + XY = 0 + XY = XY$ (b) $X + XY = X(1 + Y) = X(1) = X$ (c) $XY + XY' = X(Y + Y') = X(1) = X$ (d) $(A + B)(A + B') = AA + AB' + AB + BB' = A + AB' + AB + BB'$ $= A(1 + B + B') + 0 = A(1) = A$

Q2	Multiply out and simplify to obtain a sum of products: (a) $(A - B)(C + B)(D' + B)(ACD' - E)$ (b) $(A' + B - C')(A' + C' + D)(B' + D')$
Sol 2	**(a)** $(A + B)(C + B)(D' + B)(ACD' + E)$ $= (AC + B)(D' + B)(ACD' + E)$ By Th. 8D $= (ACD' + B)(ACD' + E)$ By Th. 8D $= ACD' + BE$ By Th. 8D **(b)** $(A' + B + C')(A' + C' + D)(B' + D')$ $= (A' + C' + BD)(B' + D')$ {By Th. 8D with $X = A' + C'$} $= A'B' + B'C' + B'BD + A'D' + C'D' + BDD'$ $= A'B' + A'D' + C'B' + C'D'$

Q3	Simplify each of the following expressions by applying *one* of the theorems. State the theorem used. (a) $(A' + B' + C)(A' + B' + C)'$ (b) $AB(C' - D) - B(C' + D)$ (c) $AB + (C' + D)(AB)'$ (d) $(A'BF - CD')(A'BF - CEG)$ (e) $[AB' + (C - D)' - E'F](C + D)$ (f) $A'(B + C)(D'E + F)' + (D'E - F)$

Sol 3

(a) $(A' + B' + C)(A' + B' + C)' = 0$ By Th. 5D

(c) $AB + (C' + D)(AB)' = AB + C' + D$
By Th. 11D

(e) $[AB' + (C + D)' + E'F](C + D)$
$= AB'(C + D) + E'F(C + D)$ By Th. 8

(b) $AB(C' + D) + B(C' + D) = B(C' + D)$ By Th. 10

(d) $(A'BF + CD')(A'BF + CEG) = A'BF + CD'EG$
By Th. 8D

(f) $A'(B + C)(D'E + F)' + (D'E + F)$
$= A'(B + C) + D'E + F$ By Th. 11D

Q4

Use *only* DeMorgan's relationships and Involution to find the complements of the following functions:
(a) $f(A, B, C, D) = [A + (BCD)'][(AD)' + B(C' - A)]$
(b) $f(A, B, C, D) = AB'C - (A' + B + D)(ABD' + B')$

Sol 4

(a)
$$\begin{aligned} f' &= \{[A + (BCD)'][(AD)' + B(C' + A)]\}' \\ &= [A + (BCD)']' + [(AD)' + B(C' + A)]' \\ &= A'(BCD)'' + (AD)''[B(C' + A)]' \\ &= A'BCD + AD[B' + (C' + A)'] \\ &= A'BCD + AD[B' + C''A'] \\ &= A'BCD + AD[B' + CA'] \end{aligned}$$

(b)
$$\begin{aligned} f' &= [AB'C + (A' + B + D)(ABD' + B')]' \\ &= (AB'C)'[(A' + B + D)(ABD' + B')]' \\ &= (A' + B'' + C')[(A' + B + D)' + (ABD')'B''] \\ &= (A' + B + C')[A''B'D' + (A' + B' + D'')B] \\ &= (A' + B + C')[AB'D' + (A' + B' + D)B] \end{aligned}$$

Q5	Factor each of the following expressions to obtain a product of sums: (a) $W + U'YV$ (b) $TW - UY' - V$ (c) $A'B'C + B'CD' - B'E'$ (d) $ABC + ADE' + ABF'$
Sol 5	**(a)** $W + U'YV = (W + U')(W + Y)(W + V)$ **(c)** $A'B'C + B'CD' + B'E' = B'(A'C + CD' + E')$ $= B'[E' + C(A' + D')]$ $= B'(E' + C)(E' + A' + D')$ **(b)** $TW + UY' + V$ $= (T+U+Z)(T+Y'+V)(W+U+V)(W+Y'+V)$ **(d)** $ABC + ADE' + ABF' = A(BC + DE' + BF')$ $= A[DE' + B(C + F')]$ $= A(DE' + B)(DE' + C + F')$ $= A(B + D)(B + E')(C + F' + D)(C + F' + E')$

Q6	Simplify the following expressions to a minimum sum of products. Only individual variables should be complemented. (a) $[(XY')' - (X' + Y)'Z]$ (b) $(X - (Y'(Z + W)')')'$ (c) $[(A' - B')' - (A'B'C)' + C'D]'$ (d) $(A - B)CD + (A + B)'$

Sol 6

(a) $[(XY')' + (X' + Y)'Z] = X' + Y + (X' + Y)'Z$
$= X' + Y' + Z$ By Th. 11D with $Y = (X' + Y)$

(c) $[(A' + B')' + (A'B'C)' + C'D]'$
$= (A' + B')A'B'C(C + D') = A'B'C$

(b) $(X + (Y'(Z + W)')')' = X'Y'(Z + W)' = X'Y'Z'W'$

(d) $(A + B)\,CD + (A + B)' = CD + (A + B)'$
{By Th. 11D with $Y = (A + B)'$}
$= CD + A'B'$

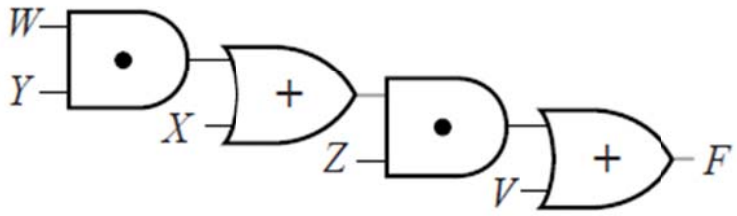
Q7

Draw a circuit that uses two OR gates and two AND gates to realize the followin function:

$$F = (V - W + X)(V + X + Y)(V + Z)$$

Sol 7

$F = (\underline{V + X} + W)\,(\underline{V + X} + Y)\,(V + Z)$
$= (V + X + WY)(V + Z) = V + Z\,(X + WY)$
By Th. 8D with $X = V$



Q8

Prove the following equations using truth tables:

(d) $(A + C)(AB + C') = AB + AC'$
(e) $W'X\,Y + WZ = (W' + Z)(W + XY)$

Sol 8

(d)

A B C	A+C	AB+C′	(A+C)(AB+C′)	AB	AC′	AB+AC′
0 0 0	0	1	0	0	0	0
0 0 1	1	0	0	0	0	0
0 1 0	0	1	0	0	0	0
0 1 1	1	0	0	0	0	0
1 0 0	1	1	1	0	1	1
1 0 1	1	0	0	0	0	0
1 1 0	1	1	1	1	1	1
1 1 1	1	1	1	1	0	1

(e)

W X Y Z	W′XY	WZ	W′XY+WZ	W′+Z	W+XY	(W′+Z)(W+XY)
0 0 0 0	0	0	0	1	0	0
0 0 0 1	0	0	0	1	0	0
0 0 1 0	0	0	0	1	0	0
0 0 1 1	0	0	0	1	0	0
0 1 0 0	0	0	0	1	0	0
0 1 0 1	0	0	0	1	0	0
0 1 1 0	1	0	1	1	1	1
0 1 1 1	1	0	1	1	1	1
1 0 0 0	0	0	0	0	1	0
1 0 0 1	0	1	1	1	1	1
1 0 1 0	0	0	0	0	1	0
1 0 1 1	0	1	1	1	1	1
1 1 0 0	0	0	0	0	1	0
1 1 0 1	0	1	1	1	1	1
1 1 1 0	0	0	0	0	1	0
1 1 1 1	0	1	1	1	1	1