Lecture (08)
Karnaugh Maps I

By:

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Minimum Forms of Switching Function

- The Karnaugh map techniques developed in this unit lead directly to minimum cost two-level circuits composed of AND and OR gates.
- An expression consisting of a sum of product terms corresponds directly to a two-level circuit composed of a group of AND gates feeding a single OR gate.
- Similarly, a product-of-sums expression corresponds to a two-level circuit composed of OR gates feeding a single AND gate

minimum sum-of-products expression:

(a) has a minimum number of terms and

(b) has a minimum number of literals.

- To obtain minimum sum-of-products from minterm procedure:
- **1.** Combine terms by using XY'+XY=X. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because (X+X=X).
- **2.** Eliminate redundant terms

Example 05

- Find a minimum sum-of-products expression for

$$F(a, b, c) = \Sigma m\,(0, 1, 2, 5, 6, 7)$$

- Combine terms by using XY’+XY=X. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because (X+X=X).

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$
$$= a'b' + b'c + bc' + ab$$

- Eliminate redundant terms

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$
$$= a'b' + bc' + ac$$

<u>*minimum product-of-sums* expression:</u>

- (a) has a minimum number of factors, and
- (b) has a minimum number of literals.

<u>Procedures</u>

- the theorem (X+Y).(X+Y’)=X is used to combine terms.

Example 06

- $F = \prod M(10, 8, 9, 1, 13, 6)$

$$(A + B' + C + D')(A + B' + C' + D')(A + B' + C' + D)(A' + B' + C' + D)(A + B + C' + D)(A' + B + C' + D)$$

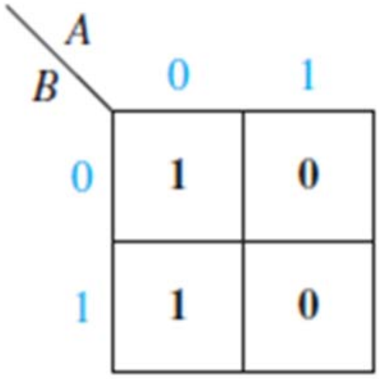
$$= (A + B' + D') \quad (A + B' + C') \quad (B' + C' + D) \quad (B + C' + D)$$

$$= (A + B' + D') \quad (A + B' + C') \quad (C' + D)$$

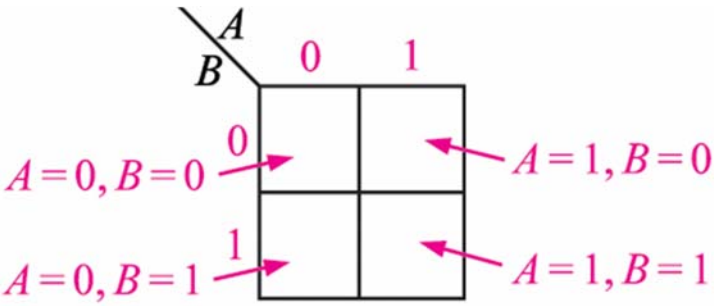
Two-Variable Karnaugh Maps

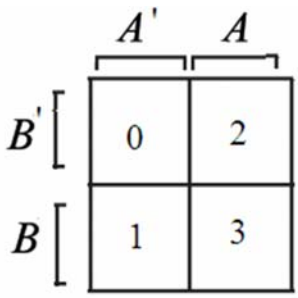
- The values of one variable are listed across the top of the map, and the values of the other variable are listed on the left side.
- Each square of the map corresponds to a pair of values for *A* and *B* as indicated.

A B	*F*
0 0	1
0 1	1
1 0	0
1 1	0



- We can read the minterms from the map
- 1’s in square 00, 01 refers to minterms





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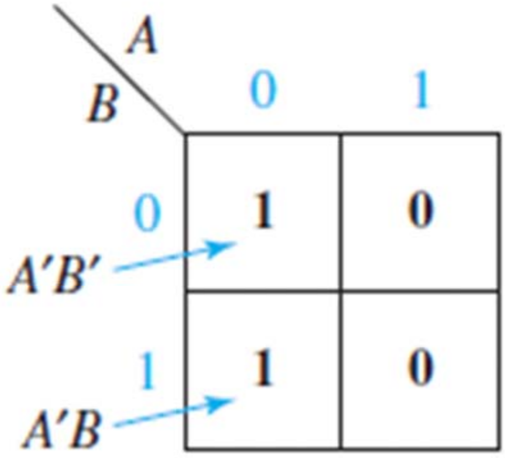
A B	F
0 0	1
0 1	1
1 0	0
1 1	0

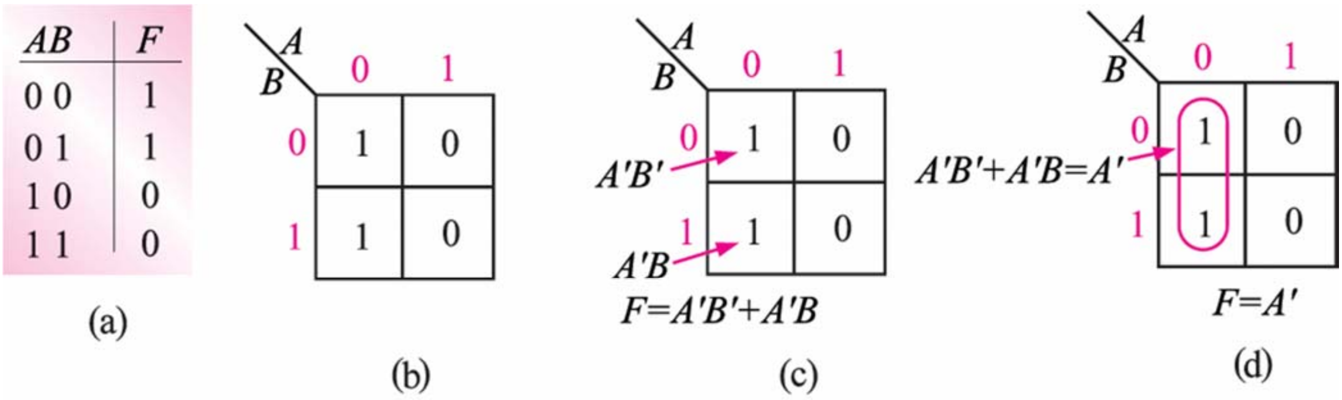


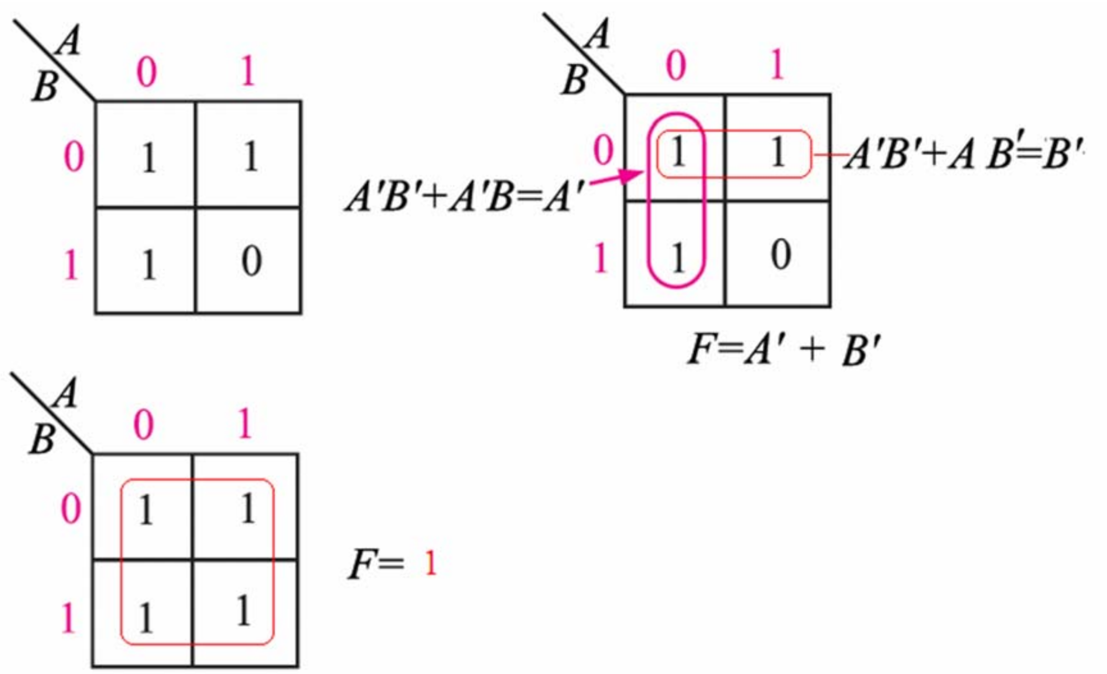
$F = A'B' + A'B$

Adjacent Cells

- Two cell which *differ in just one variable* are said to be adjacent.
- 2^k adjacent calls can be combined.

- Minterms in adjacent squares of the map can be combined since they differ in only one variable.
- Thus, *A’B’* and *A’B* combine to form *A’*

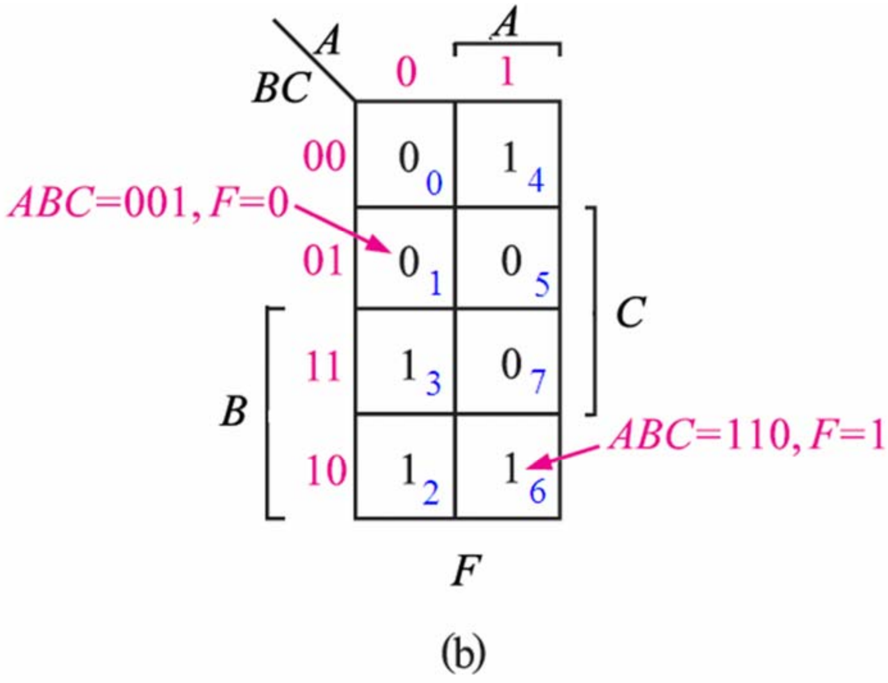


- 

3 variable Karnaugh Maps

ABC	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

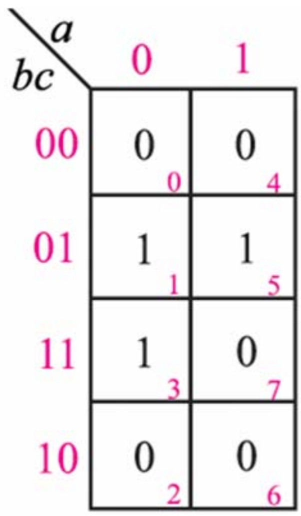
(a)



(b)

- If F is given as a minterm (maxterm) expansion, the map by placing 1's(0's) in the squares which correspond to the minterm (maxterm) and then by filling in the remaining squares with 0's(1's).

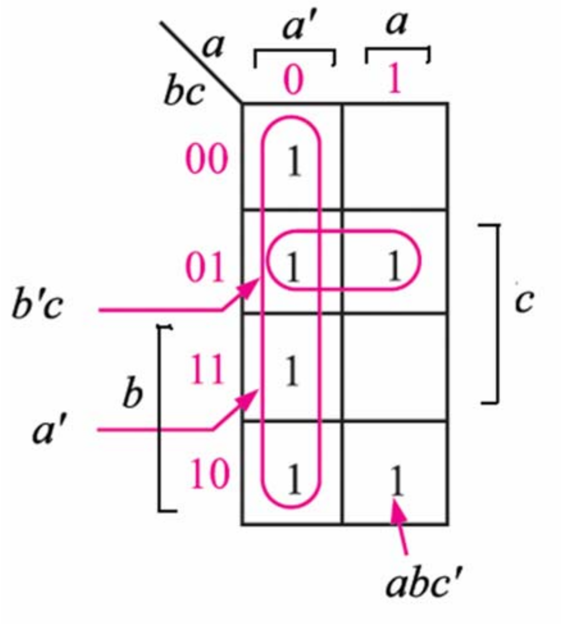
$$F(a,b,c) = m_1 + m_3 + m_5 = M_0 M_2 M_4 M_6 M_7$$



$$F(a,b,c) = \Sigma m(1, 3, 5) = \Pi M(0, 2, 4, 6, 7)$$

- If a function is given in algebraic form, plot it's Karnaugh Map

$$f(a,b,c) = abc' + b'c + a'$$



Example

- Simplify a function using Karnaugh Map $F = \Sigma m(1,3,5)$

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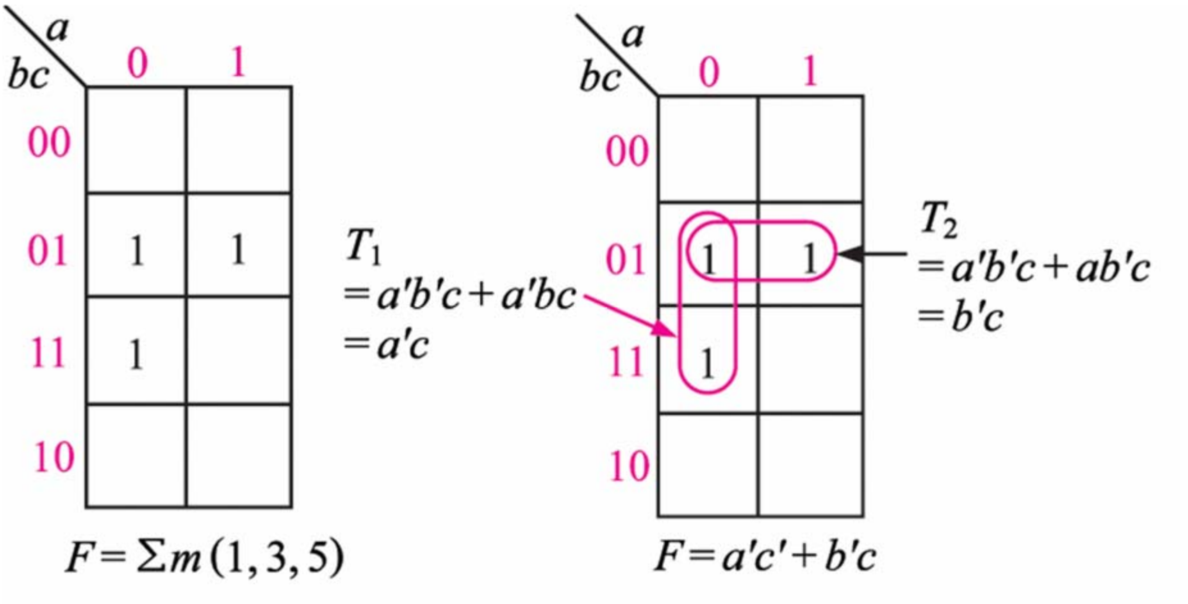
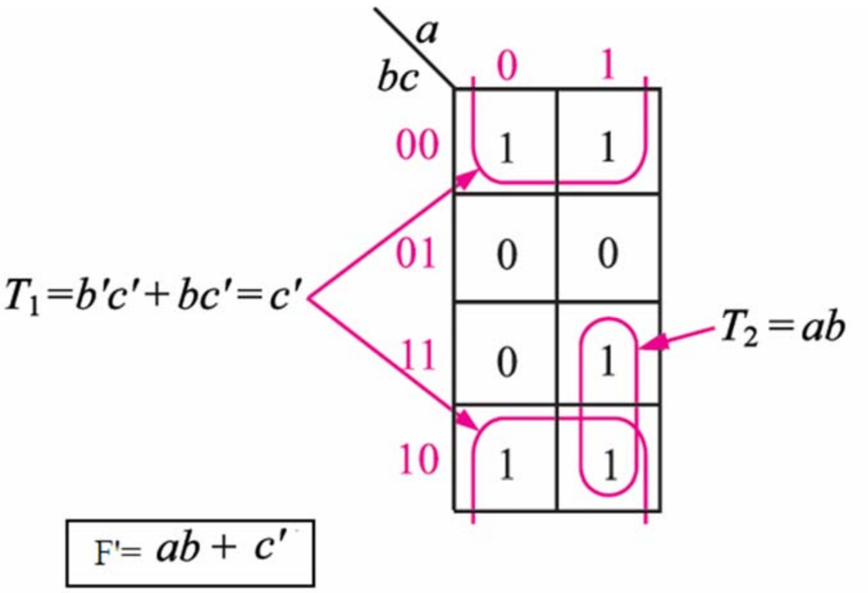
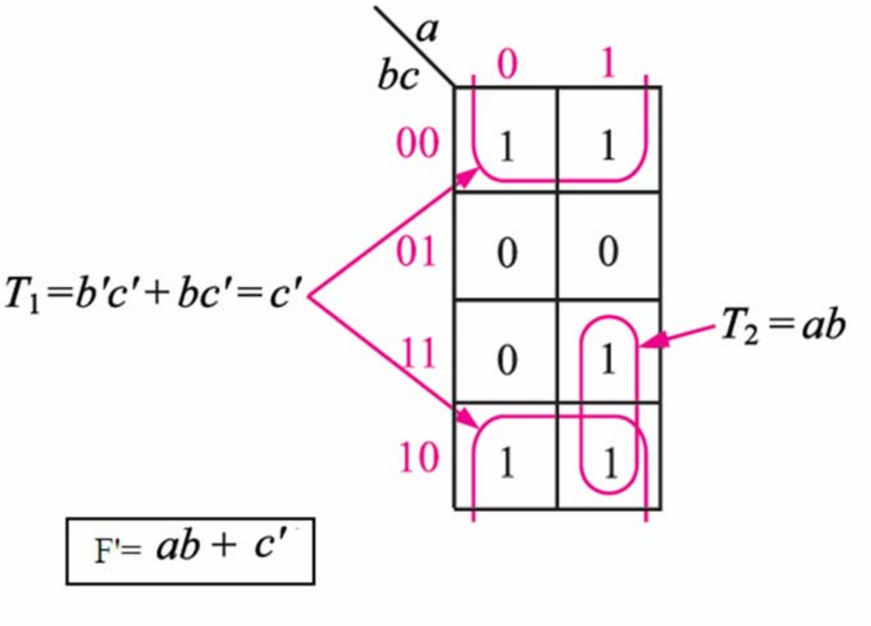


Example

- Simplify a function using Karnaugh Map Simplify the *complement* of $F = \Sigma m(1,3,5)$



- *complement* of $F = \Sigma m(1,3,5)$

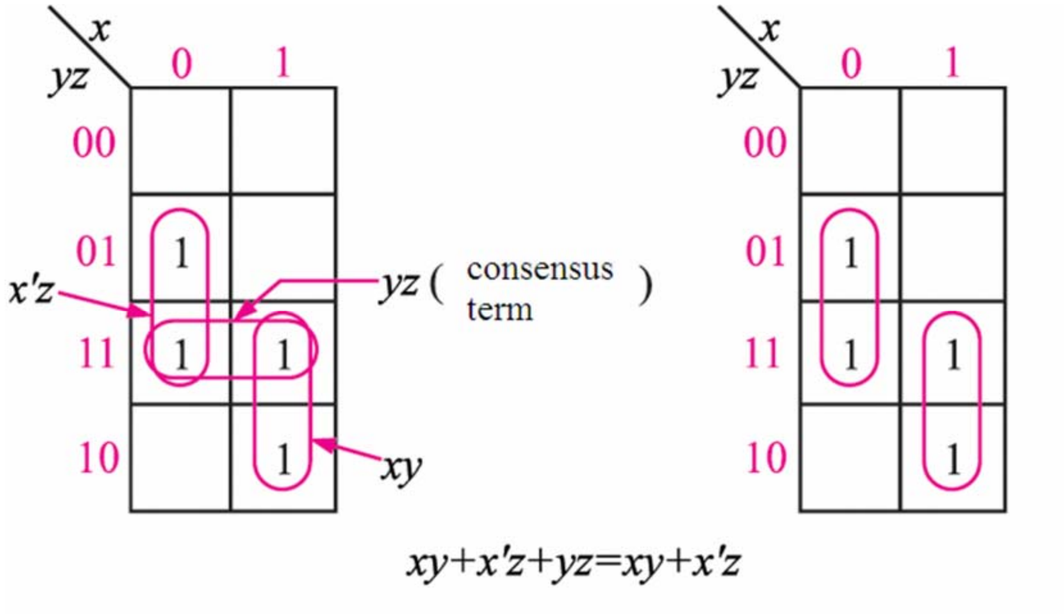


Example

- ***Illus trate* the Consensus Theorem**

$xy + x'z + yz = xy + x'z$

- $xy + x'z + yz = xy + x'z$

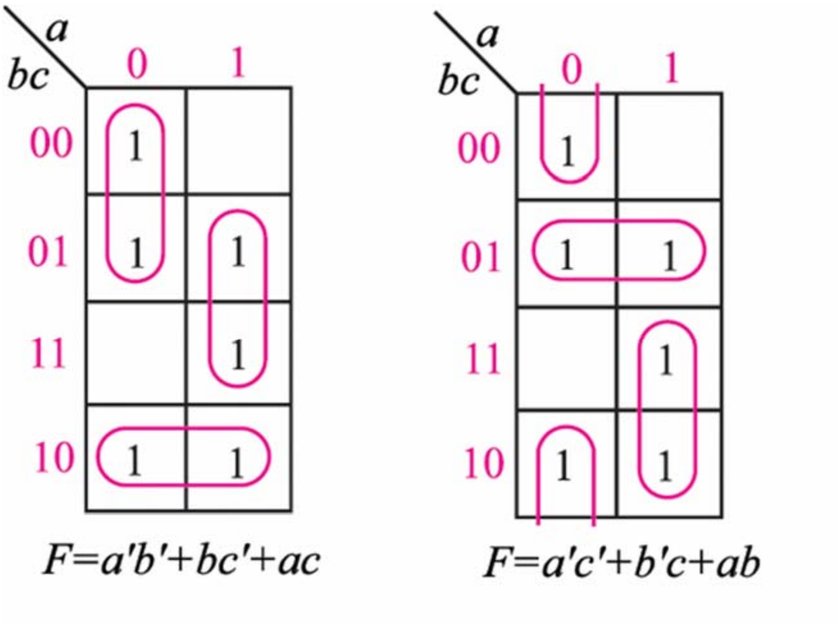


example

- Minimum sum-of-products is *not* unique.

$f = \Sigma m(0,1,2,5,6,7)$

- $f = \Sigma m(0,1,2,5,6,7)$



Function F with two minimum forms 3 terms, 6 literals

Four- Variable KarnaughMaps

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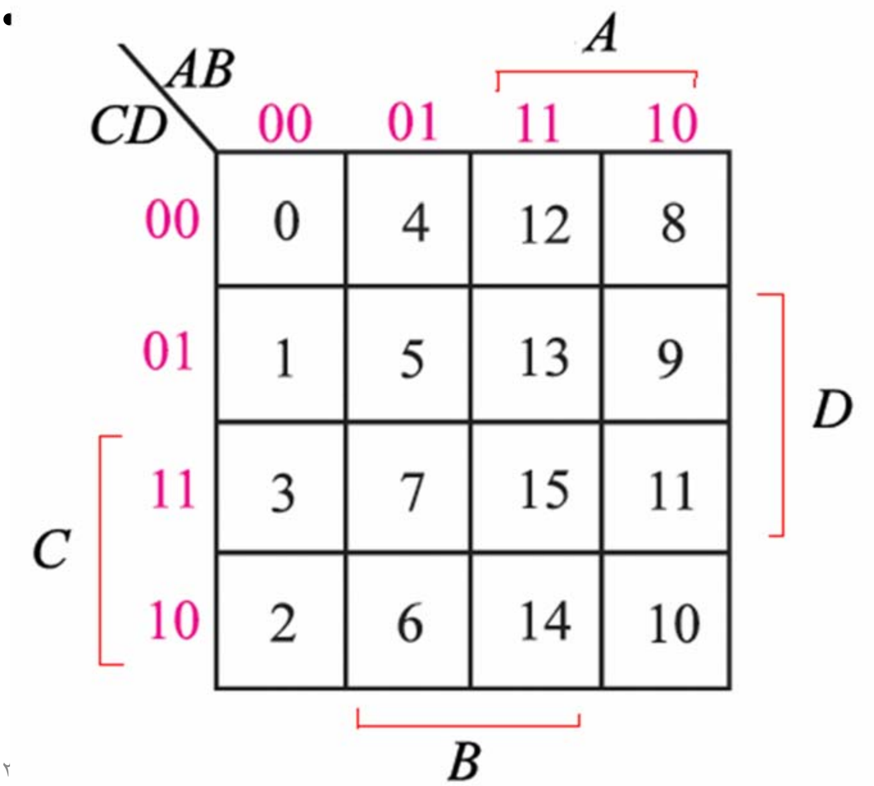
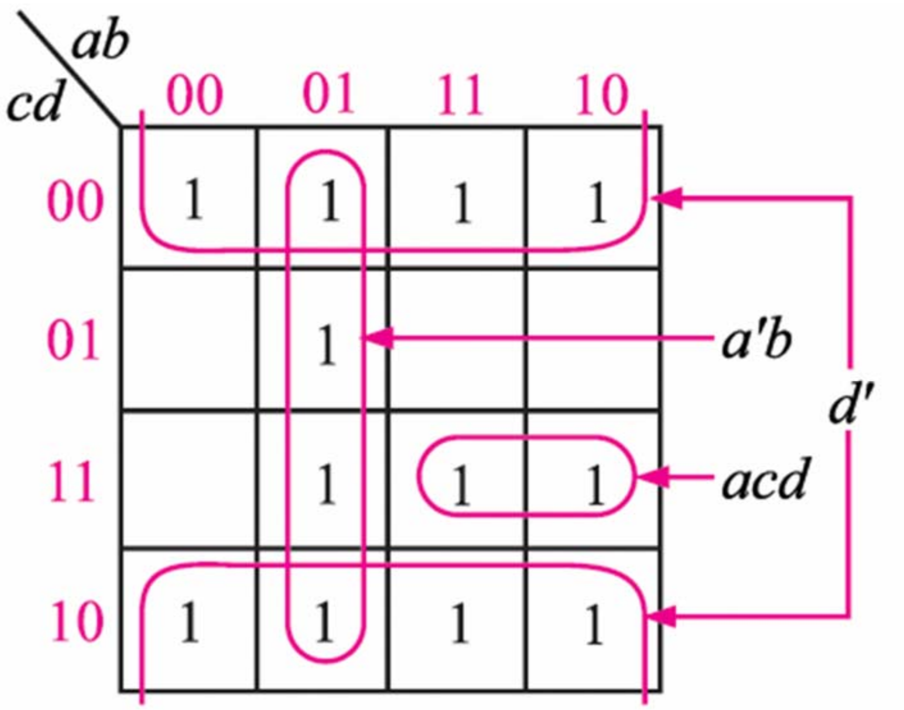


Example

- $f(a,b,c,d) = acd + a'b + d'$

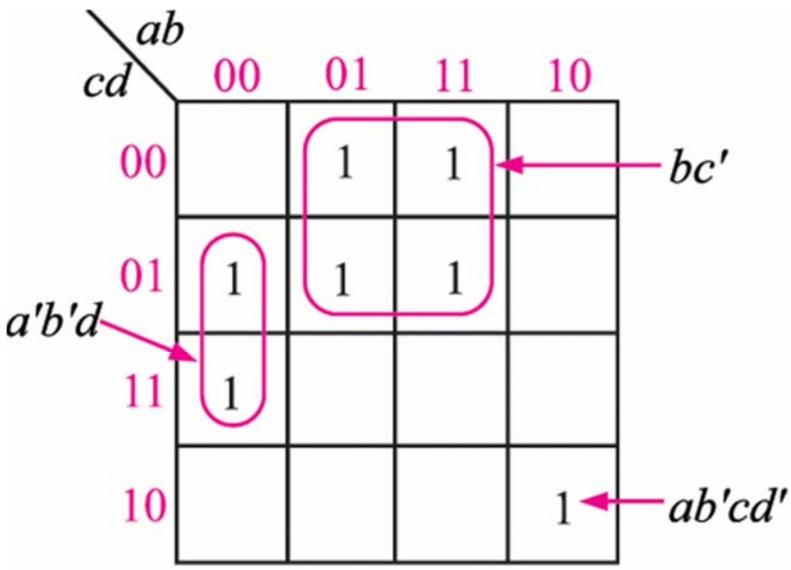
- $f(a,b,c,d) = acd + a'b + d'$



Example

- Simplify F= Σm (1,3,4,5,10,12,13)

- Simplify = Σ (1,3,4,5,10,12,13)

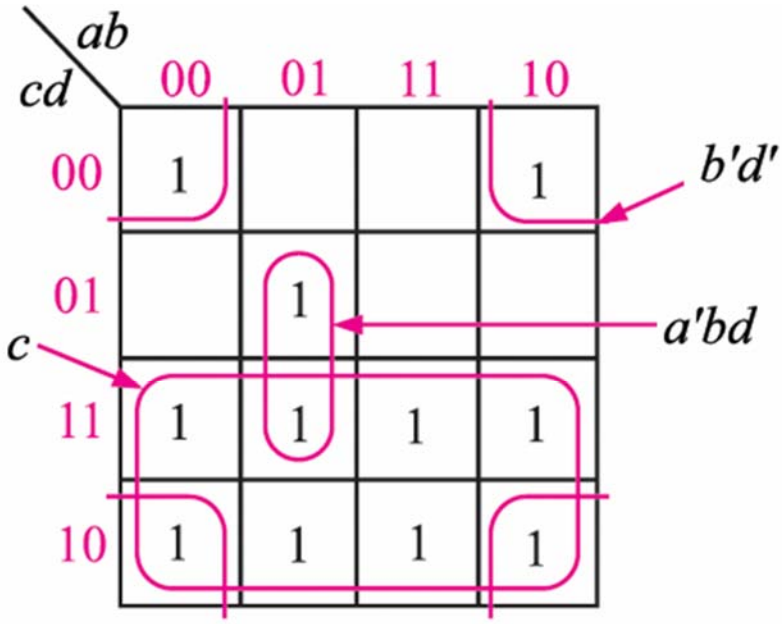


$$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$$
$$= bc' + a'b'd + ab'cd'$$

Example

- Simplify F= Σ m(0,2,3,5,6,7,8,10,11,14,15)

- F= Σ m(0,2,3,5,6,7,8,10,11,14,15)



$$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$$
$$= c + b'd' + a'bd$$

Example

- Simplify a function with don't care
- $f = \Sigma m(1,3,5,7,9) + \Sigma\, d(6,12,13)$

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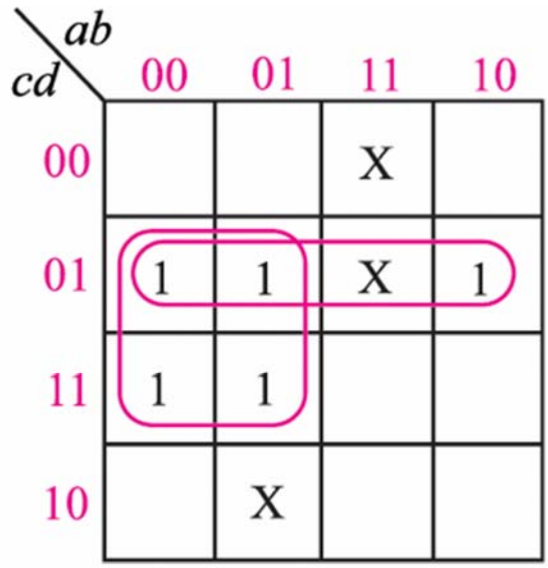


$$f = \Sigma m(1,3,5,7,9) + \Sigma d(6,12,13)$$
$$= a'd + c'd$$

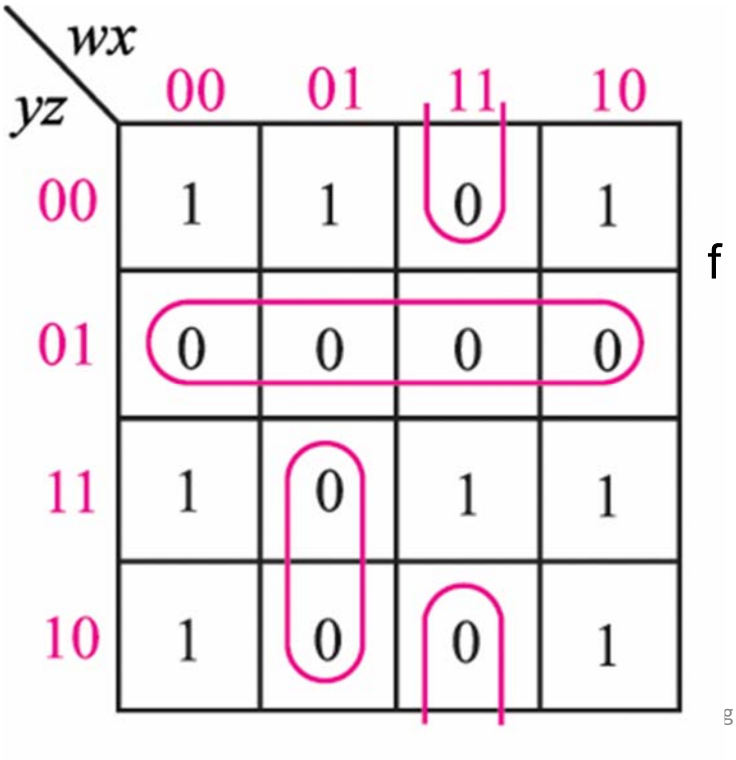
Find a minimum product-of-sums

- 1. Find a minimum sum-of-products for F’
- 2. Complement F’ using DeMorgan’s Theorem

Example

- $f = x'z' + wyz + w'y'z' + x'y$

- $f = x'z' + wyz + w'y'z' + x'y$



$f' = y'z + wxz' + w'xy$

$f = (f')' = (y+z')(w'+x'+z)(w+x'+y')$

Thanks,..

See you next week (ISA),...