



Lecture (08) Karnaugh Maps I



By:

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Minimum Forms of Switching Function

- The Karnaugh map techniques developed in this unit lead directly to minimum cost two-level circuits composed of AND and OR gates.
- An expression consisting of a sum of product terms corresponds directly to a two-level circuit composed of a group of AND gates feeding a single OR gate.
- Similarly, a product-of-sums expression corresponds to a two-level circuit composed of OR gates feeding a single AND gate

minimum sum-of-products expression:

(a) has a minimum number of terms and

(b) has a minimum number of literals.

- To obtain minimum sum-of-products from minterm procedure:
- **1.** Combine terms by using $XY' + XY = X$. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because $(X+X=X)$.
- **2.** Eliminate redundant terms

Example 05

- Find a minimum sum-of-products expression for

$$F(a, b, c) = \sum m (0, 1, 2, 5, 6, 7)$$

- Combine terms by using $XY' + XY = X$. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because $(X + X = X)$.

$$\begin{aligned}
 F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\
 &= a'b' + b'c + bc' + ab
 \end{aligned}$$

- Eliminate redundant terms

$$\begin{aligned}
 F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\
 &= a'b' + bc' + ac
 \end{aligned}$$

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minimum product-of-sums expression:

- (a) has a minimum number of factors, and
- (b) has a minimum number of literals.

Procedures

- the theorem $(X+Y).(X+Y')=X$ is used to combine terms.

Example 06

- $F = \prod M(10, 8, 9, 1, 13, 6)$

Y

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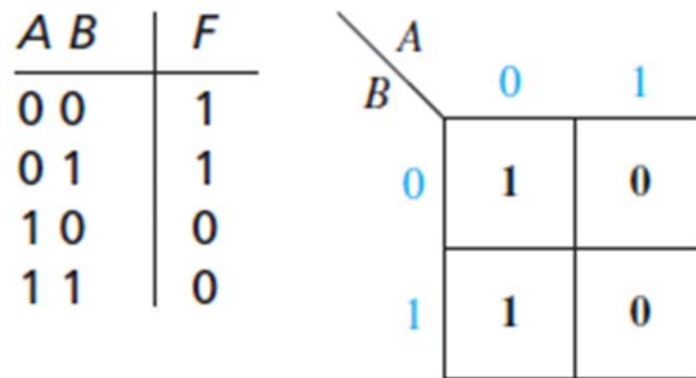
$$\begin{aligned}
 & (A + B' + C + D')(A + B' + C' + D')(A + B' + C' + D)(A' + B' + C' + D)(A + B + C' + D)(A' + B + C' + D) \\
 &= (A + B' + D') \quad (A + B' + C') \quad (B' + C' + D) \quad (B + C' + D) \\
 &= (A + B' + D') \quad (A + B' + C') \quad (C' + D)
 \end{aligned}$$

A

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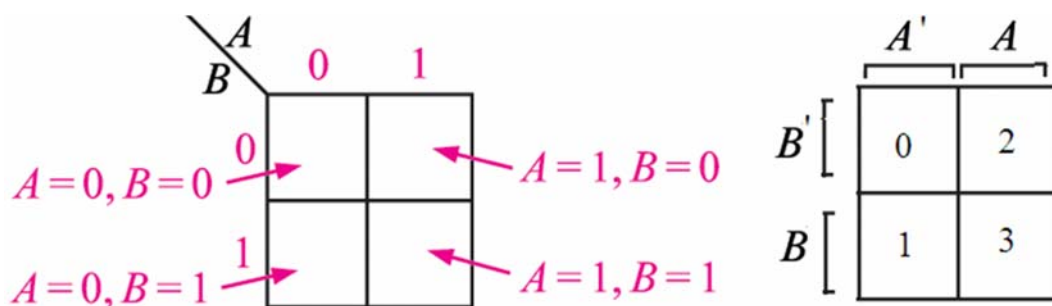
Two-Variable Karnaugh Maps

- The values of one variable are listed across the top of the map, and the values of the other variable are listed on the left side.
- Each square of the map corresponds to a pair of values for A and B as indicated.

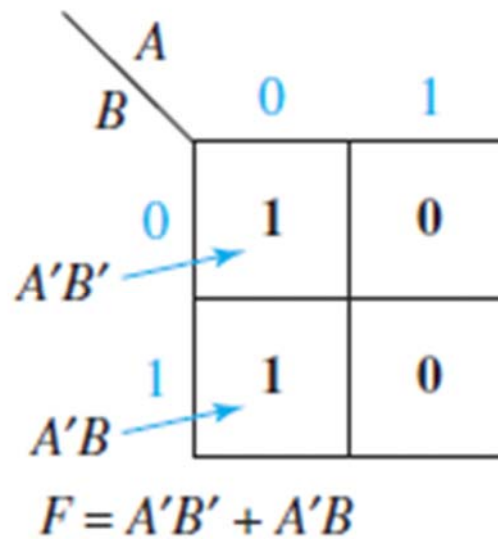


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- We can read the minterms from the map
- 1's in square 00, 01 refers to minterms



A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



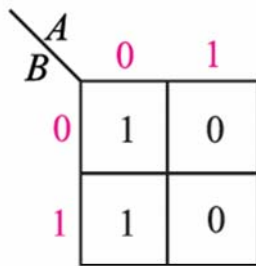
Adjacent Cells

- Two cell which *differ in just one variable* are said to be adjacent.
- 2^k adjacent calls can be combined.

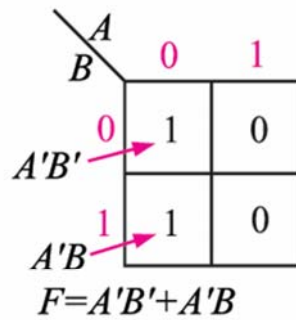
- Minterms in adjacent squares of the map can be combined since they differ in only one variable.
- Thus, $A'B'$ and $A'B$ combine to form A'

AB	F
00	1
01	1
10	0
11	0

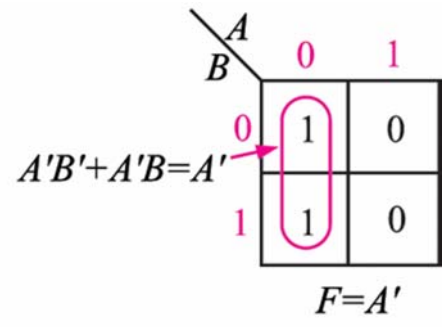
(a)



(b)



(c)



(d)

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- A 2x2 Karnaugh map with variables A and B. The top row is labeled B=0 and the bottom row is labeled B=1. The left column is labeled A=0 and the right column is labeled A=1. The cells contain values: (0,0)=1, (0,1)=1, (1,0)=1, (1,1)=0. The cells (0,0) and (1,0) are circled in pink. An arrow points from the label $A'B' + A'B = A'$ to these cells.

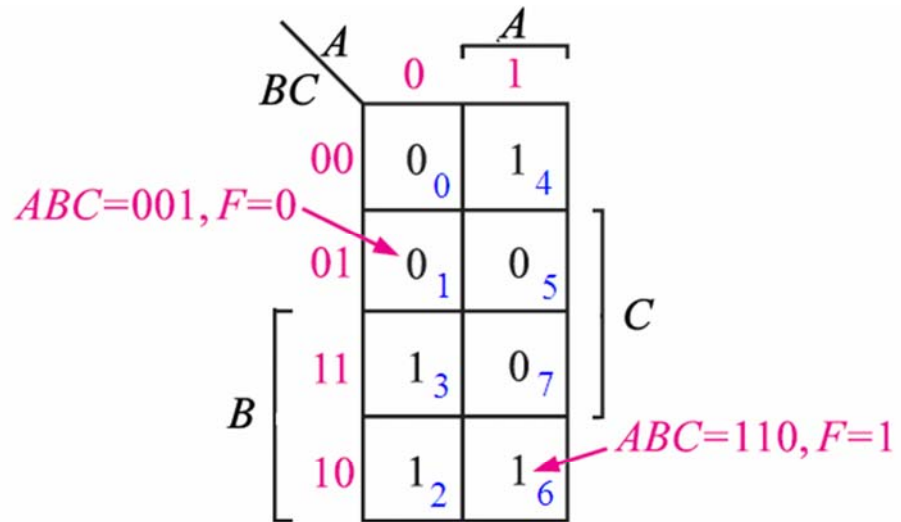
A 2x2 Karnaugh map with variables A and B. The top row is labeled B=0 and the bottom row is labeled B=1. The left column is labeled A=0 and the right column is labeled A=1. The cells contain values: (0,0)=1, (0,1)=1, (1,0)=1, (1,1)=0. The cells (0,0) and (0,1) are circled in pink. An arrow points from the label $A'B' + AB' = B'$ to these cells. Below the map is the equation $F = A' + B'$.

A 2x2 Karnaugh map with variables A and B. The top row is labeled B=0 and the bottom row is labeled B=1. The left column is labeled A=0 and the right column is labeled A=1. The cells contain values: (0,0)=1, (0,1)=1, (1,0)=1, (1,1)=1. All four cells are circled in pink. Below the map is the equation $F = 1$.

3 variable Karnaugh Maps

<i>ABC</i>	<i>F</i>
000	0
001	0
010	1
011	1
100	1
101	0
110	1
111	0

(a)

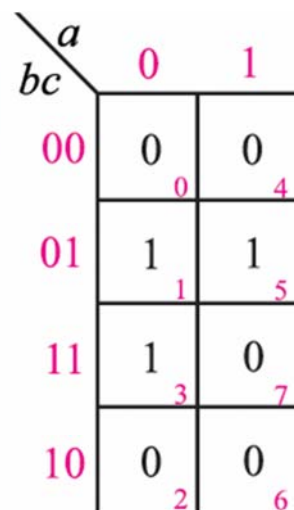


F

(b)

- If *F* is given as a minterm (maxterm) expansion, the map by placing 1's(0's) in the squares which correspond to the minterm (maxterm) and then by filling in the remaining squares with 0's(1's).

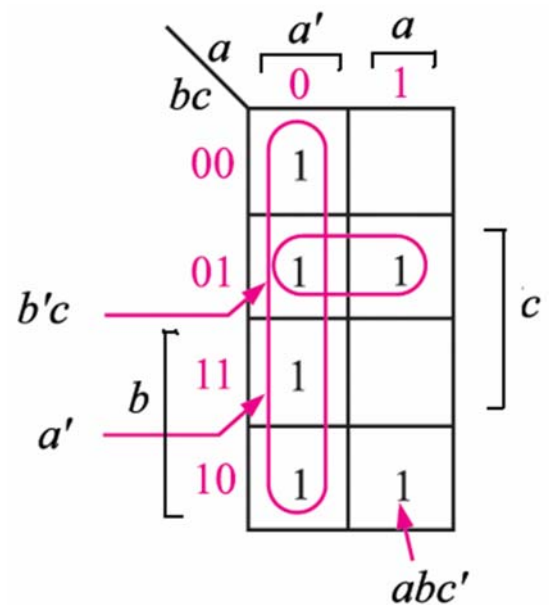
$$F(a,b,c) = m_1 + m_3 + m_5 = M_0M_2M_4M_6M_7$$



$$F(a,b,c) = \sum m(1, 3, 5) = \prod M(0, 2, 4, 6, 7)$$

- If a function is given in algebraic form, plot it's Karnaugh Map

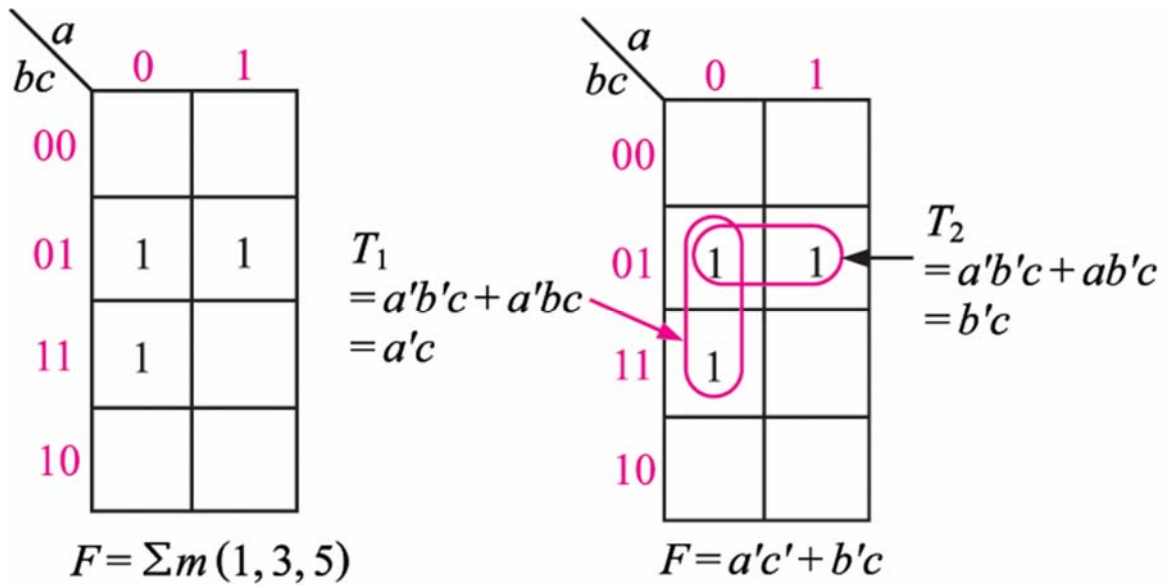
$$f(a,b,c) = abc' + b'c + a'$$



Example

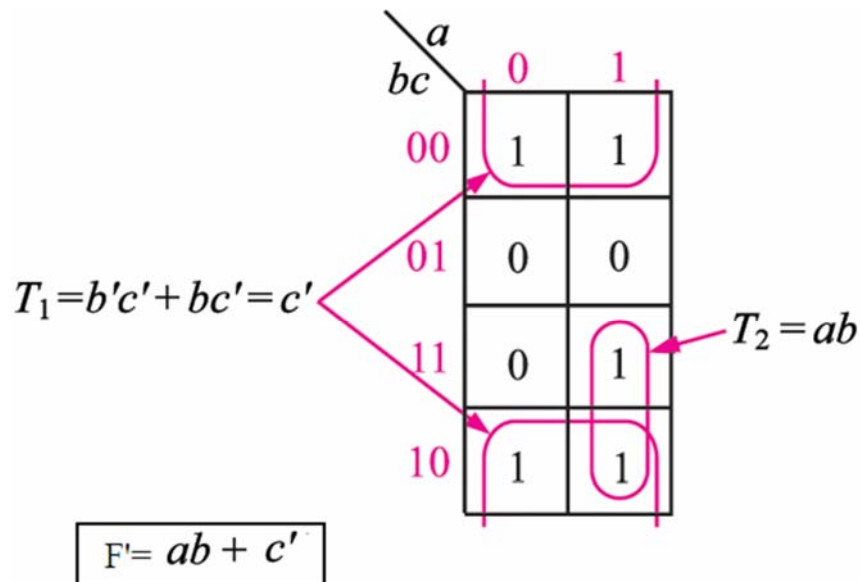
- Simplify a function using Karnaugh Map $F = \Sigma m(1,3,5)$

- $F = \Sigma m(1,3,5)$

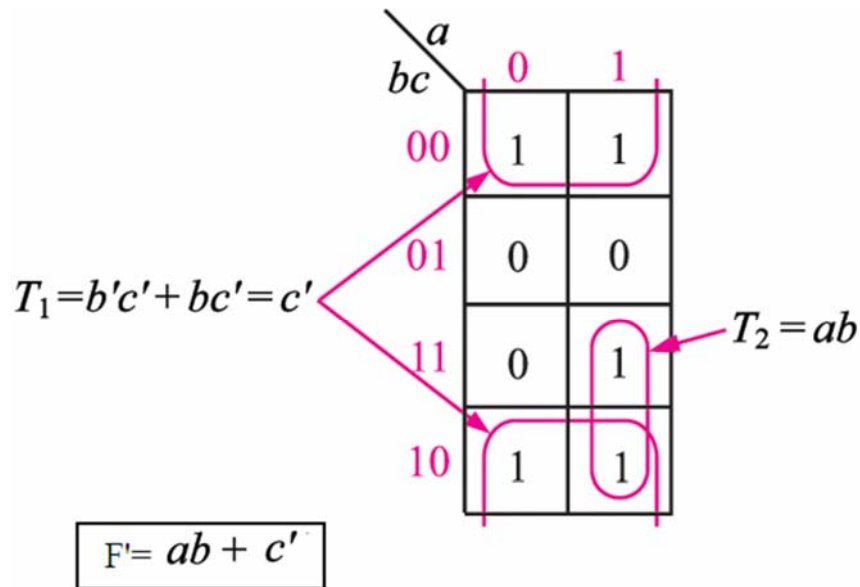


Example

- Simplify a function using Karnaugh Map Simplify the complement of $F = \Sigma m(1,3,5)$



- complement of $F = \Sigma m(1,3,5)$



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Example

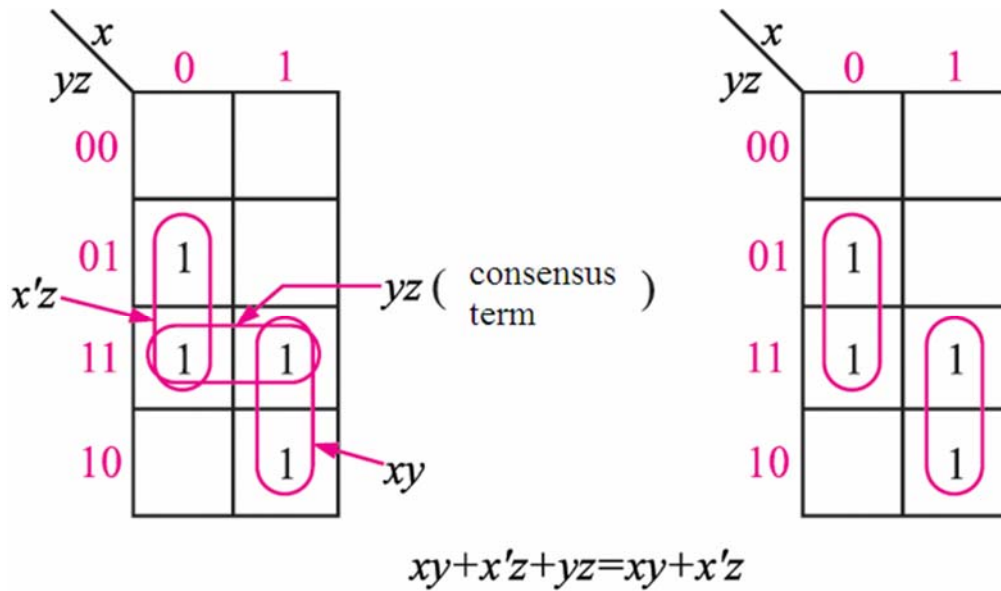
- *Illustrate* the Consensus Theorem

$$xy + x'z + yz = xy + x'z$$

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- $xy + x'z + yz = xy + x'z$

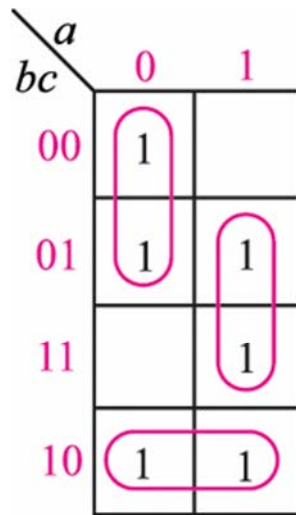


example

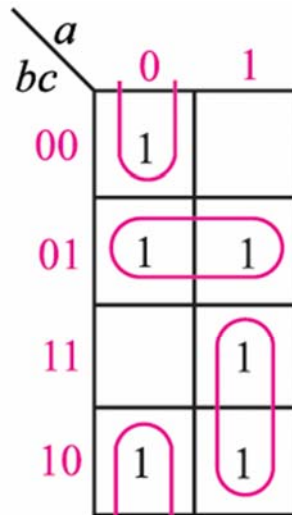
- Minimum sum-of-products is *not* unique.

$$f = \Sigma m(0,1,2,5,6,7)$$

- $f = \Sigma m(0,1,2,5,6,7)$



$$F = a'b' + bc' + ac$$

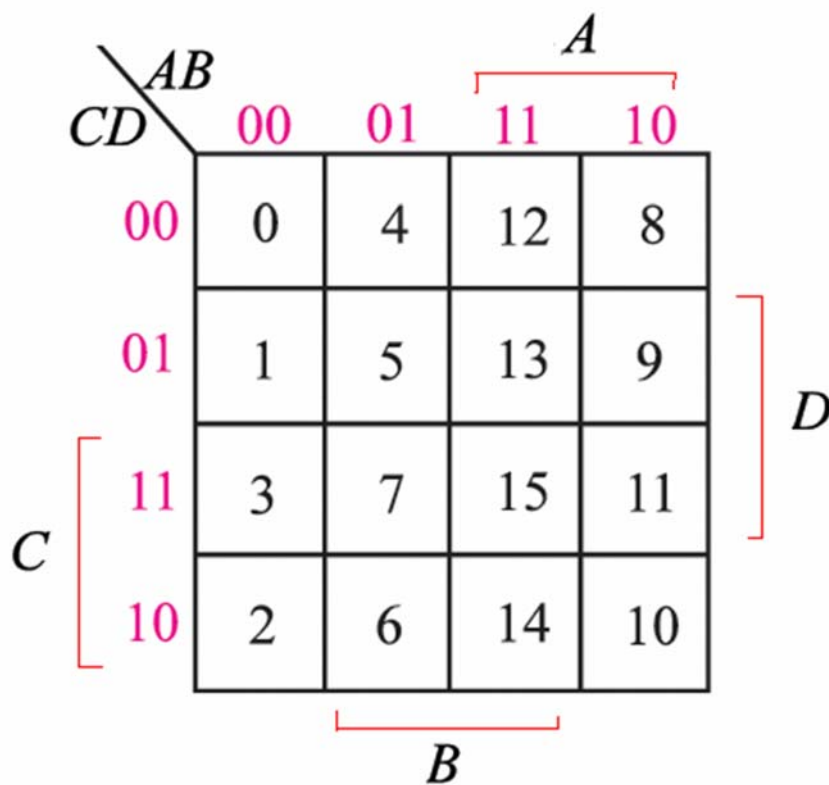


$$F = a'c' + b'c + ab$$

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Function F with two minimum forms 3 terms, 6 literals

Four- Variable Karnaugh Maps



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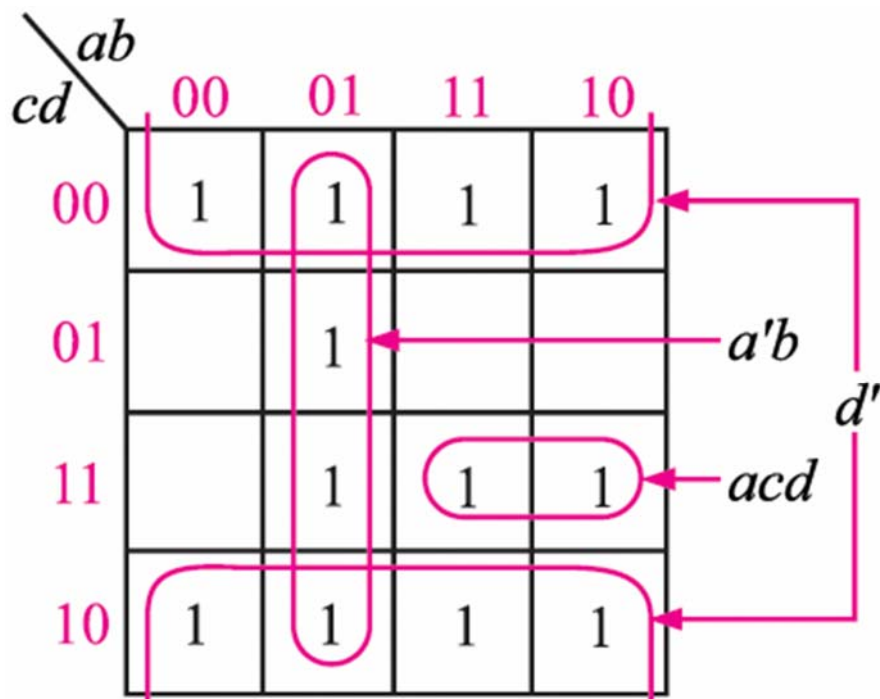
Example

- $f(a,b,c,d) = acd + a'b + d'$

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- $f(a,b,c,d) = acd + a'b + d'$



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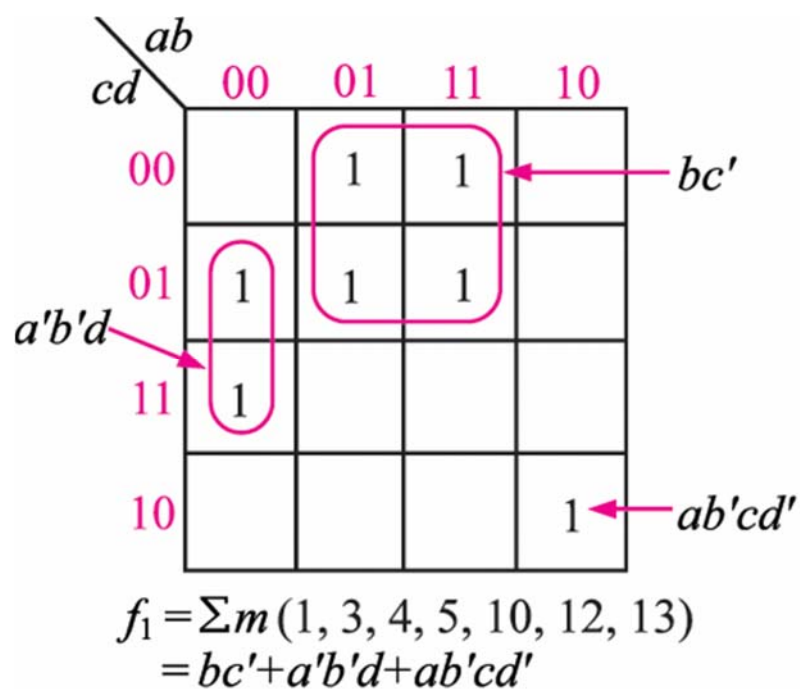
Example

- Simplify $F = \sum m(1, 3, 4, 5, 10, 12, 13)$

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- Simplify = $\Sigma(1, 3, 4, 5, 10, 12, 13)$



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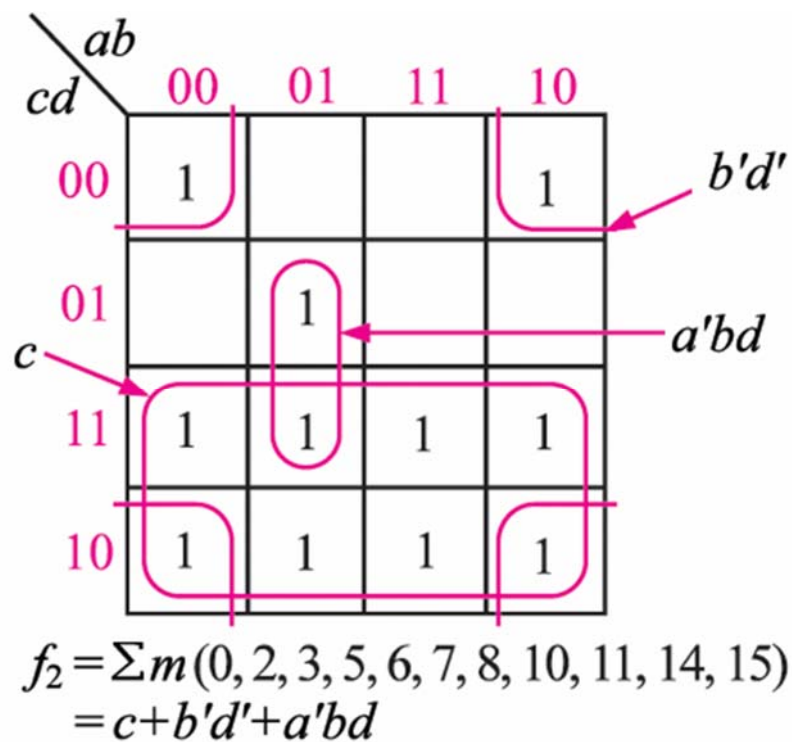
Example

- Simplify $F = \sum m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$

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- $F = \sum m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$



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Example

- Simplify a function with don't care
- $f = \sum m(1,3,5,7,9) + \sum d(6,12,13)$

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- $f = \sum m(1,3,5,7,9) + \sum d(6,12,13)$

$cd \backslash ab$	00	01	11	10
00			X	
01	1	1	X	1
11	1	1		
10		X		

$$f = \sum m(1, 3, 5, 7, 9) + \sum d(6, 12, 13)$$
$$= a'd + c'd$$

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Find a minimum product-of-sums

- 1. Find a minimum sum-of-products for F'
- 2. Complement F' using DeMorgan's Theorem

Example

- $f = x'z' + wyz + w'y'z' + x'y$

- $f = x'z' + wyz + w'y'z' + x'y$

wx \ yz	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

$$f' = y'z + wxz' + w'xy$$

$$f = (f')' = (y+z')(w'+x'+z)(w+x'+y')$$

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Thanks,..

See you next week (ISA),...