Lecture (02)

Number Systems and Conversion (2)

By:

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Arithmetic operations

```
  101101
 +100111
 -------
 1010100
```

```
  1010001
 +1000101
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 10010110
```

•

```
  101101
 -100111
 -------
  000110
```

```
  1010001
- 1000101
---------------
  0001100
```



```
d:     1011
     × 101
     ------
       1011
      0000
     1011
     ------
     110111
```

Complements of a number

- Complements are used in digital computers to **simplify the subtraction operation** and for logical manipulation.
- leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements
  - radix complement
  - diminished radix complement

1. Diminished Radix Complement

- Given a number *N* in base *r* having *n* digits,
  the (*r* - 1)'s complement of *N*
- diminished radix complement, is defined as $(r^n - 1) - N$

- For decimal numbers, $r = 10$ and $r - 1 = 9$,
- so the 9's complement of *N* is $(10^n - 1) - N$.
- 10^n represents a number that consists of a single 1 followed by *n* 0's.

More explanation:

- *Decimal r=10*
- *r-1=9 →9's complement*
- *n* = 4
- $10^4 = 10000$
- $10^4-1=9999$
- 9's complement of a decimal number is obtained by subtracting each digit from 9

decimal 9's complement examples:

- The 9's complement of 546700 is 999999 - 546700 = 453299.
- The 9's complement of 012398 is 999999 - 012398 = 987601.

Binary numbers

- $r = 2$
- $r - 1 = 1$, → 1's complement
- so the 1's complement of N is $(2^n - 1) - N$
- 2^n is represented by a binary number that consists of a 1 followed by n 0's.
- $2^n - 1$ is a binary number represented by n 1's

- Example
- $n = 4$,
- $2^4 = (10000)_2$
- $2^4 - 1 = (1111)_2$.
- the 1's complement of a binary number is obtained by subtracting each digit from 1.
- subtracting binary digits from 1, have either 1 - 0 = 1 or 1 - 1 = 0,
- which causes the bit to change from 0 to 1 or from 1 to 0, respectively

the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.

Examples

- The 1’s complement of 1011000 is 0100111.
- The 1’s complement of 0101101 is 1010010.

Octal and hexadecimal systems

- The (*r* - 1)>s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

2. Radix Complement

- The *r*'s complement of an *n*-digit number *N* in base *r* is defined as $r^n - N$ for $N > 0$

  and = 0 for $N = 0$.
- Comparing with the $(r - 1)$'s complement, we note that the r's complement is obtained by adding 1 to the $(r - 1)$'s complement:

$$r^n - N = [(r^n - 1) - N] + 1.$$

Examples

- 9's complement of 2389 → 9999-2389=7610
- 10's complement of 2389 → 10000-2389=7611

  or → 7610 + 1=7611

- The 1's complement of 101100 → 010011
- The 2's complement of 101100 → 010011 +1=01100

  or→ 1000000- 101100 = 10100
- the 10's complement of decimal 2389 is 7610 + 1 = 7611
- The 2's complement of binary 101100 is 010011 + 1 = 010100

Shortcut:

- 10's complement of *N*, can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.

- the 10's complement of 012398 is

  987602
- the 10's complement of 246700 is

  753300

Shortcut

- the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.

- the 2's complement of 1101100 is

  0010100
- the 2's complement of 0110111 is

  1001001

Number	1’s complement	2’s complement
0101	1010	1011
0110	1001	1010
0111	1000	1001

The complement of the complement restores the number to its original value .

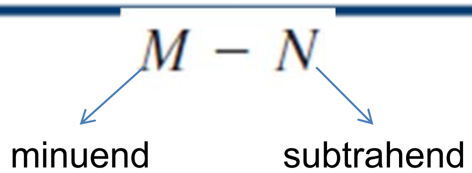
r’s complement of N is $r^n - N$,

- the complement of the complement is
- $r^n - (r^n - N) = N$

Subtraction with Complements

- The direct method of subtraction uses the borrow concept
- borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit.
- Works well with paper and pencil.
- For digital systems, we complements method

<u>Method 1</u>



- Add the minuend *M* to the *r*'s complement of the subtrahend *N*.

$$M + (r^n - N) = M - N + r^n.$$

- If *M* > *N*, the sum will produce an end carry *r n*, which can be discarded
- If *M* < *N*, the sum does not produce an end carry and is equal to r^n - (*N* - *M*), which is the *r*'s complement of (*N* - *M*). To obtain the answer in a familiar form, take the *r*'s complement of the sum and place a negative sign in front

Example 01

Using 10's complement, subtract 72532 − 3250.

Note that *M* has five digits and *N* has only four digits. Both numbers must have the same number of digits, so we write *N* as 03250

72532 − 3250.

$M \geq N$

$$
\begin{array}{rr}
M = & 72532 \\
\text{10's complement of } N = + & 96750 \\
\text{Sum} = & 169282 \\
\text{Discard end carry } 10^5 = - & \underline{100000} \\
\textit{Answer} = & 69282
\end{array}
$$

Example 02

Using 10's complement, subtract 3250 − 72532.

3250 − 72532.

$3250 < 72532$, the result is negative.

Therefore, the answer is −(10's complement of 30718)

$$
\begin{aligned}
M &= \quad 03250 \\
\text{10's complement of } N &= +\underline{27468} \\
\text{Sum} &= \quad 30718
\end{aligned}
$$

-ve 10's complement of sum = - 69282

Example 03

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

- $X = 1010100$; $Y = 1000011$,

  X – Y, and X > Y

$$
\begin{aligned}
X &= \quad 1010100 \\
\text{2's complement of } Y &= +\ 0111101 \\
\text{Sum} &= \quad 10010001 \\
\text{Discard end carry } 2^7 &= -\ \underline{10000000} \\
\textit{Answer}: X - Y &= \quad 0010001
\end{aligned}
$$

- $X = 1010100$; $Y = 1000011$,

Y – X, and Y > X

$$\begin{aligned} Y &= \quad 1000011 \\ \text{2's complement of } X &= + \ \underline{0101100} \\ \text{Sum} &= \quad 1101111 \end{aligned}$$

$$-(\text{2's complement of } 1101111) = \quad -0010001$$

Method 2

- Sub traction can be done by means of the (r - 1)'s complement.
- (r - 1)'s complement is one less than the r's complement.
- So we add one to the result, called *end-around carry.*

Example 04

Repeat previous Example , but this time using 1's complement.

$X = 1010100; \; Y = 1000011,$

- $X - Y = 1010100 - 1000011$

$$\begin{aligned} X &= 1010100 \\ \text{1's complement of } Y &= +\ \underline{0111100} \\ \text{Sum} &= 10010000 \\ \text{End-around carry} &= +\ \underline{1} \\ \textit{Answer}: X - Y &= 0010001 \end{aligned}$$

$$Y - X = 1000011 - 1010100$$

$$\begin{array}{rr} Y = & 1000011 \\ \text{1's complement of } X = & +\ \underline{0101011} \\ \text{Sum} = & 1101110 \end{array}$$

, the answer is $Y - X = -(\text{1's complement of } 1101110)$

$-0010001.$