Introduction To Engineering – Tutorial - 02

#	Student ID	Student Name	Grade (10)
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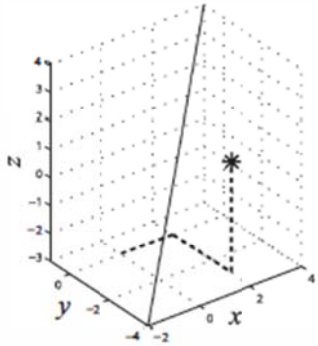
Q1

The parametric equations of a line in space are: $x = x_0 + at$, $y = y_0 + bt$, and $z = z_0 + ct$. The distance d from a point $A\ (x_A, y_A, z_A)$ to the line can be calculated by:

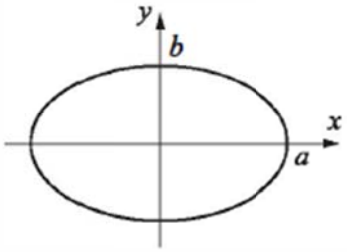
$$d = d_{A0} \sin\left[\operatorname{acos}\left(\frac{(x_A - x_0)a + (y_A - y_0)b + (z_A - z_0)c}{d_{A0}\sqrt{a^2 + b^2 + c^2}}\right)\right]$$

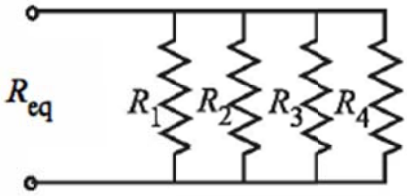


where $d_{A0} = \sqrt{(x_A - x_0)^2 + (y_A - y_0)^2 + (z_A - z_0)^2}$.

Determine the distance of the point A $(2, -3, 1)$ from the line $x = -4 + 0.6t$, $y = -2 + 0.5t$, and $z = -3 + 0.7t$. First define the variables x_0 , y_0 , z_0 , a, b, and c, then use the variable (and the coordinates of point A) to calculate the variable d_{A0} , and finally calculate d.

Sol 1

Q2	The circumference of an ellipse can be approximated by: $C = \pi[3(a+b) - \sqrt{(3a+b)(a+3b)}]$ Calculate the circumference of an ellipse with $a = 16$ in. and $b = 11$ in. 
Sol 2	

Q3	The equivalent resistance, R_{eq}, of four resistors, R_1, R_2, R_3, and R_4, that are connected in parallel is given by: $$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}}$$  Calculate R_{eq} if $R_1 = 120\Omega$, $R_2 = 220\Omega$, $R_3 = 75\Omega$, and $R_4 = 130\Omega$.
Sol 3	

Q4	Radioactive decay of carbon-14 is used for estimating the age of organic material. The decay is modeled with the exponential function $f(t) = f(0)e^{kt}$, where t is time, $f(0)$ is the amount of material at $t = 0$, $f(t)$ is the amount of material at time t, and k is a constant. Carbon-14 has a half-life of approximately 5,730 years. A sample taken from the ancient footprints of Acahualinca in Nicaragua shows that 77.45% of the initial ($t = 0$) carbon-14 is present. Determine the estimated age of the footprint. Solve the problem by writing a program in a script file. The program first determines the constant k, then calculates t for $f(t) = 0.7745f(0)$, and finally rounds the answer to the nearest year.

Sol 4	`clear, clc`
	`k=log(0.5)/5730;`
	`Age=round(log(.7745)/k)`
	
	`Age =`
	`2112`

Q5	The value B of a principal P that is deposited in a saving account with a fixed annual interest rate r after n years can be calculated by the formula: $$B = P\left(1+\frac{r}{m}\right)^{nm}$$ where m is the number of times that the interest is compounded annually. Consider a \$80,000 deposit for 5 years. Determine how much more money will be earned if the interest is compounded daily instead of yearly.
Sol 5	

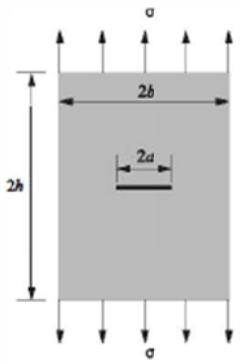
```
clear, clc
%an interest rate of 10% is assumed
P=80000; n=5; r=.1;
bonus=P*(1+ r/365)^(365*n) - P*(1+ r)^n
```

```
bonus =

  3.0479e+003

>>
```

Q6

The stress intensity factor K predicts the stress state (stress intensity) near a crack tip. For a plate with a crack and loading shown in the figure, K is given by:

$$K = \sigma\sqrt{\pi a}\left[\frac{1-\frac{a}{2b}+0.326\left(\frac{a}{b}\right)^2}{\sqrt{1-\frac{a}{b}}}\right]$$



Determine K for the case where $\sigma = 12000$ psi, $h = 5$ in., and $b = 4$ in., and $a = 1.5$ in.

Sol 6

```
clear, clc
sigma=12000; h=5; b=4; a=1.5;
K=sigma*sqrt(pi*a)*(1-a/(2*b)+0.326*(a/b)^2)/sqrt(1-a/b)
```

```
K =

  2.8283e+004
```

Q7	Stirling's approximation for large factorials is given by: $$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$ Use the formula for calculating 20!. Compare the result with the true value obtained with MATLAB's built-in function `factorial` by calculating the error ($Error = (TrueVal - ApproxVal)/\ TrueVal$).

Sol 7	`clear, clc`
	`factorial_20=sqrt(2*pi*20)*(20/exp(1))^20`
	`error=(factorial(20)-factorial_20)/factorial(20)`
	
	`factorial_20 =`
	`2.4228e+018`
	
	`error =`
	`0.0042`