

Logic Design – Tutorial

08

#	Student ID	Student Name	Grade (10)
-			



Faculty of Engineering

Q1	For each of the following functions, find all of the prime implicants, using the Quine-McCluskey method. (a) $f(a, b, c, d) = \Sigma m(1, 5, 7, 9, 11, 12, 14, 15)$																																								
Sol 1	<table><tr><td><u>1</u></td><td>0001✓</td><td>1, 5</td><td>0-01</td><td>a'c'd</td></tr><tr><td>5</td><td>0101✓</td><td><u>1, 9</u></td><td>-001</td><td>b'c'd</td></tr><tr><td>9</td><td>1001✓</td><td>5, 7</td><td>01-1</td><td>a'bd</td></tr><tr><td><u>12</u></td><td><u>1100</u>✓</td><td>9, 11</td><td>10-1</td><td>ab'd</td></tr><tr><td>7</td><td>0111✓</td><td><u>12, 14</u></td><td>11-0</td><td>abd'</td></tr><tr><td>11</td><td>1011✓</td><td>7, 15</td><td>-111</td><td>bcd</td></tr><tr><td><u>14</u></td><td><u>1110</u>✓</td><td>11, 15</td><td>1-11</td><td>acd</td></tr><tr><td>15</td><td>1111✓</td><td>14, 15</td><td>111-</td><td>abc</td></tr></table> <p>Prime implicants: $a'c'd, b'c'd, a'bd, ab'd, abd', bcd, acd, abc$</p>	<u>1</u>	0001✓	1, 5	0-01	a'c'd	5	0101✓	<u>1, 9</u>	-001	b'c'd	9	1001✓	5, 7	01-1	a'bd	<u>12</u>	<u>1100</u> ✓	9, 11	10-1	ab'd	7	0111✓	<u>12, 14</u>	11-0	abd'	11	1011✓	7, 15	-111	bcd	<u>14</u>	<u>1110</u> ✓	11, 15	1-11	acd	15	1111✓	14, 15	111-	abc
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Faculty of Engineering

Q2	<p>For this function, find a minimum sum-of-products solution, using the Quine- McCluskey method.</p> $f(a, b, c, d) = \sum m(1, 3, 4, 5, 6, 7, 10, 12, 13) + \sum d(2, 9, 15)$																																																									
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		1	3	4	5	6	7	10	12	13
1, 3, 5, 7	a'd	x	x		x		x			
1, 5, 9, 13	c'd	x			x					x
2, 3, 6, 7	a'c		x			x	x			
4, 5, 6, 7	a'b			x	x	x	x			
4, 5, 12, 13	bc'			x	x				x	x
5, 7, 13, 15	bd				x		x			x
2, 10	b'cd'							x		

$$f = \underline{bc'} + \underline{b'cd'} + a'd + a'b$$

$$f = \underline{bc'} + \underline{b'cd'} + c'd + a'c$$

$$f = \underline{bc'} + \underline{b'cd'} + a'c + a'd$$

Q3	<p>For each of the following functions, find all of the prime implicants using the Quine-McCluskey method.</p> <p>Using a prime implicant chart, find <i>all</i> minimum sum-of-products solutions for each of the functions given in</p> <p>(a) $f(a, b, c, d) = \sum m(0, 3, 4, 5, 7, 9, 11, 13)$</p>																																								
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		0	3	4	5	7	9	11	13
0, 4	a'c'd'	⊗		×					
4, 5	a'bc'			×	×				
3, 7	a'cd		×			×			
3, 11	b'cd		×					×	
5, 7	a'bd				×	×			
5, 13	bc'd				×				×
9, 11	ab'd						×	×	
9, 13	ac'd							×	×

$f = a'c'd' + a'cd + ab'd + bc'd$
 $f = a'c'd' + ac'd + a'bd + b'cd$

Q4	(a) Using the Quine-McCluskey, method find all prime implicants of $f(A, B, C, D) = \sum m(1, 3, 5, 6, 8, 9, 12, 14, 15) + \sum d(4, 10, 13)$. Identify all essential prime implicants and find all minimum sum-of-products expressions. (b) Repeat Part (a) for f' .																																																																																																																																																																																																																																																																																								
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4	0-00*	4	0100√	2, 10	-010*	10	1010√	10, 11	101-*	7	0111*			11	1011√			13	1101*				0	2	7	11	8	9	12	14	15	(7)			⊗							(13)										(0, 2)	×	×								(0, 4)	×									(2, 10)			×							(10, 11)				⊗					
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Faculty of Engineering

Q5	Find all prime implicants of the following function, and then find all minimum solutions using Petrick's method: $F(A, B, C, D) = \sum m(7, 12, 14, 15) + \sum d(1, 3, 5, 8, 10, 11, 13)$
Sol 5	Prime implicants: $AC, AD', AB, CD, BD, A'D$ Minimum solutions: $(AD' + CD); (AD' + BD);$ $(AB + BD); (AB + CD); (AB + A'D)$