Logic Design – Tutorial 07

#	Student ID	Student Name	Grade (10)
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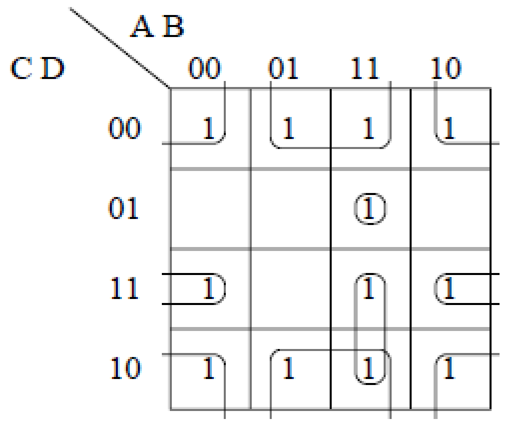
Q1	(a) Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.) $$F(A,B,C,D) = BD' + B'CD + ABC + ABC'D + B'D'$$ (b) Find the minimum sum of products. (c) Find the minimum product of sums.

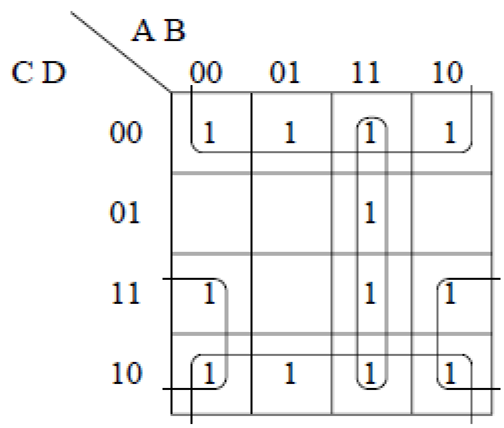
Sol 1

(a)



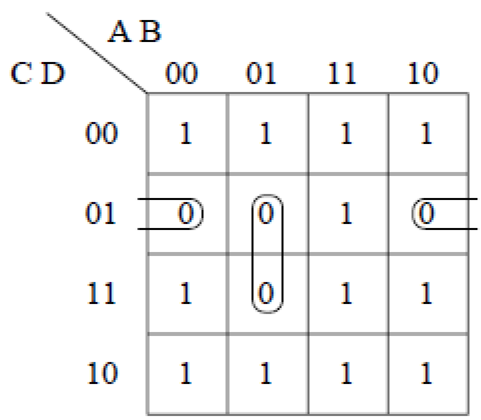
$F = BD' + B'CD + ABC + ABC'D + B'$

(b)



$F = D' + B'C + AB$

(c)



$F = (A + B' + D')(B + C + D')$

Q2

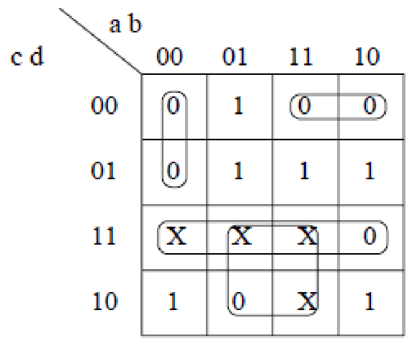
Find the minimum sum of products and the minimum product of sums for each function:

(a) $f(a, b, c, d) = \Pi M(0, 1, 6, 8, 11, 12) \cdot \Pi D(3, 7, 14, 15)$

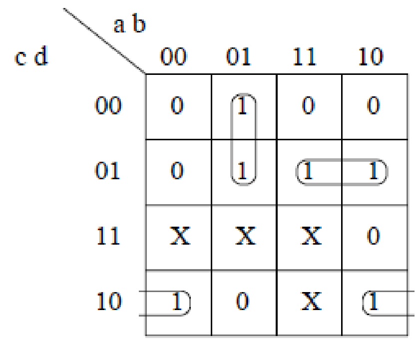
(b) $f(a, b, c, d) = \Sigma m(1, 3, 4, 11) + \Sigma d(2, 7, 8, 12, 14, 15)$

Sol 2

(a)

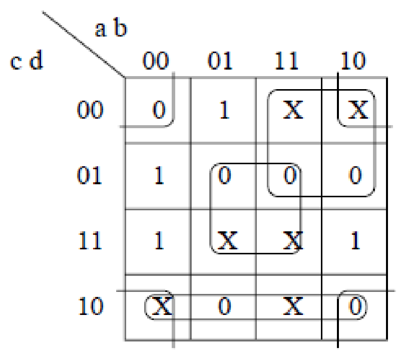


f = (c'+ d')(b'+ c')(a + b + c)(a'+ c + d)



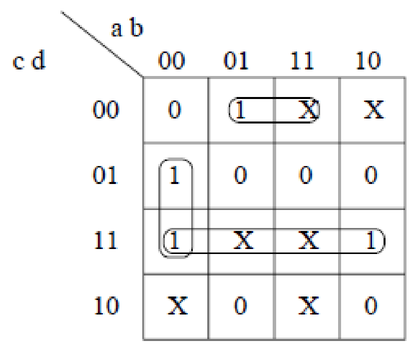
f = a'bc' + ac'd + b'cd'

(b)



f = (a'+ c)(b'+ d')(b + d)(c'+ d)

Alt: f = (a'+ c)(b'+ d')(b + d)(b'+ c')



f = a'b'd + bc'd' + cd

Q3

Given $F = AB'D' + A'B + A'C + CD$.

(a) Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebric notation).

(b) Use a Karnaugh map to find the minimum sum-of-products form for F'.

(c) Find the minimum product of sums for F.

Sol 3

(a)

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

F = A B'D' + A'B + A'C + C D

$F = \prod M(0, 1, 9, 12, 13, 14) = (A + B + C + D)$
$(A + B + C + D')\ (A' + B' + C + D)$
$(A' + B' + C + D')\ (A' + B' + C' + D)$
$(A' + B + C + D')$

(b)

CD \ AB	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	0	0	0	0
10	0	0	1	0

F' = A'B'C' + A B D' + A C'D

(c)

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

F = (A'+ B'+ D)(A + B + C)(A'+ C + D')

Q4

Find the minimum sum-of-products expressions for each of these functions.

(a) $f_1(A, B, C) = m_1 + m_2 + m_5 + m_7$

(b) $f_2(d, e, f) = \Sigma\, m(1, 5, 6, 7)$

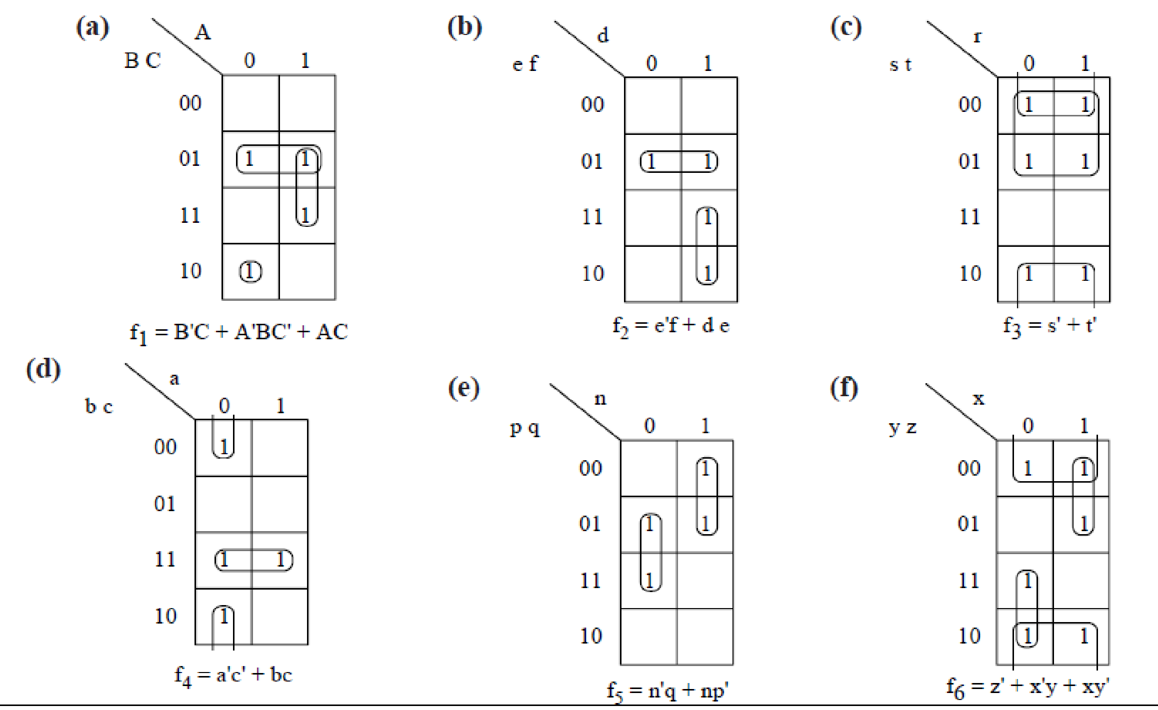
(c) $f_3(r, s, t) = rs' + r's' + st'$

(d) $f_4(a, b, c) = m_0 + m_2 + m_3 + m_7$

(e) $f_5(n, p, q) = \Sigma\, m(1, 3, 4, 5)$

(f) $f_6(x, y, z) = M_1 M_7$

Sol 4



Q5

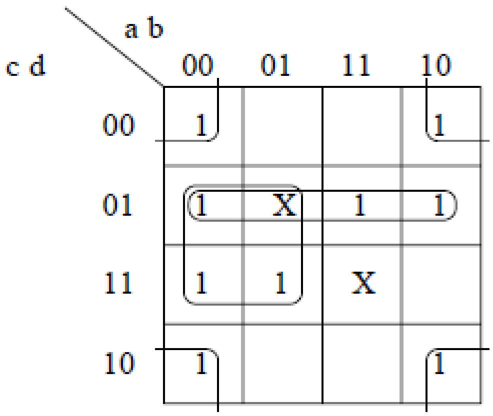
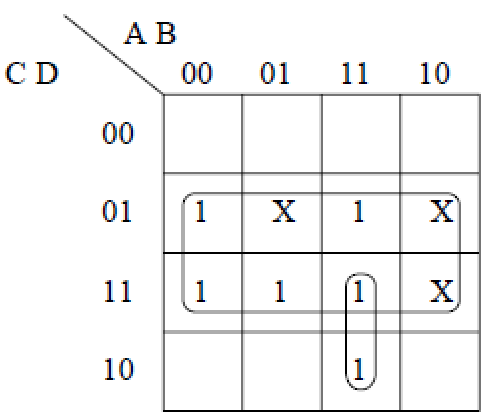
Simplify the following expression first by using a map and then by using Boolean algebra. Use the map as a guide to determine which theorems to apply to which terms for the algebraic simplification.

$$F = a'b'c' + a'c'd + bcd + abc + ab'$$

Sol 5

cd \ ab	00	01	11	10
00	1			1
01	1	1		1
11		1	1	1
10			1	1

$$\begin{aligned}
F &= a'b'c' + a'c'd + bcd + abc + ab' \\
&= (a'b'c' + ab') + a'c'd + bcd + (abc + ab') \\
&= (a'c' + a)b' + (a'c'd + bcd) + a(bc + b') \\
&= (c' + a)b' + (a'c'd + bcd + a'bd) + a(c + b') \\
&= (b'c' + a'bd + a'c'd) + (bcd + a'bd + ac) + ab' \\
&= (b'c' + ac + ab') + a'bd \\
&= b'c' + ac + a'bd
\end{aligned}$$

Q6	A logic *circuit* realizes the function $F(a, b, c, d) = a'b' + a'cd + ac'd + ab'd'$. Assuming that $a = c$ never occurs when $b = d = 1$, find a simplified expression for F.
Sol 6	 F = b'd' + a'd + c'd Notice that *abcd* = 0101 and 1111 never occur, so minterms 5 and 15 are don't cares.
Q7	Assuming that the inputs $ABCD$ = 0101, $ABCD$ = 1001, $ABCD$ = 1011 never occur, find a simplified expression for $$F = A'BC'D + A'B'D + A'CD + ABD + ABC$$
Sol 7	 F = D + AB C

Q 8

A logic circuit realizing the function *f* has four inputs *a, b, c, d*. The three inputs *a, b*, and *c* are the binary representation of the digits 0 through 7 with *a* being the most significant bit. The input *d* is an odd-parity bit; that is, the value of *d* is such that *a, b, c*, and *d* always contains an odd number of 1's. (For example, the digit 1 is represented by *abc* = 001 and *d* = 0, and the digit 3 is represented by *abcd* = 0111.) The function *f* has value 1 if the input digit is a prime number. (A number is prime if it is divisible only by itself and 1; 1 is considered to be prime, and 0 is not.)

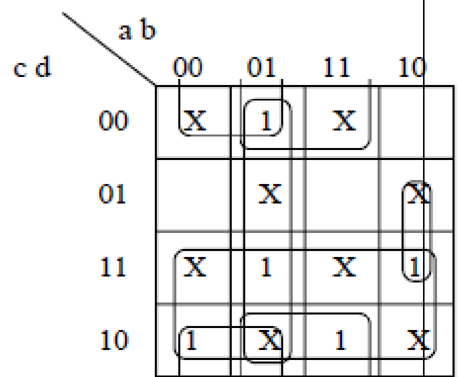
(a) Draw a Karnaugh map for *f*.
(b) Find all prime implicants of *f*.
(c) Find all minimum sum of products for *f*.
(d) Find all prime implicants of *f'*.
(e) Find all minimum product of sums for *f*.

So 18

(a)

cd \ ab	00	01	11	10
00	X	1	X	
01		X		X
11	X	1	X	1
10	1	X	1	X

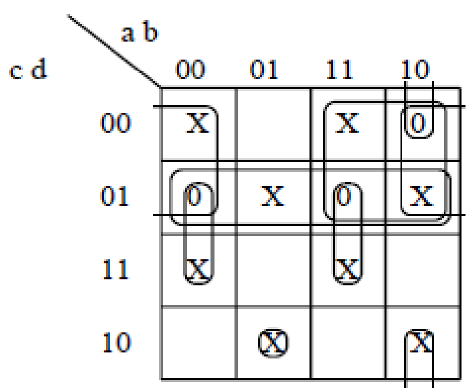
(b) & (c)



PIs: bd', a'b, a'd', c, ab'd

f = b d' + c or
= a'b + c or
= a'd' + c

(d) & (e)



PIs: (c + d'), (a'+ c), (b + c), (a + b +d'),
(a + b'+c'+d), (a'+b'+d'), (a'+b+d)

f = (c + d')(a'+ c) or
= (b + c)(c + d') or
= (b + c)(a'+ c)