Logic Design – Tutorial 06

#	Student ID	Student Name	Grade (10)
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Q1

Work parts (a) through (d) with the given truth table.

A	B	C	F_1	F_2	F_3	F_4
0	0	0	1	1	0	1
0	0	1	X	0	0	0
0	1	0	0	1	X	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	X	0	1	0
1	1	0	0	X	X	X
1	1	1	1	X	1	X

(a) Find the simplest expression for F_1, and specify the values for the don't-cares that lead to this expression.
(b) Repeat for F_2.
(c) Repeat for F_3.
(d) Repeat for F_4.

Sol 1

If don't cares are changed to (1, 1), respectively,

$$F_1 = A'B'C' + ABC + A'B'C + AB'C$$
$$= A'B' + AC$$

If don't cares are changed to (1, 0), respectively

$$F_2 = A'B'C' + A'BC' + AB'C' + ABC' = C'$$

If don't cares are changed to (1, 1), respectively

$$F_3 = (A + B + C)(A + B + C') = A + B$$

If don't cares are changed to (0, 1), respectively

$$F_4 = A'B'C' + A'BC + AB'C' + ABC$$
$$= B'C' + BC$$

Q2

A combinational circuit has four inputs (A, B, C, D) and three outputs (X, Y, Z). XYZ represents a binary number whose value equals the number of 1's at the input. For example if $ABCD = 1011$, $XYZ = 011$.

(a) Find the minterm expansions for X, Y, and Z.

(b) Find the maxterm expansions for Y and Z.

Sol 2

A B C D	1's	X Y Z
0 0 0 0	0	0 0 0
0 0 0 1	1	0 0 1
0 0 1 0	1	0 0 1
0 0 1 1	2	0 1 0
0 1 0 0	1	0 0 1
0 1 0 1	2	0 1 0
0 1 1 0	2	0 1 0
0 1 1 1	3	0 1 1
1 0 0 0	1	0 0 1
1 0 0 1	2	0 1 0
1 0 1 0	2	0 1 0
1 0 1 1	3	0 1 1
1 1 0 0	2	0 1 0
1 1 0 1	3	0 1 1
1 1 1 0	3	0 1 1
1 1 1 1	4	1 0 0

$$X = ABCD$$

$$Y = A'B'CD + A'BC'D + A'BCD' + A'BCD + AB'C'D + AB'CD' + AB'CD + ABC'D' + ABC'D + ABCD'$$

$$Z = A'B'C'D + A'B'CD' + A'BC'D' + A'BCD + AB'C'D' + AB'CD + ABC'D + ABCD'$$

$$Y = (A + B + C + D)\,(A + B + C + D')$$
$$(A + B + C' + D)\,(A + B' + C + D)$$
$$(A' + B + C + D)\,(A' + B' + C' + D')$$

$$Z = (A + B + C + D)\,(A + B' + C + D')$$
$$(A + B' + C' + D)\,(A' + B + C + D')$$
$$(A' + B + C' + D)\,(A' + B' + C + D)$$
$$(A' + B' + C' + D')$$

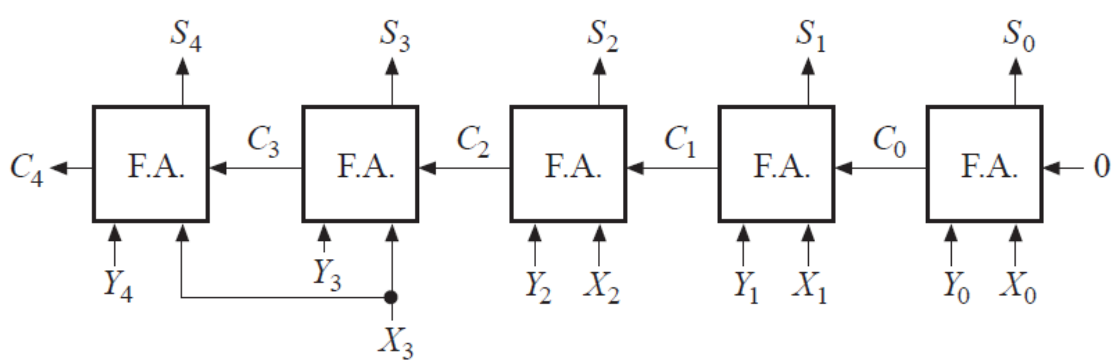
Q3

Design a circuit which will add a 4-bit binary number to a 5-bit binary number. Use five full adders. Assume negative numbers are represented in 2's complement. (*Hint:* How do you make a 4-bit binary number into a 5-bit binary number, without making a negative number positive or a positive number negative? Try writing

down the representation for −3 as a 3-bit 2's complement number, a 4-bit 2's complement number, and a 5-bit 2's complement number. Recall that one way to find the 2's complement of a binary number is to complement *all* bits to the left of the first 1.)

Sol 3

Notice that the sign bit X_3 of the 4-bit number is extended to the leftmost full adder as well.



Q4

(a) Write the switching function $f(x, y) = x + y$ as a sum of minterms and as a product of maxterms.

(b) Consider the Boolean algebra of four elements {0, 1, *a*, *b*} specified by the following operation tables and the Boolean function $f(x, y) = ax + by$ where *a* and *b* are two of the elements in the Boolean algebra. Write $f(x, y)$ in a sum-of-minterms form.

(c) Write the Boolean function of part (b) in a product-of-maxterms form.

(d) Give a table of combinations for the Boolean function of Part (b). (*Note*: The table of combinations has 16 rows, not just 4.)

(e) Which four rows of the table of combinations completely specify the function of Part (b)? Verify your answer.

	′
0	1
1	0
a	*b*
b	*a*

+	0	1	*a*	*b*
0	0	1	*a*	*b*
1	1	1	1	1
a	*a*	1	*a*	1
b	*b*	1	1	*b*

•	0	1	*a*	*b*
0	0	0	0	0
1	0	1	*a*	*b*
a	0	*a*	*a*	0
b	0	*b*	0	*b*

Sol 4	(a) $f = x(y+y')+y(x+x') = xy+xy'+x'y$ (sum-of-minterms) $f = x+y$ already in product-of-maxterms form (b) $f = ax+by = ax(y+y')+by(x+x')$ $= axy+axy'+bxy+bx'y = (a+b)xy+axy'+bx'y$ $= xy+axy'+bx'y$ (c) $f' = (a'+x')(b'+y') = (b+x')(a+y')$ $= ab+ax'+by'+x'y' = ax'(y+y')+by'(x+x')+x'y'$ $= ax'y+ax'y'+by'x+by'x'+x'y'$ $= ax'y+by'x+x'y'(a+b+1) = ax'y+by'x+x'y'$ so $f = (a'+x+y')(b'+x'+y)(x+y)$ $= (b+x+y')(a+x'+y)(x+y)$ Alternatively, $f = ax+by = (a+by)(x+by) = (a+b)(a+y)(x+b)(x+y)$ $= (a+xx'+y)(b+yy'+x)(x+y)$ $= (a+x+y)(a+x'+y)(b+x+y)(b+x+y')(x+y)$ $= [(a+x+y)(b+x+y)(x+y)](a+x'+y)(b+x+y')$ $= (ab+x+y)(a+x'+y)(b+x+y')$ $= (x+y)(a+x'+y)(b+x+y')$

(d)

$x\ y$	f	$x\ y$	f
0 0	0	a 0	a
0 1	b	a 1	1
0 a	0	a a	a
0 b	b	a b	1
1 0	a	b 0	0
1 1	1	b 1	b
1 a	a	b a	0
1 b	1	b b	b

(e)

f(x,y) is completely specified by the coefficients of the minterms in the sum of minterms expression. These coefficients are determined by the value of the function for xy = 00, 01, 10 and 11.