Lecture (07) Karnauph map

By:

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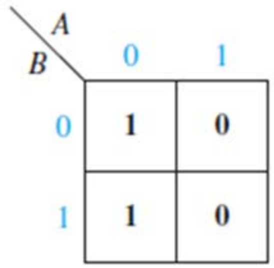
Two- and Three-Variable Karnaugh Maps

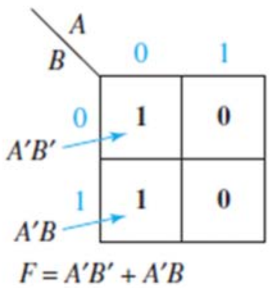
- Two variables karnaugh map
- The values of one variable are listed across the top of the map, and the values of the other variable are listed on the left side.
- Each square of the map corresponds to a pair of values for *A* and *B* as indicated.

A B	F
0 0	1
0 1	1
1 0	0
1 1	0



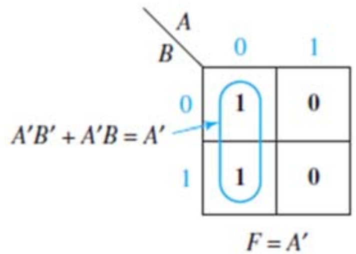
- We can read the minterms from the map
- 1's in square 00, 01 refers to minterms

A B	F
0 0	1
0 1	1
1 0	0
1 1	0



- Minterms in adjacent squares of the map can be combined since they differ in only one variable.
- Thus, *A'B'* and *A'B* combine to form *A'*

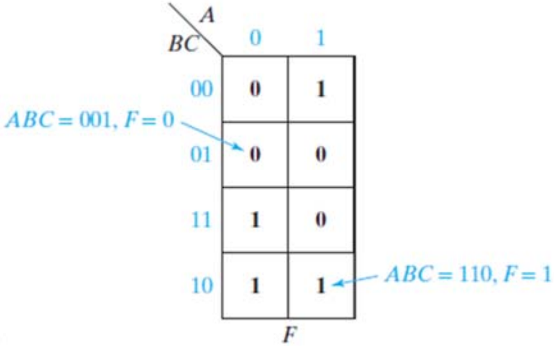
A B	F
0 0	1
0 1	1
1 0	0
1 1	0



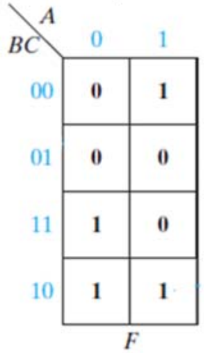
Three variables karnaugh map

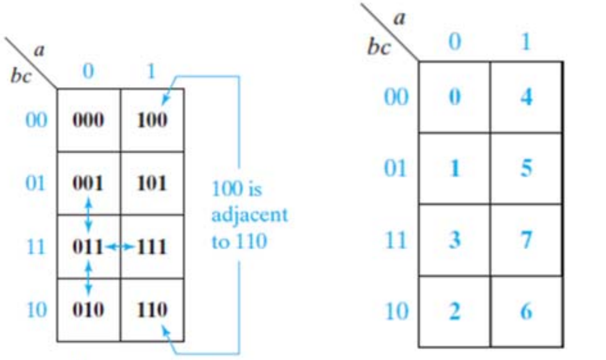
- The value of one variable (A) is listed across the top of the map, and the values of the other two
- variables (B, C) are listed along the side of the map

A B C	F
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0



- adjacent squares of the map differ in only one variable therefore can be combined using the uniting theorem $XY' + XY = X$
- the top and bottom rows of the map are defined to be adjacent

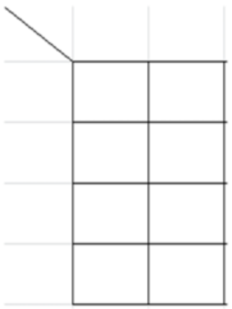


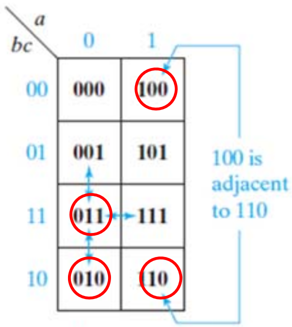


(a) Binary notation

(b) Decimal notation

- 011 ($a'bc$) is adjacent to the minterms with 010 ($a'bc'$), which it can be combined using the uniting theorem $XY' + XY = X$.



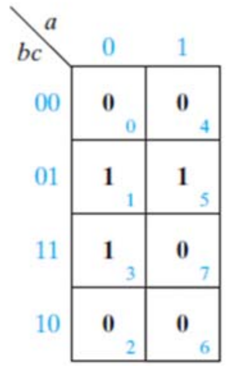


(a) Binary notation

Plotting minterms in karnaugh map

- minterm expansion of a function, it can be plotted on a map by placing 1's in the squares which correspond to minterms of the function and 0's in the remaining squares

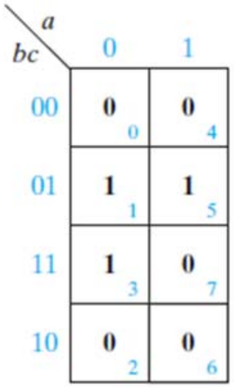
$$F(a, b, c) = m_1 + m_3 + m_5$$



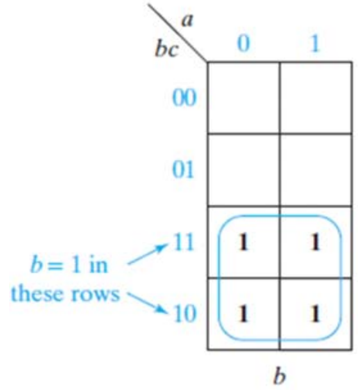
Plotting maxterms in karnaugh map

- the maxterms and then by filling in 0 the remaining squares with 1’s. Thus, $F(a, b, c) = M0M2M4M6M7$

Karnaugh Map of
$F(a, b, c) =$
$\Sigma m(1, 3, 5) =$
$\Pi M(0, 2, 4, 6, 7)$



Plotting products in karnaugh map



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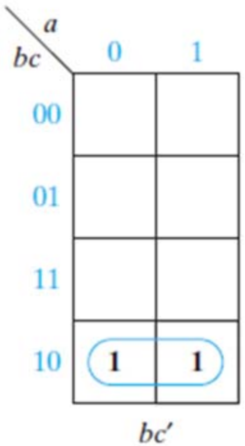

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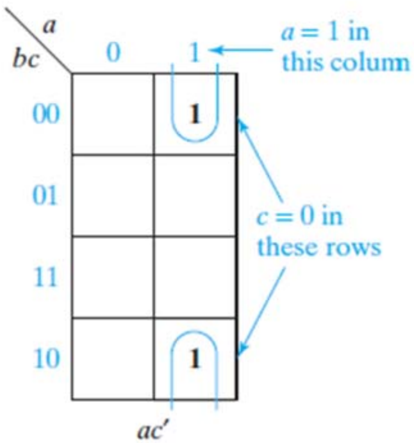
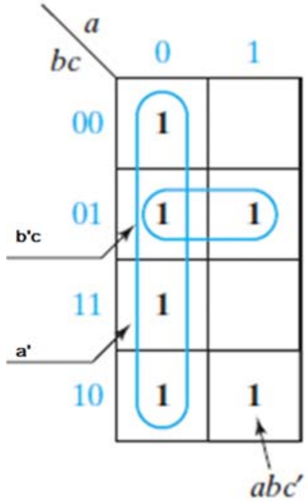


Plotting function algebraic expression in karnaugh map

- If a function is given in algebraic form
- If the algebraic expression is converted to sum-of-products form, then each product term can be plotted directly as a group of 1's on the map.

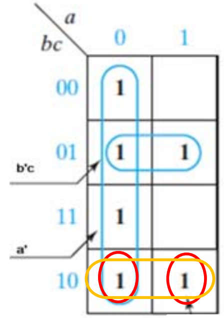
$$f(a, b, c) = abc' + b'c + a'$$



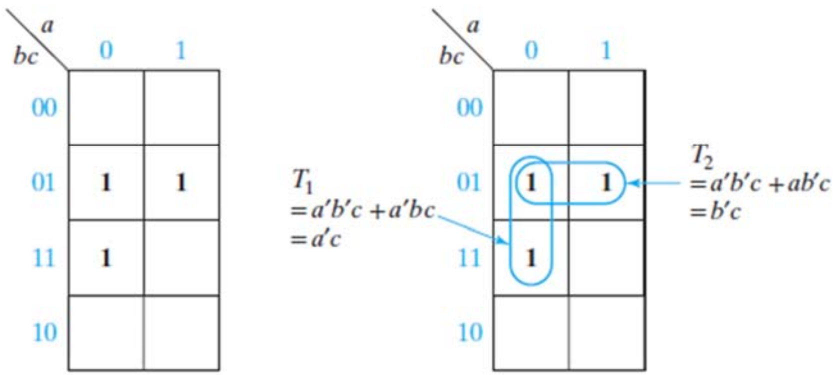
$$f(a, b, c) = abc' + b'c + a'$$

Simplifying fucntion using karnaugh map

- Terms in adjacent squares on the map differ in only one variable and can be combined using the uniting theorem *XY′* + *XY* = *X*
  - abc' and a'bc' cobine to form bc'
- F(a,b,c)= bc' + b'c + a'



- Another example $F = a'b'c + a'bc + ab'c$



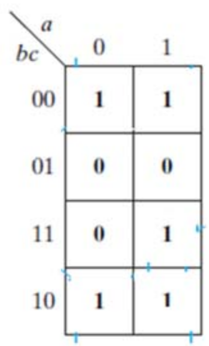
$F = \Sigma m(1, 3, 5)$

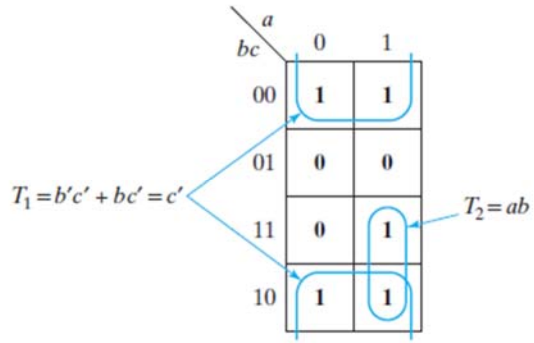
(a) Plot of minterms

$F = a'c + b'c$

(b) Simplified form of F

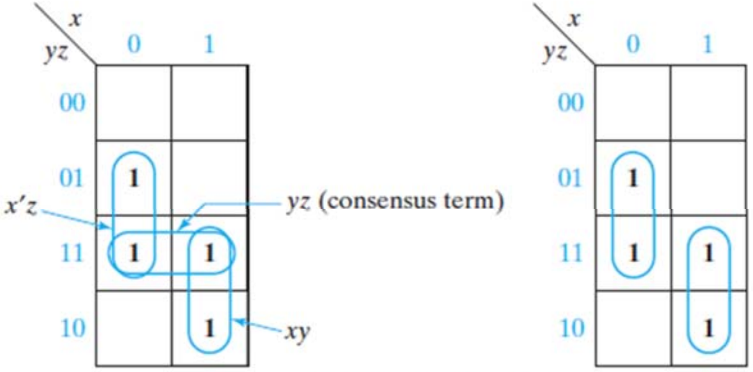
- Another example $F = a'b'c' + ab'c' + abc + a'bc' + abc'$





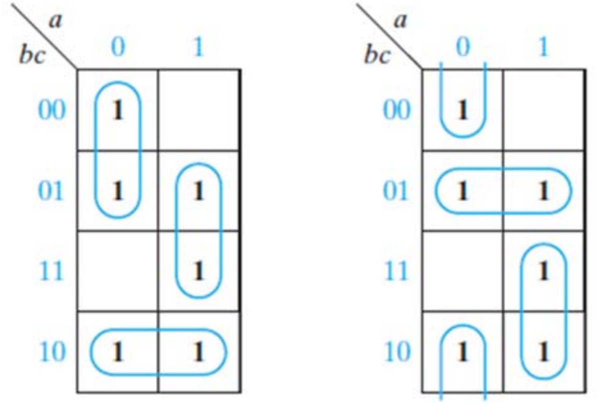
$$F = a'b'c' + ab'c' + abc + a'bc' + abc'$$

- Another example $XY + X'Z + YZ$



$xy + x'z + yz = xy + x'z$

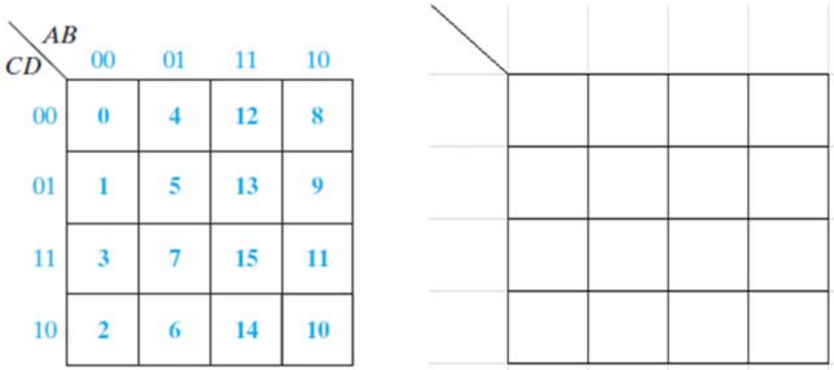
- Another example $F = \Sigma\, m(0, 1, 2, 5, 6, 7)$.



$F = a'b' + bc' + ac$ $\quad$ $F = a'c' + b'c + ab$

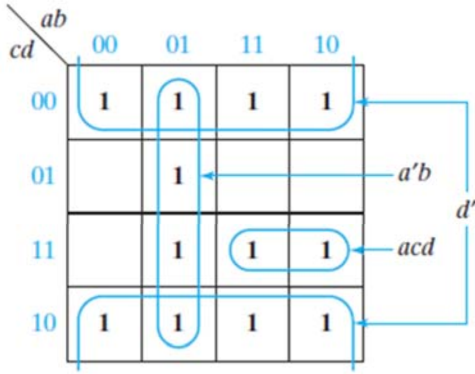
Four-Variable Karnaugh Maps

- Each minterm is located adjacent to the four terms with which it can combine



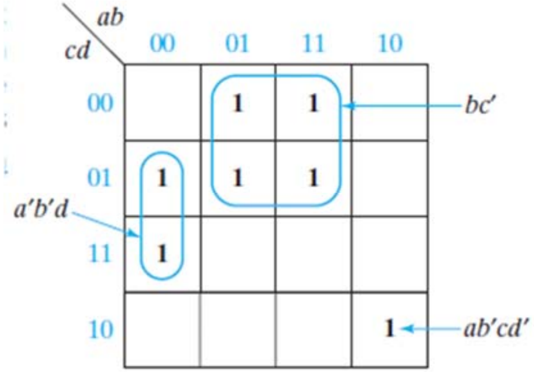
- $f(a, b, c, d) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15)$

Plot of
$acd + a'b + d'$



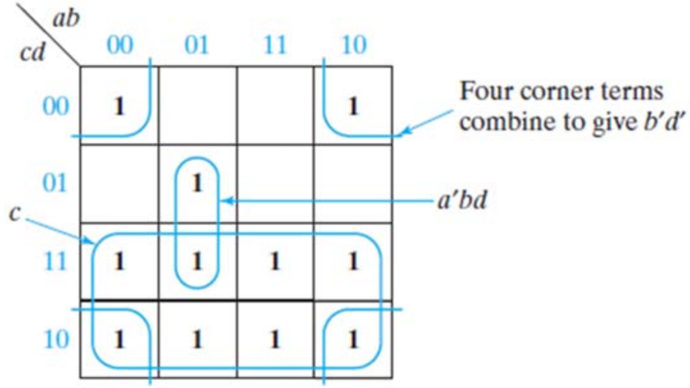
$f(a, b, c, d) = acd + a'b + d'$

$f_1 = \Sigma\, m(1, 3, 4, 5, 10, 12, 13)$



$= bc' + a'b'd + ab'cd'$

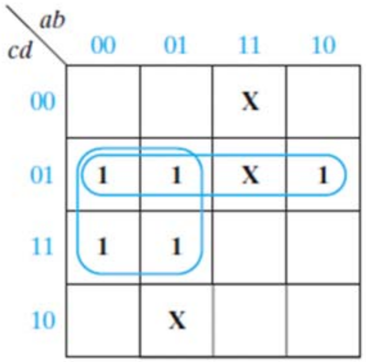
$f_2 = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$



$= c + b'd' + a'bd$

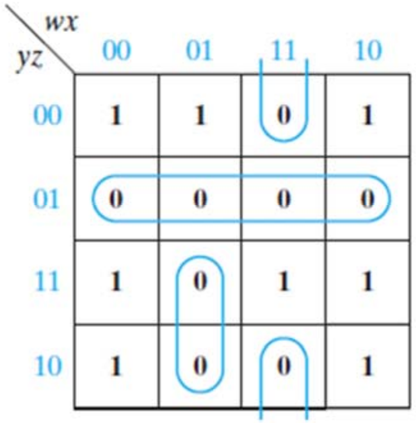
- The Karnaugh map method is easily extended to functions with don't-care terms

$f = \Sigma\, m(1, 3, 5, 7, 9) + \Sigma\, d(6, 12, 13)$



$= a'd + c'd$

- A minimum product of sums can also be obtained from the map. $f = x'z' + wyz + w'y'z' + x'y$

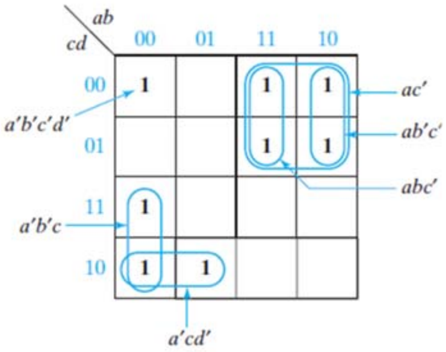


$f = (y + z')(w' + x' + z)(w + x' + y')$

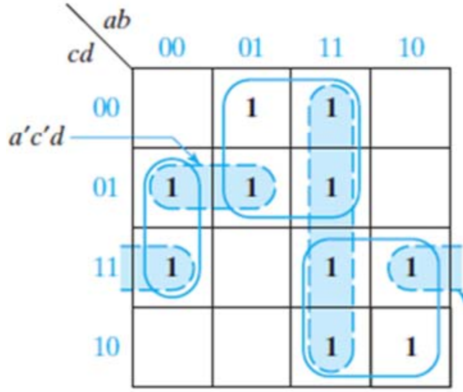
Determination of Minimum Expressions Using Essential Prime Implicants

- Any single 1 or any group of 1's which can be combined together on a map of the function *F* represents a product term which is called an *implicant* of *F*
- A product term implicant is called a *prime implicant* if it cannot be combined with another term to eliminate a variable

- a'b'c, a'cd', and ac' are prime implicants
- because they cannot be combined with other terms to eliminate a variable.
- a'b'c'd' is not a prime implicant because it can be combined with a'b'cd' or ab'c'd'.
- abc', nor ab'c'is a prime implicant because these terms can be combined together to form ac



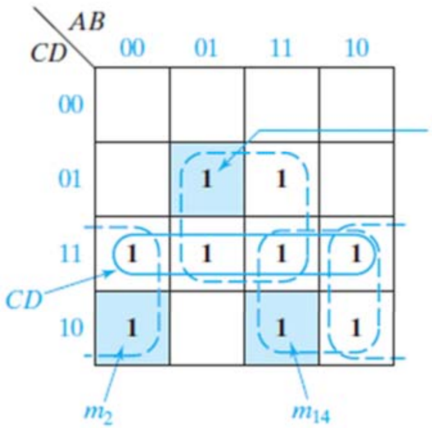
- The minimum sum-of-products expression for a function consists of some (but not necessarily all) of the prime implicants of a function.



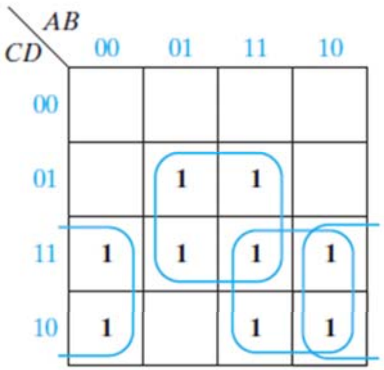
All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

Minimum solution: $F = a'b'd + bc' + ac$

- Systematic method to find minimum solution:

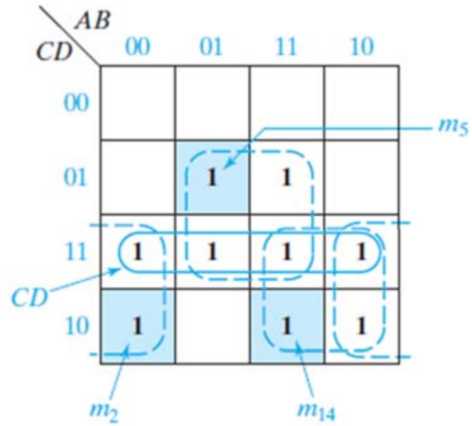


$f = CD + BD + B'C + AC$



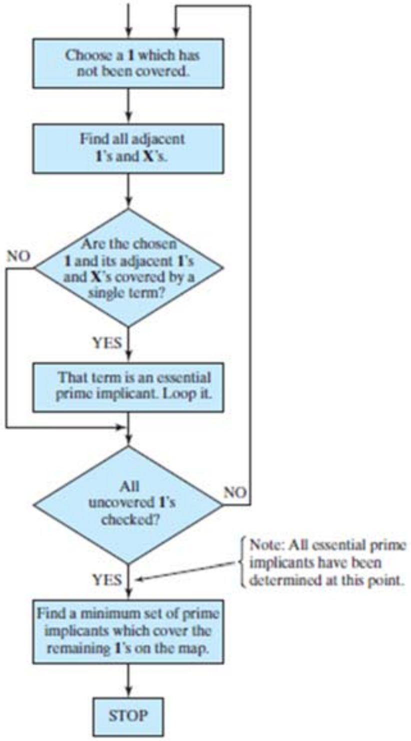
$f = BD + B'C + AC$

- If a minterm is covered by only one prime implicant, that prime implicant is said to be *essential*
- *B'C is* an essential prime implicant because *m*2 is not covered by any other prime implicant
- *BD is* an essential prime implicant because *m*5
- *CD* is *not* essential because each of the 1's in *CD* can be covered by another prime implicant



$f = CD + BD + B'C + AC$

- procedure used to obtain a minimum sum of products from a Karnaugh map:

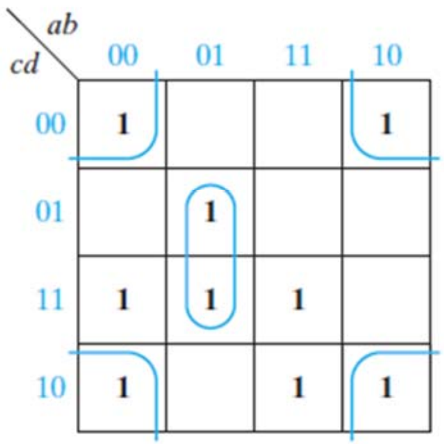


Example

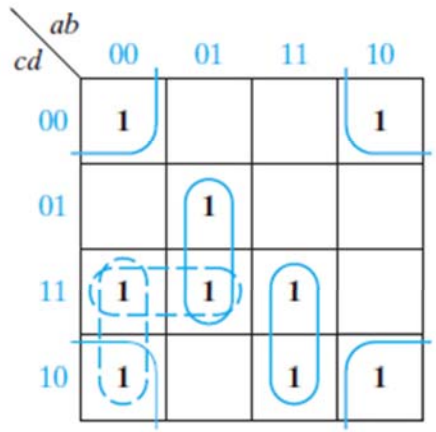
- Determine the minimum sum of products and minimum product of sums for

$$f = b'c'd' + bcd + acd' + a'b'c + a'bc'd$$

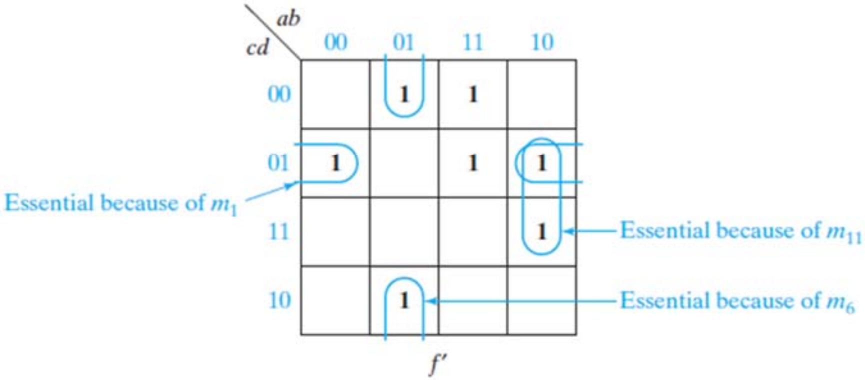
$$f = b'c'd' + bcd + acd' + a'b'c + a'bc'd$$



$$f = b'c'd' + bcd + acd' + a'b'c + a'bc'd$$



$$f = b'd' + a'bd + abc + \left\{\begin{matrix} a'cd \\ or \\ a'b'c \end{matrix}\right\}$$



$$f' = b'c'd + a'bd' + ab'd + abc'$$
$$f = (b + c + d')(a + b' + d)(a' + b + d')(a' + b' + c)$$

Thanks,..

See you next week (ISA),...