Lecture (02)

Complements

By:

Dr. Ahmed ElShafee

Complements of a number

- Complements are used in digital computers to **simplify the subtraction operation** and for logical manipulation.
- leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements
  - radix complement
  - diminished radix complement

1. Diminished Radix Complement

- Given a number *N* in base *r* having *n* digits,
  the $(r - 1)$'s complement of *N*
- diminished radix complement, is defined as $(r^n - 1) - N$

- For decimal numbers, $r = 10$ and $r - 1 = 9$,
- so the 9's complement of *N* is $(10^n - 1) - N$.
- 10^n represents a number that consists of a single 1 followed by *n* 0's.

More explanation:

- *Decimal r=10*
- *r-1=9 →9's complement*
- $n = 4$
- $10^4 = 10000$
- $10^4-1=9999$
- 9's complement of a decimal number is obtained by subtracting each digit from 9

decimal 9's complement examples:

- The 9's complement of 546700 is 999999 - 546700 = 453299.
- The 9's complement of 012398 is 999999 - 012398 = 987601.

Binary numbers

- $r = 2$
- $r - 1 = 1$, → 1's complement
- so the 1's complement of N is $(2^n - 1) – N$
- 2^n is represented by a binary number that consists of a 1 followed by n 0's.
- $2^n – 1$ is a binary number represented by n 1's

- Example
- $n = 4$,
- $2^4 = (10000)_2$
- $2^4 - 1 = (1111)_2$.
- the 1's complement of a binary number is obtained by subtracting each digit from 1.
- subtracting binary digits from 1, have either 1 - 0 = 1 or 1 - 1 = 0,
- which causes the bit to change from 0 to 1 or from 1 to 0, respectively

the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.

Examples

- The 1's complement of 1011000 is 0100111.
- The 1's complement of 0101101 is 1010010.

Octal and hexadecimal systems

- The (r - 1)>s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

2. Radix Complement

- The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N > 0$

  and = 0 for $N = 0$.
- Comparing with the (r - 1)'s complement, we note that the r's complement is obtained by adding 1 to the (r - 1)'s complement:

$$r^n - N = [(r^n - 1) - N] + 1.$$

Examples

- 9's complement of 2389 → 9999-2389=7610
- 10's complement of 2389 → 10000-2389=7611

  or → 7610 + 1=7611

- The 1's complement of 101100 → 010011
- The 2's complement of 101100 → 010011 +1=01100

  or→ 1000000- 101100 = 10100
- the 10's complement of decimal 2389 is 7610 + 1 = 7611
- The 2's complement of binary 101100 is 010011 + 1 = 010100

<u>Shortcut</u>:

- 10's complement of N, can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.

- the 10's complement of 012398 is

  987602
- the 10's complement of 246700 is

  753300

Shortcut

- the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.

- the 2's complement of 1101100 is
  0010100
- the 2's complement of 0110111 is
  1001001

Number	1's complement	2's complement
0101	1010	1011
0110	1001	1010
0111	1000	1001

The complement of the complement restores the number to its original value .

r's complement of N is $r^n - N$,

- the complement of the complement is
- $r^n - (r^n - N) = N$

Subtraction with Complements

- The direct method of subtraction uses the borrow concept
- borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit.
- Works well with paper and pencil.
- For digital systems, we complements method

Method 1

$M - N$

minuend (M) — subtrahend (N)

- Add the minuend *M* to the *r*'s complement of the subtrahend *N*.

$$M + (r^n - N) = M - N + r^n.$$

- If $M > N$, the sum will produce an end carry r^n, which can be discarded
- If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the *r*'s complement of $(N - M)$. To obtain the answer in a familiar form, take the *r*'s complement of the sum and place a negative sign in front

Example 01

Using 10's complement, subtract $72532 - 3250$.

Note that *M* has five digits and *N* has only four digits. Both numbers must have the same number of digits, so we write *N* as 03250

$72532 - 3250.$

$M \geq N$

$M =$	72532
10's complement of $N =$	+ 96750
Sum =	169282
Discard end carry $10^5 =$	− 100000
Answer =	69282

Example 02

Using 10's complement, subtract $3250 - 72532$.

$3250 - 72532.$

$3250 < 72532$, the result is negative.

Therefore, the answer is $-$(10's complement of 30718)

$$
\begin{aligned}
M &= \quad 03250 \\
\text{10's complement of } N &= +\,\underline{27468} \\
\text{Sum} &= \quad 30718
\end{aligned}
$$

-ve 10's complement of sum = - 69282

Example 03

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction **(a)** $X - Y$ and **(b)** $Y - X$ by using 2's complements.

- $X = 1010100$; $Y = 1000011$,

  $X - Y$, and $X > Y$

$$
\begin{aligned}
X &= \quad 1010100 \\
\text{2's complement of } Y &= +\ \ 0111101 \\
\text{Sum} &= \quad 10010001 \\
\text{Discard end carry } 2^7 &= -\underline{10000000} \\
\textit{Answer}: X - Y &= \quad 0010001
\end{aligned}
$$

- $X = 1010100$; $Y = 1000011$,

  $Y - X$, and $Y > X$

$$
\begin{aligned}
Y &= \quad 1000011 \\
\text{2's complement of } X &= +\ \underline{0101100} \\
\text{Sum} &= \quad 1101111
\end{aligned}
$$

$$-(\text{2's complement of } 1101111) = \quad -0010001$$

Method 2

- Sub traction can be done by means of the (*r* - 1)'s complement.
- (*r* - 1)'s complement is one less than the *r*'s complement.
- So we add one to the result, called *end-around carry.*

Example 04

Repeat previous Example , but this time using 1's complement.

$X = 1010100;\ Y = 1000011,$

- $X - Y = 1010100 - 1000011$

$$\begin{aligned} X &= \quad 1010100 \\ \text{1's complement of } Y &= +\underline{\ 0111100} \\ \text{Sum} &= \quad 10010000 \\ \text{End-around carry} &= +\underline{\qquad\quad 1} \\ \textit{Answer}: X - Y &= \quad 0010001 \end{aligned}$$

$Y - X = 1000011 - 1010100$

$$\begin{aligned} Y &= \quad 1000011 \\ \text{1's complement of } X &= +\underline{\ 0101011} \\ \text{Sum} &= \quad 1101110 \end{aligned}$$

, the answer is $Y - X = -(\text{1's complement of } 1101110)$

$-0010001.$

Signed binary numbers

signed-magnitude system

- number by changing its sign,
- a negative number is indicated by a minus sign and a positive number by a plus sign.
- Because of hardware limitations, computers must represent everything with binary digits.
- We represent the sign with a bit placed in the leftmost position of the number, sign bit 0 for positive and 1 for negative.
- If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number.

- If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number
- the string of bits 01001 can be considered as 9 (unsigned binary) or as +9 (signed binary) because the leftmost bit is 0.
- string of bits 11001 represents the binary equivalent of 25 when considered as an unsigned number and the binary equivalent of -9 when considered as a signed number.

<u>signed complement system</u>,

- When arithmetic operations are implemented in a computer, it is more convenient to use a different system
- for representing negative numbers, a negative number is indicated by its complement.
- Since positive numbers always start with 0 (plus) in the leftmost position, the complement will always start with a 1, indicating a negative number
- system can use either the 1’s or the 2’s complement, but the 2’s complement is the most common.

- As an example,

+9		00001001
−9	signed-magnitude representation:	10001001
	signed-1’s-complement representation:	11110110
	signed-2’s-complement representation:	11110111

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

- The signed-magnitude system is used in ordinary arithmetic
- the signed-complement system is normally used in computer systems

Arithmetic Addition

<u>signed-magnitude system</u>

The addition of two numbers in the signed-magnitude system:

- are the same, we add the two magnitudes and give the sum the common sign
- signs are different, subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude

<u>signed-complement system</u>

- does not require a comparison or subtraction, only addition.
- Addition of two numbers of signed- 2's-complement form,
  - addition of the two numbers, including their sign bits.
  - A carry out of the sign-bit position is discarded.
- <u>*Note: negative numbers must be initially in 2's-complement*</u>

+ 6	00000110	− 6	11111010
+13	00001101	+13	00001101
+19	00010011	+ 7	00000111

+ 6	00000110	− 6	11111010
−13	11110011	−13	11110011
− 7	11111001	−19	11101101

$M - N$

minuend subtrahend

Arithmetic Subtraction

- Take the 2's complement of the subtrahend (including the sign bit).
- add it to the minuend (including the sign bit).

$(-6) - (-13) = +7.$

-6	**Unsigned**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**0**
	1's	1	1	1	1	1	0	0	1
	2's	1	1	1	1	1	0	1	0

-13	**unsigned**	**0**	**0**	**0**	**0**	**1**	**1**	**0**	**1**
	1's	1	1	1	1	0	0	1	0
	2's	1	1	1	1	0	0	1	1

-ve sign	**2's**	**0**	**0**	**0**	**0**	**1**	**1**	**0**	**1**

	1	**1**	**1**	**1**	**1**	**0**	**1**	**0**
	+							
	0	0	0	0	1	1	0	1
	=							
1	0	0	0	0	0	1	1	1

7

Thanks,..

See you next week (ISA),…