

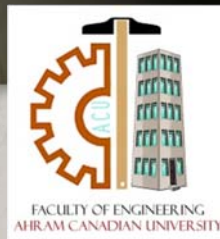


Lecture (02) Complements

By:

Dr. Ahmed ElShafee

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Complements of a number

- Complements are used in digital computers to **simplify the subtraction operation** and for logical manipulation.
- leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements
 - radix complement
 - diminished radix complement

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1. Diminished Radix Complement

- Given a number N in base r having n digits, the $(r - 1)$'s complement of N
- diminished radix complement, is defined as $(r^n - 1) - N$
- For decimal numbers, $r = 10$ and $r - 1 = 9$,
- so the 9's complement of N is $(10^n - 1) - N$.
- 10^n represents a number that consists of a single 1 followed by n 0's.

More explanation:

- *Decimal* $r=10$
- $r-1=9 \rightarrow 9$'s complement
- $n = 4$
- $10^4 = 10000$
- $10^4 - 1 = 9999$
- 9's complement of a decimal number is obtained by subtracting each digit from 9

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decimal 9's complement examples:

- The 9's complement of 546700 is $999999 - 546700 = 453299$.
- The 9's complement of 012398 is $999999 - 012398 = 987601$.

Binary numbers

- $r = 2$
- $r - 1 = 1$, \rightarrow 1's complement
- so the 1's complement of N is $(2^n - 1) - N$
- 2^n is represented by a binary number that consists of a 1 followed by n 0's.
- $2^n - 1$ is a binary number represented by n 1's

Example

- $n = 4$,
- $2^4 = (10000)_2$
- $2^4 - 1 = (11111)_2$.
- the 1's complement of a binary number is obtained by subtracting each digit from 1.
- subtracting binary digits from 1, have either $1 - 0 = 1$ or $1 - 1 = 0$,
- which causes the bit to change from 0 to 1 or from 1 to 0, respectively

the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.

Examples

- The 1's complement of 1011000 is 0100111.
- The 1's complement of 0101101 is 1010010.

Octal and hexadecimal systems

- The $(r - 1)$'s complement of octal or hexadecimal numbers is obtained by subtracting each digit from 7 or F (decimal 15), respectively.

2. Radix Complement

- The r 's complement of an n -digit number N in base r is defined as $r^n - N$ for $N > 0$ and $= 0$ for $N = 0$.
- Comparing with the $(r - 1)$'s complement, we note that the r 's complement is obtained by adding 1 to the $(r - 1)$'s complement:

$$r^n - N = [(r^n - 1) - N] + 1.$$

Examples

- 9's complement of 2389 $\rightarrow 9999 - 2389 = 7610$
- 10's complement of 2389 $\rightarrow 10000 - 2389 = 7611$
or $\rightarrow 7610 + 1 = 7611$
- The 1's complement of 101100 $\rightarrow 010011$
- The 2's complement of 101100 $\rightarrow 010011 + 1 = 01100$
or $\rightarrow 1000000 - 101100 = 10100$
- the 10's complement of decimal 2389 is $7610 + 1 = 7611$
- The 2's complement of binary 101100 is $010011 + 1 = 010100$

Shortcut:

- 10's complement of N , can be formed also by leaving all least significant 0's unchanged, subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.
- the 10's complement of 012398 is
987602
- the 10's complement of 246700 is
753300

Shortcut

- the 2's complement can be formed by leaving all least significant 0's and the first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.
- the 2's complement of 1101100 is
0010100
- the 2's complement of 0110111 is
1001001

Number	1's complement	2's complement
0101	1010	1011
0110	1001	1010
0111	1000	1001

Subtraction with Complements

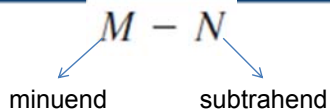
- The direct method of subtraction uses the borrow concept
- borrow a 1 from a higher significant position when the minuend digit is smaller than the subtrahend digit.
- Works well with paper and pencil.
- For digital systems, we complements method

The complement of the complement restores the number to its original value .

r 's complement of N is $r^n - N$,

- the complement of the complement is
- $r^n - (r^n - N) = N$

Method 1



- Add the minuend M to the r 's complement of the subtrahend N .

$$M + (r^n - N) = M - N + r^n.$$

- If $M > N$, the sum will produce an end carry r^n , which can be discarded
- If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r 's complement of $(N - M)$. To obtain the answer in a familiar form, take the r 's complement of the sum and place a negative sign in front

Example 01

Using 10's complement, subtract $72532 - 3250$.

Note that M has five digits and N has only four digits. Both numbers must have the same number of digits, so we write N as 03250

$$72532 - 3250$$

$$M \geq N$$

$$\begin{array}{r} M = \quad 72532 \\ 10\text{'s complement of } N = + 96750 \\ \text{Sum} = \quad 169282 \\ \text{Discard end carry } 10^5 = - 100000 \\ \text{Answer} = \quad 69282 \end{array}$$

Example 02

Using 10's complement, subtract $3250 - 72532$.

$$3250 - 72532.$$

3250 < 72532, the result is negative.

Therefore, the answer is $-(10\text{'s complement of } 30718)$

$$M = 03250$$

$$10\text{'s complement of } N = + \underline{27468}$$

$$\text{Sum} = 30718$$

$$\text{-ve } 10\text{'s complement of sum} = -69282$$

Example 03

Given the two binary numbers $X = 1010100$ and $Y = 1000011$, perform the subtraction
(a) $X - Y$ and (b) $Y - X$ by using 2's complements.

-
- $X = 1010100$; $Y = 1000011$,
 $X - Y$, and $X > Y$

$$X = 1010100$$

$$2\text{'s complement of } Y = + \underline{0111101}$$

$$\text{Sum} = 10010001$$

$$\text{Discard end carry } 2^7 = -\underline{10000000}$$

$$\text{Answer: } X - Y = 0010001$$

-
- $X = 1010100$; $Y = 1000011$,
 $Y - X$, and $Y > X$

$$Y = 1000011$$

$$2\text{'s complement of } X = + \underline{0101100}$$

$$\text{Sum} = 1101111$$

$$-(2\text{'s complement of } 1101111) = -0010001$$

Method 2

- Subtraction can be done by means of the $(r - 1)$'s complement.
- $(r - 1)$'s complement is one less than the r 's complement.
- So we add one to the result, called end-around carry.

Example 04

Repeat previous Example , but this time using 1's complement.

$$X = 1010100; Y = 1000011,$$

$$X - Y = 1010100 - 1000011$$

$$\begin{array}{r} X = \quad 1010100 \\ \text{1's complement of } Y = + \underline{0111100} \\ \text{Sum} = \quad 10010000 \\ \text{End-around carry} = + \quad \underline{\quad 1} \\ \text{Answer: } X - Y = \quad 0010001 \end{array}$$

$$Y - X = 1000011 - 1010100$$

$$\begin{array}{r} Y = \quad 1000011 \\ \text{1's complement of } X = + \underline{0101011} \\ \text{Sum} = \quad 1101110 \\ \text{, the answer is } Y - X = -(1\text{'s complement of } 1101110) \\ \quad \quad \quad -0010001. \end{array}$$

Signed binary numbers

signed-magnitude system

- number by changing its sign,
- a negative number is indicated by a minus sign and a positive number by a plus sign.
- Because of hardware limitations, computers must represent everything with binary digits.
- We represent the sign with a bit placed in the leftmost position of the number, sign bit 0 for positive and 1 for negative.
- If the binary number is signed, then the leftmost bit represents the sign and the rest of the bits represent the number.

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- If the binary number is assumed to be unsigned, then the leftmost bit is the most significant bit of the number
- the string of bits 01001 can be considered as 9 (unsigned binary) or as +9 (signed binary) because the leftmost bit is 0.
- string of bits 11001 represents the binary equivalent of 25 when considered as an unsigned number and the binary equivalent of -9 when considered as a signed number.

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signed complement system,

- When arithmetic operations are implemented in a computer, it is more convenient to use a different system
- for representing negative numbers, a negative number is indicated by its complement.
- Since positive numbers always start with 0 (plus) in the leftmost position, the complement will always start with a 1, indicating a negative number
- system can use either the 1's or the 2's complement, but the 2's complement is the most common.

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- As an example,

+9		00001001
-9	signed-magnitude representation:	10001001
	signed-1's-complement representation:	11110110
	signed-2's-complement representation:	11110111

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Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	—	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	—	—

- The signed-magnitude system is used in ordinary arithmetic
- the signed-complement system is normally used in computer systems

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Arithmetic Addition

signed-magnitude system

The addition of two numbers in the signed-magnitude system:

- are the same, we add the two magnitudes and give the sum the common sign
- signs are different, subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude

signed-complement system

- does not require a comparison or subtraction, only addition.
- Addition of two numbers of signed- 2's-complement form,
 - addition of the two numbers, including their sign bits.
 - A carry out of the sign-bit position is discarded.
- Note: negative numbers must be initially in 2's-complement

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$$\begin{array}{r}
 + 6 \quad 00000110 \\
 +13 \quad 00001101 \\
 \hline
 +19 \quad 00010011 \\
 \\
 + 6 \quad 00000110 \\
 -13 \quad 11110011 \\
 \hline
 - 7 \quad 11111001
 \end{array}
 \qquad
 \begin{array}{r}
 - 6 \quad 11111010 \\
 +13 \quad 00001101 \\
 \hline
 + 7 \quad 00000111 \\
 \\
 - 6 \quad 11111010 \\
 -13 \quad 11110011 \\
 \hline
 -19 \quad 11101101
 \end{array}$$

$$M - N$$

minuend subtrahend

Arithmetic Subtraction

- Take the 2's complement of the subtrahend (including the sign bit).
- add it to the minuend (including the sign bit).


$$(-6) - (-13) = +7.$$

-6	Unsigned	0	0	0	0	0	1	1	0
	1's	1	1	1	1	1	0	0	1
	2's	1	1	1	1	1	0	1	0

-13	unsigned	0	0	0	0	1	1	0	1
	1's	1	1	1	1	0	0	1	0
	2's	1	1	1	1	0	0	1	1

-ve sign	2's	0	0	0	0	1	1	0	1
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	1	1	1	1	1	0	1	0
	+							
	0	0	0	0	1	1	0	1
	=							
1	0	0	0	0	0	1	1	1
	7							



Thanks,..
See you next week (ISA),...

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