

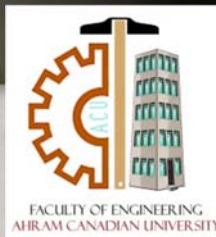


# Lecture (01)

# Digital Systems and Binary Numbers

By:

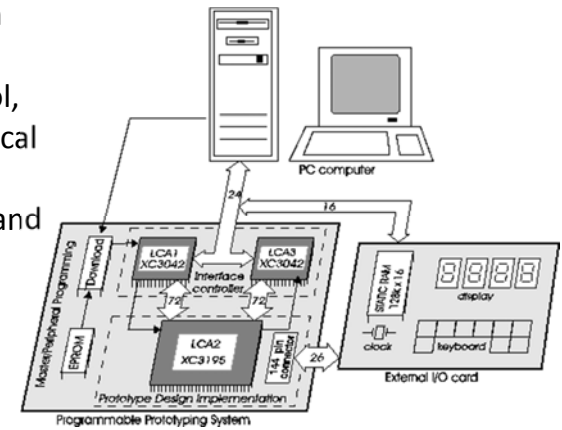
Dr. Ahmed ElShafee



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## Digital systems

- Digital systems are used in communication, business transactions, traffic control, spacecraft guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises
- Most, if not all, of these devices have a special-purpose digital computer embedded within them.



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- These devices follow a sequence of instructions, called a program, that operates on given data.
- One characteristic of digital systems is their ability to represent and manipulate discrete elements of information.
- Any set that is restricted to a finite number of elements contains discrete information.



- Examples of discrete sets are the 10 decimal digits, the 26 letters of the alphabet, the 52 playing cards, and the 64 squares of a chessboard
- Discrete elements of information are represented in a digital system by physical quantities called signals.
- Electrical signals such as voltages and currents are the most common.

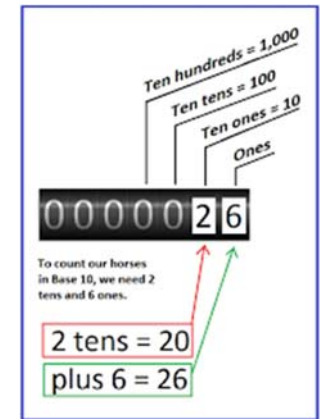


- electronic digital systems use just two discrete values and are therefore said to be *binary*.
- A binary digit, called a *bit*, has two values: 0 and 1.
- Discrete elements of information are represented with groups of bits called *binary codes*.
- For example, the decimal digits 0 through 9 are represented in a digital system with a code of four bits (e.g., the number 7 is represented by 0111)



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- we could write  $(0111)_2$  to indicate that the pattern 0111 is to be interpreted in a binary system, and  $(0111)_{10}$  to indicate that the reference system is decimal.
- Then  $0111_2 = 7_{10}$ , which is not the same as  $0111_{10}$ , or one hundred eleven.



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## Numbering systems

- A decimal number 7323 is a shorthand notation for what should be written as

$$7 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

- In general, a number with a decimal point is represented by a series of coefficients:

$$a_5a_4a_3a_2a_1a_0. a_{-1}a_{-2}a_{-3}$$

- The coefficients  $a_j$  are any of the 10 digits (0, 1, 2, ..., 9), subscript value  $j$  gives the place value and, hence, the power of 10 by which the coefficient must be multiplied.

$$10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

- $a_3 = 7$ ,  $a_2 = 3$ ,  $a_1 = 2$ , and  $a_0 = 3$ .

- The decimal number system is said to be of *base*, or *radix*, 10 because it uses 10 digits and the coefficients are multiplied by powers of 10.

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### The *binary* system

- The coefficients have only two possible values: 0 and 1.
- coefficient  $a_j$  is multiplied by a power of the radix, e.g.,  $2^j$ , and the results are added to obtain the decimal equivalent of the number
- The radix (float) point distinguishes positive powers of 2 from negative powers of 2.
- $11010.11_2 =$   
 $1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$

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- There are many different number system, number expressed in a base- $r$  system has coefficients multiplied by powers of  $r$ : stems,

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \dots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} \\ + a_{-2} \cdot r^{-2} + \dots + a_{-m} \cdot r^{-m}$$

- The coefficients  $a_j$  range in value from 0 to  $r - 1$

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- the conversion from binary to decimal can be obtained by adding only the numbers with powers of two corresponding to the bits that are equal to 1. For example,

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

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### Other numbering systems

- An example of a base-5 number is  
 $(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$
- The coefficient values for base 5 can be only 0, 1, 2, 3, and 4.  
 $(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$
- An example of base-8 number (octal system)

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### hexadecimal (base-16) number system

- The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10
- The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15, respectively

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46,687)_{10}$$

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### Why the hexadecimal is important in computer systems?

- The hexadecimal system is used commonly by designers to represent long strings of bits in the addresses, instructions, and data in digital systems.
- For example, B65F is used to represent 1011011001010000

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### Binary system prefixes:

- In computer work,  $2^{10}$  is referred to as K (kilo),  $2^{20}$  as M (mega),  $2^{30}$  as G (giga), and  $2^{40}$  as T (tera).
- $4K = 2^{12} = 4,096$  and  $16M = 2^{24} = 16,777,216$ .
- Computer capacity is usually given in bytes.
- A *byte* is equal to eight bits which presents a one keyboard character
- A computer hard disk with four gigabytes of storage has a capacity of  $4G = 2^{32}$  bytes
- A terabyte is 1024 gigabytes, = 240 bytes.

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### Arithmetic operations

101101	1010001
+100111	+1000101
<hr/>	<hr/>
1010100	10010110

$$\begin{array}{r} 101101 \\ -100111 \\ \hline 000110 \end{array}$$

$$\begin{array}{r} 1010001 \\ -1000101 \\ \hline 0001100 \end{array}$$

$$\begin{array}{r} \text{d: } 1011 \\ \times 101 \\ \hline 1011 \\ 0000 \\ 1011 \\ \hline 110111 \end{array}$$

### Powers of Two

$n$	$2^n$	$n$	$2^n$	$n$	$2^n$
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

## Number base conversion

- Convert decimal 41 to binary
- Convert decimal 41 to binary.
- First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of 1/2.
- Then the quotient is again divided by 2 to give a new quotient and remainder.
- The process is continued until the integer quotient becomes 0.
- The *coefficients* of the desired binary number are obtained from the *remainders* as follows

## Example

	Integer Quotient		Remainder	Coefficient
$41/2 =$	20	+	$\frac{1}{2}$	$a_0 = 1$
$20/2 =$	10	+	0	$a_1 = 0$
$10/2 =$	5	+	0	$a_2 = 0$
$5/2 =$	2	+	$\frac{1}{2}$	$a_3 = 1$
$2/2 =$	1	+	0	$a_4 = 0$
$1/2 =$	0	+	$\frac{1}{2}$	$a_5 = 1$

Therefore, the answer is  $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$ .

- Convert decimal 153 to octal.
- The required base  $r$  is 8.
- First, 153 is divided by 8 to give an integer quotient of 19 and a remainder of 1.
- Then 19 is divided by 8 to give an integer quotient of 2 and a remainder of 3.
- Finally, 2 is divided by 8 to give a quotient of 0 and a remainder of 2.

## Example

	quotient		reminder	Coefficient
$153/8$	19	+	1	$a_0=1$
$19/8$	2	+	3	$a_1=3$
$2/8$	0	+	2	$a_2=2$

$(153)_{10} = (231)_8$

- Convert  $(0.6875)_{10}$  to binary
- First, 0.6875 is multiplied by 2 to give an integer and a fraction.
- Then the new fraction is multiplied by 2 to give a new integer and a new fraction.
- The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

	Integer	Fraction	Coefficient
0.6875x2	1	+ 0.3750	$a_{-1}=1$
0.3750x2	0	+ 0.7500	$a_{-2}=0$
0.7500 x2	1	+ 0.5000	$a_{-3}=1$
0.5000 x2	1	+ 0.0000	$a_{-4}=1$

the answer is  $(0.6875)_{10} = (0. a_{-1} a_{-2} a_{-3} a_{-4})_2 = (0.1011)_2$ .

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## Example

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

- Convert  $(0.513)_{10}$  to octal.

	Integer	Fraction	Coefficient
0.513x8	4	+ 0.104	$a_{-1}=4$
0.104x8	0	+ 0.832	$a_{-2}=0$
0.832 x8	6	+ 0.656	$a_{-3}=6$
0.656 x8	5	+ 0.248	$a_{-4}=5$
0.248x8	1	+ 0.984	$a_{-5}=1$
0.984	7	0.872	$a_{-6}=7$

- $(0.513)_{10} = (0.406517 \dots)_8$

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- By combining previous examples:

$$(41.6875)_{10} = (101001.1011)_2$$

$$(153.513)_{10} = (231.406517)_8$$

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## Octal and Hexadecimal numbers

- The conversion from and to binary, octal, and hexadecimal plays an important role in digital computers, because shorter patterns of hex characters are easier to recognize than long patterns
- $2^3 = 8$  and  $2^4 = 16$ , each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

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## Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

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## The conversion from binary to octal

- partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.
- The corresponding octal digit is then assigned to each group.

$$(10\ 110\ 001\ 101\ 011 \cdot 111\ 100\ 000\ 110)_2 = (26153.7406)_8$$

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## Conversion from binary to hexadecimal

- similar, except that the binary number is divided into groups of *four* digits:

$$(10\ 1100\ 0110\ 1011 \cdot 1111\ 0010)_2 = (2C6B.F2)_{16}$$

$$2\ C\ 6\ B\ F\ 2$$

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## Conversion from octal to binary

- is done by reversing the preceding procedure.
- Each octal digit is converted to its three-digit binary equivalent.

$$(673.124)_8 = (110\ 111\ 011 \cdot 001\ 010\ 100)_2$$

$$6\ 7\ 3\ 1\ 2\ 4$$

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### Conversion from hexadecimal to binary

- is done by reversing the preceding procedure.
- each hexadecimal digit is converted to its four-digit binary equivalent

$$(306.D)_{16} = (0011 \ 0000 \ 0110 \cdot \ 1101)_2$$

3            0            6            D

- 
- that's why we need to use hex and octal,

Easy conversion between binary  $\leftrightarrow$  octal , and binary  $\leftrightarrow$  hexadecimal,

- Binary numbers are difficult to work with because they require three or four times as many digits as their decimal equivalents.
- For example, the binary number 111111111111 is equivalent to decimal 4095.



**Thanks,..  
See you next week (ISA),...**