# Lecture (01) Digital Systems and Binary Numbers

By:

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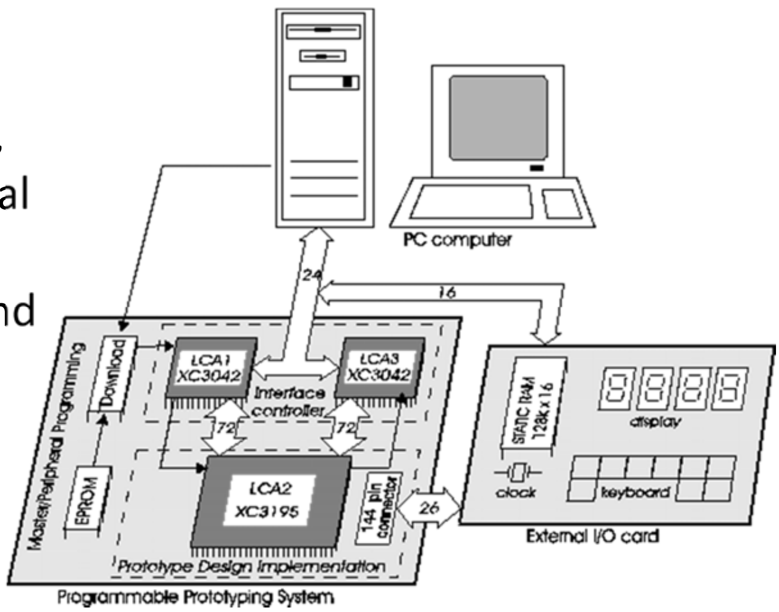
# Digital systems

- Digital systems are used in communication, business transactions, traffic control, spacecraft guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises
- Most, if not all, of these devices have a special-purpose digital computer embedded within them.



- These devices follow a sequence of instructions, called a program, that operates on given data.
- One characteristic of digital systems is their ability to represent and manipulate discrete elements of information.
- Any set that is restricted to a finite number of elements contains discrete information.

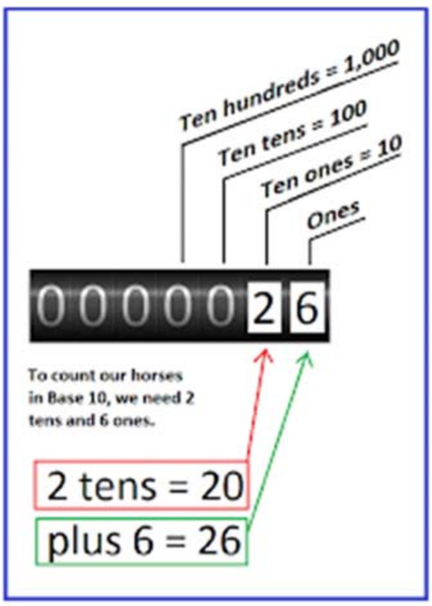


- Examples of discrete sets are the 10 decimal digits, the 26 letters of the alphabet, the 52 playing cards, and the 64 squares of a chessboard
- Discrete elements of information are represented in a digital system by physical quantities called signals.
- Electrical signals such as voltages and currents are the most common.

- electronic digital systems use just two discrete values and are therefore said to be *binary*.
- A binary digit, called a *bit,* has two values: 0 and 1.
- Discrete elements of information are represented with groups of bits called *binary codes*.
- For example, the decimal digits 0 through 9 are represented in a digital system with a code of four bits (e.g., the number 7 is represented by 0111).




- we could write $(0111)_2$ to indicate that the pattern 0111 is to be interpreted in a binary system, and $(0111)_{10}$ to indicate that the reference system is decimal.
- Then $0111_2 = 7_{10}$, which is not the same as $0111_{10}$, or one hundred eleven.



# Numbering systems

- A decimal number 7323 is a shorthand notation for what should be written as

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

- In general, a number with a decimal point is represented by a series of coefficients:

$$a_5a_4a_3a_2a_1a_0.\ a_{-1}a_{-2}a_{-3}$$

- The coefficients *aj* are any of the 10 digits (0, 1, 2, c, 9), subscript value *j* gives the place value and, hence, the power of 10 by which the coefficient must be multiplied.

$$10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

- $a_3$ = 7, $a_2$ = 3, $a_1$ = 9, and $a_0$ = 2.



- The decimal number system is said to be of *base,* or *radix,* 10 because it uses 10 digits and the coefficients are multiplied by powers of 10.

**The *binary* system**

- The coefficients have only two possible values: 0 and 1.
- coefficient *aj* is multiplied by a power of the radix, e.g., $2^j$, and the results are added to obtain the decimal equivalent of the number
- The radix (float) point distinguishes positive powers of 2 from negative powers of 2.
- $11010.11_2$ =

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$



- There are many different number system, number expressed in a base-*r* system has coefficients multiplied by powers of *r*: stems,

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

- The coefficients *aj* range in value from 0 to *r* - 1

- the conversion from binary to decimal can be obtained by adding only the numbers with powers of two corresponding to the bits that are equal to 1. For example,

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$



**Other numbering systems**

- An example of a base-5 number is

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

- The coefficient values for base 5 can be only 0, 1, 2, 3, and 4.

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

- An example of base-8 number (octal system)

***hexadecimal* (base-16) number system**

- The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10
- The letters A, B, C, D, E, and F are used for the digits 10, 11, 12, 13, 14, and 15, respectively

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46,687)_{10}$$



- **Why the hexadecimal is important in computer systems?**
- The hexadecimal system is used commonly by designers to represent long strings of bits in the addresses, instructions, and data in digital systems.
- For example, B65F is used to represent 1011011001010000

## Binary system prefixes:

- In computer work, $2^{10}$ is referred to as K (kilo), $2^{20}$ as M (mega), $2^{30}$ as G (giga), and $2^{40}$ as T (tera).
- 4K = $2^{12}$ = 4,096 and 16M = $2^{24}$ = 16,777,216.

- Computer capacity is usually given in bytes.
- A *byte* is equal to eight bits which presents a one keyboard character

- A computer hard disk with four gigabytes of storage has a capacity of 4G = $2^{32}$ bytes
- A terabyte is 1024 gigabytes, = 240 bytes.



## Arithmetic operations

```
  101101
 +100111
 -------
 1010100
```

```
  1010001
 +1000101
 --------------
 10010110
```

•

```
  101101
 -100111
 -------
  000110
```

```
  1010001
- 1000101
---------------
  0001100
```



d:

```
    1011
   × 101
  ------
    1011
   0000
  1011
  ------
  110111
```

## Powers of Two

| $n$ | $2^n$ | $n$ | $2^n$ | $n$ | $2^n$ |
|---|---|---|---|---|---|
| 0 | 1 | 8 | 256 | 16 | 65,536 |
| 1 | 2 | 9 | 512 | 17 | 131,072 |
| 2 | 4 | 10 | 1,024 (1K) | 18 | 262,144 |
| 3 | 8 | 11 | 2,048 | 19 | 524,288 |
| 4 | 16 | 12 | 4,096 (4K) | 20 | 1,048,576 (1M) |
| 5 | 32 | 13 | 8,192 | 21 | 2,097,152 |
| 6 | 64 | 14 | 16,384 | 22 | 4,194,304 |
| 7 | 128 | 15 | 32,768 | 23 | 8,388,608 |



# Number base conversion

- Convert decimal 41 to binary
- Convert decimal 41 to binary.
- First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of 1/2.
- Then the quotient is again divided by 2 to give a new quotient and remainder.
- The process is continued until the integer quotient becomes 0.
- The *coefficients* of the desired binary number are obtained from the *remainders* as follows

| | Integer Quotient | | Remainder | Coefficien |
|---|---|---|---|---|
| $41/2 =$ | 20 | + | $\frac{1}{2}$ | $a_0 = 1$ |
| $20/2 =$ | 10 | + | 0 | $a_1 = 0$ |
| $10/2 =$ | 5 | + | 0 | $a_2 = 0$ |
| $5/2 =$ | 2 | + | $\frac{1}{2}$ | $a_3 = 1$ |
| $2/2 =$ | 1 | + | 0 | $a_4 = 0$ |
| $1/2 =$ | 0 | + | $\frac{1}{2}$ | $a_5 = 1$ |

Therefore, the answer is $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$.



# Example

- Convert decimal 153 to octal.
- The required base *r* is 8.
- First, 153 is divided by 8 to give an integer quotient of 19 and a remainder of 1.
- Then 19 is divided by 8 to give an integer quotient of 2 and a remainder of 3.
- Finally, 2 is divided by 8 to give a quotient of 0 and
- a remainder of 2.

| | quotient | | reminder | Coefficient |
|---|---|---|---|---|
| 153/8 | 19 | + | 1 | a0=1 |
| 19/8 | 2 | + | 3 | a1=3 |
| 2/8 | 0 | + | 2 | a2=2 |

$(153)_{10} = (231)_8$



# Example

- Convert $(0.6875)_{10}$ to binary
- First, 0.6875 is multiplied by 2 to give an integer and a fraction.
- Then the new fraction is multiplied by 2 to give a new integer and a new fraction.
- The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

| | Integer | | Fraction | Coefficient |
|---|---|---|---|---|
| 0.6875x2 | 1 | + | 0.3750 | $a_{-1}$=1 |
| 0.3750x2 | 0 | + | 0.7500 | $a_{-2}$=0 |
| 0.7500 x2 | 1 | + | 0.5000 | $a_{-3}$=1 |
| 0.5000 x2 | 1 | + | 0.0000 | $a_{-4}$=1 |

the answer is $(0.6875)_{10} = (0.\ a_{-1}\ a_{-2}\ a_{-3}\ a_{-4})_2 = (0.1011)_2$.



# Example

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5.248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

- Convert (0.513)10 to octal.

| | Integer | | Fraction | Coefficient |
|---|---|---|---|---|
| 0.513x8 | 4 | + | 0.104 | $a_{-1}$=4 |
| 0.104x8 | 0 | + | 0.832 | $a_{-2}$=0 |
| 0.832 x8 | 6 | + | 0.656 | $a_{-3}$=6 |
| 0.656 x8 | 5 | + | 0.248 | $a_{-4}$=5 |
| 0.248x8 | 1 | + | 0.984 | $a_{-5}$=1 |
| 0.984 | 7 | | 0.872 | $a_{-6}$=7 |

- $(0.513)_{10} = (0.406517\ ....)_8$

- By combining previous examples:

$$(41.6875)_{10} = (101001.1011)_2$$

$$(153.513)_{10} = (231.406517)_8$$



# Octal and Hexadecimal numbers

- The conversion from and to binary, octal, and hexadecimal plays an important role in digital computers, because shorter patterns of hex characters are easier to recognize than long patterns
- $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

*Numbers with Different Bases*

| Decimal (base 10) | Binary (base 2) | Octal (base 8) | Hexadecimal (base 16) |
|---|---|---|---|
| 00 | 0000 | 00 | 0 |
| 01 | 0001 | 01 | 1 |
| 02 | 0010 | 02 | 2 |
| 03 | 0011 | 03 | 3 |
| 04 | 0100 | 04 | 4 |
| 05 | 0101 | 05 | 5 |
| 06 | 0110 | 06 | 6 |
| 07 | 0111 | 07 | 7 |
| 08 | 1000 | 10 | 8 |
| 09 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |



**The conversion from binary to octal**

- partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right.
- The corresponding octal digit is then assigned to each group.

$$(10\quad 110\quad 001\quad 101\quad 011\quad \cdot\quad 111\quad 100\quad 000\quad 110)_2 = (26153.7406)_8$$
$$2\quad 6\quad 1\quad 5\quad 3\qquad 7\quad 4\quad 0\quad 6$$

## Conversion from binary to hexadecimal

- similar, except that the binary number is divided into groups of *four* digits:

$$
\begin{array}{cccccccc}
(10 & 1100 & 0110 & 1011 & \cdot & 1111 & 0010)_2 = (2C6B.F2)_{16} \\
2 & C & 6 & B & & F & 2
\end{array}
$$



## Conversion from octal to binary

- is done by reversing the preceding procedure.
- Each octal digit is converted to its three-digit binary equivalent.

$$
\begin{array}{cccccccc}
(673.124)_8 = (110 & 111 & 011 & \cdot & 001 & 010 & 100)_2 \\
6 & 7 & 3 & & 1 & 2 & 4
\end{array}
$$

## Conversion from hexadecimal to binary

- is done by reversing the preceding procedure.
- each hexadecimal digit is converted to its four-digit binary equivalent

$$(306.D)_{16} = (0011 \quad 0000 \quad 0110 \quad \cdot \quad 1101)_2$$
$$\quad\quad\quad\quad\quad 3 \quad\quad 0 \quad\quad 6 \quad\quad\quad\quad D$$



- that's why we need to use hex and octal,

Easy conversion between binary ←→ octal , and binary ←→hexadecimal,

- Binary numbers are difficult to work with because they require three or four times as many digits as their decimal equivalents.
- For example, the binary number 111111111111 is equivalent to decimal 4095.

# Thanks,..

# See you next week (ISA),…