

Logic Design– Assignment 05

#	Student ID	Student Name	Grade (10)
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Delivery Date	
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١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

Q1

A flow rate sensing device used on a liquid transport pipeline functions as follows. The device provides a 5-bit output where all five bits are zero if the flow rate is less than 10 gallons per minute. The first bit is 1 if the flow rate is at least 10 gallons per minute; the first and second bits are 1 if the flow rate is at least 20 gallons per minute; the first, second, and third bits are 1 if the flow rate is at least 30 gallons per minute; and so on. The five bits, represented by the logical variables A , B , C , D , and E , are used as inputs to a device that provides two outputs Y and Z .

- (a) Write an equation for the output Y if we want Y to be 1 iff the flow rate is less than 30 gallons per minute.
 (b) Write an equation for the output Z if we want Z to be 1 iff the flow rate is at least 20 gallons per minute but less than 50 gallons per minute.

Sol 1

A	B	C	D	E		y	z
0	0	0	0	0	(less than 10 gpm)	+	
1	0	0	0	0	(at least 10 gpm)	+	
1	1	0	0	0	(at least 20 gpm)	+	+
1	1	1	0	0	(at least 30 gpm)		+
1	1	1	1	0	(at least 40 gpm)		+
1	1	1	1	1	(at least 50 gpm)		

(a) $Y = A'B'C'D'E' + AB'C'D'E' + ABC'D'E'$

(b) $Z = ABC'D'E' + ABCD'E' + ABCDE'$

Q2

- (a) How many switching functions of two variables (x and y) are there?
 (b) Give each function in truth table form and in reduced algebraic form.

Sol 2

... (a) $2^{2^n} = 2^{2^2} = 2^4 = 16$

... (b)

x y	z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}
0 0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0 1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1 0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1 1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0 $x'y'$ xy x' xy' y' $xy+xy'$ $x'+y'$ xy $xy'+xy$ y $x'+y$ x $x+y'$ $x+y$ 1

Q3	<p>Given: $F(a, b, c) = abc' + b'$.</p> <p>(a) Express F as a minterm expansion. (Use m-notation.)</p> <p>(b) Express F as a maxterm expansion. (Use M-notation.)</p> <p>(c) Express F' as a minterm expansion. (Use m-notation.)</p> <p>(d) Express F' as a maxterm expansion. (Use M-notation.)</p>
Sol 3	<p>.....</p> <p>... (a) $F = abc' + b'(a + a')(c + c') = abc' + ab'c + ab'c'$ $+ a'b'c + a'b'c'$; $F = \sum m(0, 1, 4, 5, 6)$</p> <p>... ..</p> <p>... (b) Remaining terms are maxterms: $F = \prod M(2, 3, 7)$</p> <p>... ..</p> <p>... (c) Maxterms of F are minterms of F':</p> <p>... $F' = \sum m(2, 3, 7)$</p> <p>... ..</p> <p>... (d) Minterms of F are maxterms of F':</p> <p>... $F' = \prod M(0, 1, 4, 5, 6)$</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p>

Q4

Given $f(a, b, c, d) = acd + bd' + a'c'd + ab'cd + a'b'cd'$

- (a) Express f as a minterm expansion (use m -notation).
- (b) Express f as maxterm expansion (use M -notation).
- (c) Express f' as a minterm expansion (use m -notation).
- (d) Express f' as a maxterm expansion (use M -notation).

Sol 4

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a	b	c	d	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

(a) $f = \sum m(1, 2, 4, 5, 6, 11, 12, 14, 15)$

(b) $f = \prod M(0, 3, 7, 8, 9, 10, 13)$

(c) $f' = \sum m(0, 3, 7, 8, 9, 10, 13)$

(d) $f' = \prod M(1, 2, 4, 5, 6, 11, 12, 14, 15)$

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Q5	<p>Find both the minterm expansion and maxterm expansion for the following functions, using <i>algebraic manipulations</i>:</p> <p>(a) $f(A, B, C, D) = AB + A'CD$</p> <p>(b) $f(A, B, C, D) = (A + B + D')(A' + C)(C + D)$</p>
Sol 5	<p>.....</p> <p>... (a) $f(A,B,C,D) = AB + A'CD = ABC'D' + ABC'D$</p> <p style="padding-left: 40px;">$+ ABCD' + ABCD + A'B'CD + A'BCD$</p> <p>... ..</p> <p style="padding-left: 40px;">$= (A+A'CD)(B+A'CD) = (A+C)(A+D)(A'+B)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(B+C)(B+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$f(A,B,C,D) = (A+B'+C+D')(A+B'+C+D)$</p> <p style="padding-left: 40px;">$(A+B+C+D')(A+B+C+D)(A+B'+C'+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A+B'+C+D)(A+B+C'+D)(A+B+C+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A'+B+C'+D')(A'+B+C'+D)(A'+B+C+D')$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A'+B+C+D)(A'+B+C+D')(A'+B+C+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A+B+C+D')(A+B+C+D)(A'+B+C'+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A'+B+C+D)(A+B+C'+D)(A+B+C+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$= (A+B'+C+D')(A+B+C+D')(A+B'+C'+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A+B'+C+D)(A+B+C'+D) (A+B+C+D)$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A'+B+C'+D')(A'+B+C'+D)(A'+B+C+D')$</p> <p>... ..</p> <p style="padding-left: 40px;">$(A'+B+C+D)$</p> <p>... ..</p> <p style="padding-left: 40px;"><i>Note: Consensus could have been applied twice to</i></p> <p>... ..</p> <p style="padding-left: 40px;"><i>write $f = (A+C)(A+D)(A'+B)$ and save some work.</i></p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p> <p>.....</p>

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(b) $f(A,B,C,D) = (A+B+D')(A'+C)(C+D)$

... $= (A+B+D')(A'D+C) = AC+A'BD+BC+CD'$

... $= AC(B+B')(D+D')+A'BD(C+C')$

... $+BC(A+A')(D+D')+(A+A')(B+B')CD'$

... $= ABCD+ABCD'+AB'CD+AB'CD'+A'BCD$

... $+A'BC'D+ABCD+ABCD'+ABCD+A'BCD'$

... $+ABCD'+AB'CD'+A'BCD'+A'B'CD'$

... $= ABCD+ABCD'+AB'CD+AB'CD'+A'BCD$

... $+A'BC'D+A'BCD'+A'B'CD'$

... $f(A,B,C,D) = (A+B+CC'+D')(A'+BB'+C+DD')$

... $(AA'+BB'+C+D)$

... $= (A+B+C+D')(A+B+C'+D')(A'+B+C+D)$

... $(A'+B+C+D')(A'+B'+C+D)(A'+B'+C+D')$

... $(A+B+C+D) (A+B'+C+D)(A'+B+C+D)$

... $(A'+B'+C+D)$

... $= (A+B+C+D')(A+B+C'+D')(A'+B+C+D)$

... $(A'+B+C+D')(A'+B'+C+D)(A'+B'+C+D')$

... $(A+B+C+D)(A+B'+C+D)$

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Q6

Given $F(A, B, C, D) = \sum m(1, 2, 5, 6, 10, 15)$.

- (a) Find the minterm expansion for F (both decimal and algebraic form).
- (b) Find the maxterm expansion for F (both decimal and algebraic form).

Sol 6

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(a) $F(A, B, C, D) = \sum m(0, 3, 4, 7, 8, 9, 11, 12, 13, 14) = \frac{A'B'C'D'}{m_0} + \frac{A'B'CD}{m_3} + \frac{A'BCD'}{m_4} + \frac{A'BCD}{m_7} + \frac{AB'C'D'}{m_8} + \frac{AB'C'D}{m_9} + \frac{AB'CD'}{m_{11}} + \frac{ABC'D'}{m_{12}} + \frac{ABC'D}{m_{13}} + \frac{ABCD'}{m_{14}}$

(b) $F(A, B, C, D) = \prod M(1, 2, 5, 6, 10, 15) = \frac{(A+B+C+D')}{M_1} \frac{(A+B+C'+D)}{M_2} \frac{(A+B'+C+D')}{M_3} \frac{(A+B'+C'+D)}{M_6} \frac{(A'+B+C'+D)}{M_{10}} \frac{(A'+B'+C'+D')}{M_{15}}$

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