Electromagnetic Fields – Tutorial 08

#	Student ID	Student Name	Grade (10)
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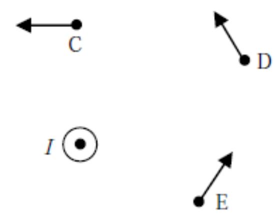
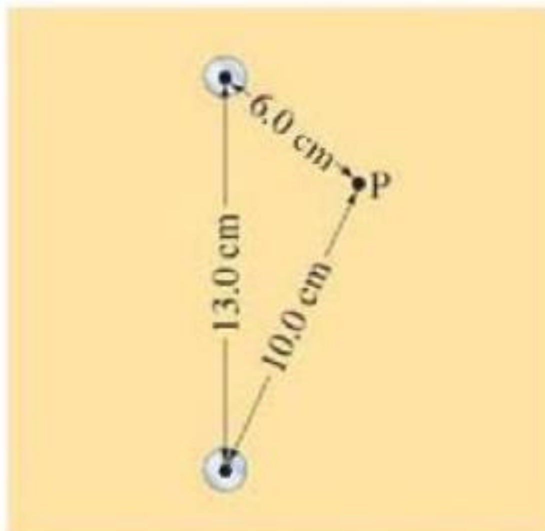
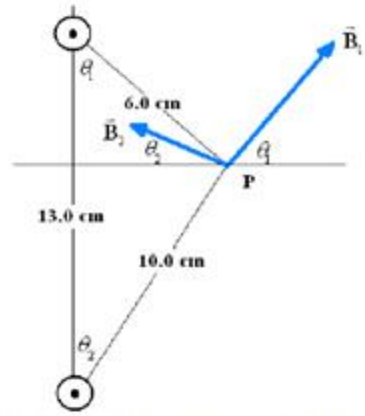
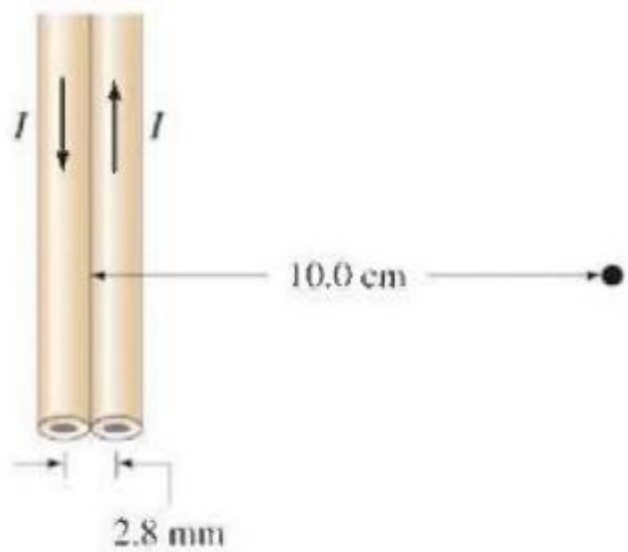
Q1	In Fig. a long straight wire carries current I out of the page toward the viewer. Indicate, with appropriate arrows, the direction of $\vec{\mathbf{B}}$ at each of the points C, D, and E in the plane of the page. 
Sol 1	To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle. 

Q 2	Jumper cables used to start a stalled vehicle often carry a 65-A current. How strong is the magnetic field 3.5 cm from one cable? Compare to the Earth's magnetic field (5.0×10^{-5} T).
Sol 2	We assume the jumper cable is a long straight wire, and use Eq. $B_{\text{cable}} = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(65\text{A})}{2\pi(0.035\,\text{m})} = 3.714 \times 10^{-4}\text{T} \approx \boxed{3.7 \times 10^{-4}\text{T}}$ Compare this to the Earth's field of $0.5 \times 10^{-4}\text{T}$. $B_{\text{cable}}/B_{\text{Earth}} = \dfrac{3.714 \times 10^{-4}\text{T}}{5.0 \times 10^{-5}\text{T}} = 7.43$, so $\boxed{\text{the field of the cable is over 7 times that of the Earth.}}$

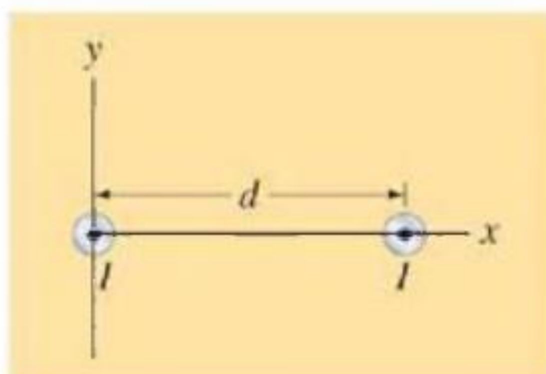
Q3	Determine the magnitude and direction of the force between two parallel wires 25 m long and 4.0 cm apart, each carrying 35 A in the same direction.
Sol 3	Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. to calculate the magnitude of the force. $F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 = \frac{(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A})}{2\pi}\frac{(35\,\mathrm{A})^2}{(0.040\,\mathrm{m})}(25\,\mathrm{m}) = \boxed{0.15\,\mathrm{N},\ \text{attractive}}$

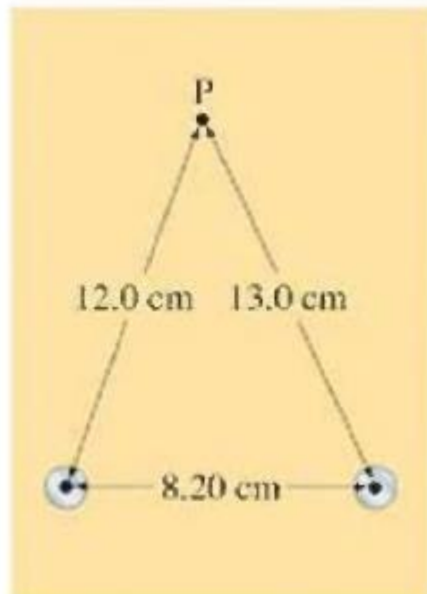
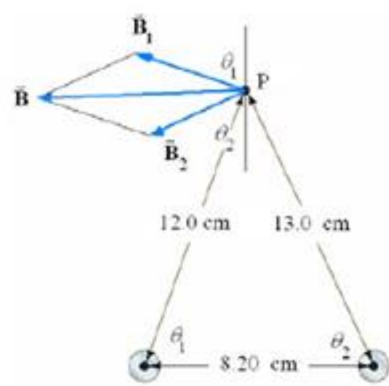
Q4	Two long thin parallel wires 13.0 cm apart carry 35-A currents in the same direction. Determine the magnetic field vector at a point 10.0 cm from one wire and 6.0 cm from the other 

Sol 4	Since the magnetic field from a current carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define $\vec{B}_1$ as the field from the top wire, and $\vec{B}_2$ as the field from the bottom wire. We use Eq. to calculate the magnitude of each individual field.  $B_1=\frac{\mu_0 I}{2\pi r_1}=\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(35\,\mathrm{A})}{2\pi(0.060\,\mathrm{m})}=1.17\times10^{-4}\,\mathrm{T}$ $B_2=\frac{\mu_0 I}{2\pi r_2}=\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(35\,\mathrm{A})}{2\pi(0.100\,\mathrm{m})}=7.00\times10^{-5}\,\mathrm{T}$ We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal. $\theta_1=\cos^{-1}\left(\frac{(0.060\,\mathrm{m})^2+(0.130\,\mathrm{m})^2-(0.100\,\mathrm{m})^2}{2(0.060\,\mathrm{m})(0.130\,\mathrm{m})}\right)=47.7^\circ$ $\theta_2=\cos^{-1}\left(\frac{(0.100\,\mathrm{m})^2+(0.130\,\mathrm{m})^2-(0.060\,\mathrm{m})^2}{2(0.100\,\mathrm{m})(0.130\,\mathrm{m})}\right)=26.3^\circ$ Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle. $B_{\mathrm{net}\,x}=B_1\cos(\theta_1)-B_2\cos\theta_2=(1.174\times10^{-4}\,\mathrm{T})\cos47.7^\circ-(7.00\times10^{-5}\,\mathrm{T})\cos26.3^\circ=1.626\times10^{-5}\,\mathrm{T}$ $B_{\mathrm{net}\,y}=B_1\sin(\theta_1)+B_2\sin\theta_1=(1.17\times10^{-4}\,\mathrm{T})\sin47.7^\circ+(7.00\times10^{-5}\,\mathrm{T})\sin26.3^\circ=1.18\times10^{-4}\,\mathrm{T}$ $B=\sqrt{B_{net,x}^2+B_{net,y}^2}=\sqrt{(1.626\times10^{-5}\ \mathrm{T})^2+(1.18\times10^{-4}\ \mathrm{T})^2}=1.19\times10^{-4}\ \mathrm{T}$ $\theta=\tan^{-1}\frac{B_{net,y}}{B_{net,y}}=\tan^{-1}\frac{1.18\times10^{-4}\ \mathrm{T}}{1.626\times10^{-5}\ \mathrm{T}}=82.2^\circ$ $\vec{\mathbf{B}}=1.19\times10^{-4}\ \mathrm{T}\ @\ 82.2^\circ\approx\boxed{1.2\times10^{-4}\mathrm{T}\ @\ 82^\circ}$

Q5	A long pair of insulated wires serves to conduct 28.0 A of dc current to and from an instrument. If the wires are of negligible diameter but are 2.8 mm apart, what is the magnetic field 10.0 cm from their midpoint, in their plane ? Compare to the magnetic field of the Earth. 
Sol 5	The fields created by the two wires will oppose each other, so the net field is the difference of the magnitudes of the two fields. The positive direction for the fields is taken to be into the paper, and so the closer wire creates a field in the positive direction, and the farther wire creates a field in the negative direction. Let *d* be the separation distance of the wires. $B_{\text{net}}=\frac{\mu_0 I}{2\pi r_{\text{closer}}}-\frac{\mu_0 I}{2\pi r_{\text{farther}}}=\frac{\mu_0 I}{2\pi}\left(\frac{1}{r_{\text{closer}}}-\frac{1}{r_{\text{farther}}}\right)=\frac{\mu_0 I}{2\pi}\left(\frac{1}{r-\frac{1}{2}d}-\frac{1}{r+\frac{1}{2}d}\right)$

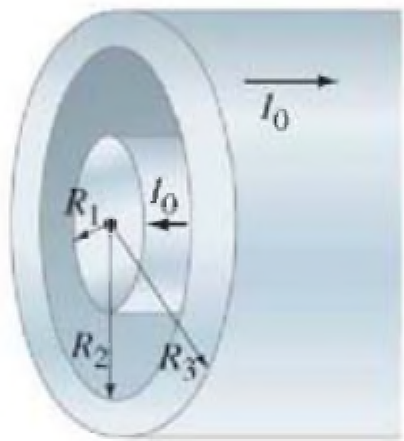
	$= \frac{\mu_0 I}{2\pi}\left(\frac{d}{(r-\frac{1}{2}d)(r+\frac{1}{2}d)}\right)$ $= \frac{(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A})(28.0\,\mathrm{A})}{2\pi}\left(\frac{0.0028\,\mathrm{m}}{(0.10\,\mathrm{m}-0.0014\,\mathrm{m})(0.10\,\mathrm{m}+0.0014\,\mathrm{m})}\right)$ $= 1.568\times10^{-6}\,\mathrm{T} \approx \boxed{1.6\times10^{-6}\,\mathrm{T}}$ Compare this to the Earth's field of $0.5\times10^{-4}\,\mathrm{T}$. $B_{net}/B_{Earth} = \frac{1.568\times10^{-6}\,\mathrm{T}}{0.5\times10^{-4}\,\mathrm{T}} = 0.031$ The field of the wires is about 3% that of the Earth.

Q6	Let two long parallel wires, a distance d apart, carry equal currents I in the same direction. One wire is at $x = 0$, the other at $x = d$, Fig. . Determine $\vec{\mathbf{B}}$ along the x axis between the wires as a function of x. 
Sol 6	The left wire will cause a field on the x axis that points in the y direction, and the right wire will cause a field on the x axis that points in the negative y direction. The distance from the left wire to a point on the x axis is x, and the distance from the right wire is $d-x$. $\vec{\mathbf{B}}_{net} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi(d-x)}\hat{\mathbf{j}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{1}{x}-\frac{1}{d-x}\right)\hat{\mathbf{j}}} = \boxed{\frac{\mu_0 I}{2\pi}\left(\frac{d-2x}{x(d-x)}\right)\hat{\mathbf{j}}}$

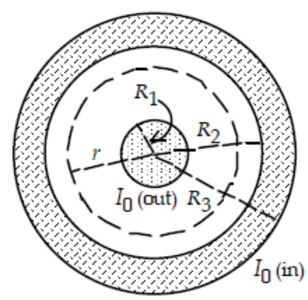
Q7	Two long parallel wires 8.20 cm apart carry 16.5-A currents in the same direction. Determine the magnetic field vector at a point P, 12.0 cm from one wire and 13.0 cm from the other. See Fig. [*Hint*: Use the law of cosines.] 
Sol 7	The net magnetic field is the vector sum of the magnetic fields produced by each current carrying wire. Since the individual magnetic fields encircle the wire producing it, the field is perpendicular to the radial line from the wire to point P. We let $\vec{\mathbf{B}}_1$ be the field from the left wire, and $\vec{\mathbf{B}}_2$ designate the field from the right wire. The magnitude of the magnetic field vectors is calculated from Eq. .  $$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(16.5\,\mathrm{A})}{2\pi(0.12\,\mathrm{m})} = 2.7500\times10^{-5}\,\mathrm{T}$$ $$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(16.5\mathrm{A})}{2\pi(0.13\mathrm{m})} = 2.5385\times10^{-5}\,\mathrm{T}$$ We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the horizontal. Since the magnetic fields are perpendicular to the radial lines, these angles are the same as the angles the magnetic fields make with the vertical. $$\theta_1 = \cos^{-1}\left(\frac{(0.12\,\mathrm{m})^2 + (0.082\,\mathrm{m})^2 - (0.13\,\mathrm{m})^2}{2(0.12\,\mathrm{m})(0.082\,\mathrm{m})}\right) = 77.606^\circ$$ $$\theta_2 = \cos^{-1}\left(\frac{(0.13\,\mathrm{m})^2 + (0.082\,\mathrm{m})^2 - (0.12\,\mathrm{m})^2}{2(0.13\,\mathrm{m})(0.082\,\mathrm{m})}\right) = 64.364^\circ$$

	Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle. $B_{\text{net }x} = -B_1\sin(\theta_1) - B_2\sin\theta_2 = -(2.7500\times10^{-5}\text{T})\sin 77.606° - (2.5385\times10^{-5}\text{T})\sin 64.364°$ $= -49.75\times10^{-6}\text{T}$ $B_{\text{net }y} = B_1\cos(\theta_1) - B_2\cos\theta_1 = (2.7500\times10^{-5}\text{T})\cos 77.606° - (2.5385\times10^{-5}\text{T})\cos 64.364°$ $= -5.080\times10^{-6}\text{T}$ $B = \sqrt{B_{net,x}^2 + B_{net,y}^2} = \sqrt{(-49.75\times10^{-6}\ \text{T})^2 + (-5.080\times10^{-6}\ \text{T})^2} = 5.00\times10^{-5}\ \text{T}$ $\theta = \tan^{-1}\frac{B_{net,y}}{B_{net,y}} = \tan^{-1}\frac{-5.08\times10^{-6}\ \text{T}}{-49.75\times10^{-6}\ \text{T}} = 5.83°$ $\boxed{\vec{\mathbf{B}} = 5.00\times10^{-5}\ \text{T @ } 5.83° \text{ below the negative } x\text{-axis}}$

Q8	A 32-cm-long solenoid, 1.8 cm in diameter, is to produce a 0.30-T magnetic field at its center. If the maximum current is 4.5 A, how many turns must the solenoid have?
Sol 8	The field inside a solenoid is given by Eq. 28-4. $B = \frac{\mu_0 IN}{\ell} \rightarrow N = \frac{B\ell}{\mu_0 I} = \frac{(0.30\,\text{T})(0.32\,\text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.5\,\text{A})} = \boxed{1.7\times10^4\ \text{turns}}$

Q9	(II) A coaxial cable consists of a solid inner conductor of radius R_1, surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3. The conductors carry equal and opposite currents I_0 distributed uniformly across their cross sections. Determine the magnetic field at a distance R from the axis for: (*a*) $R < R_1$; (*b*) $R_1 < R < R_2$; (*c*) $R_2 < R < R_3$; (*d*) $R > R_3$. (*e*) Let $I_0 = 1.50$ A, $R_1 = 1.00$ cm, $R_2 = 2.00$ cm, and $R_3 = 2.50$ cm. Graph B from $R = 0$ to $R = 3.00$ cm. 
Sol 9	see below

Because of the cylindrical symmetry, the magnetic fields will be circular. In each case, we can determine the magnetic field using Ampere's law with concentric loops. The current densities in the wires are given by the total current divided by the cross-sectional area.



$$J_{inner} = \frac{I_0}{\pi R_1^2} \qquad J_{outer} = -\frac{I_0}{\pi\left(R_3^2 - R_2^2\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by the current density of the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{encl} = \mu_0\left(J_{inner}\pi R^2\right)$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^2}{\pi R_1^2} \rightarrow \boxed{B = \frac{\mu_0 I_0 R}{2\pi R_1^2}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{encl} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

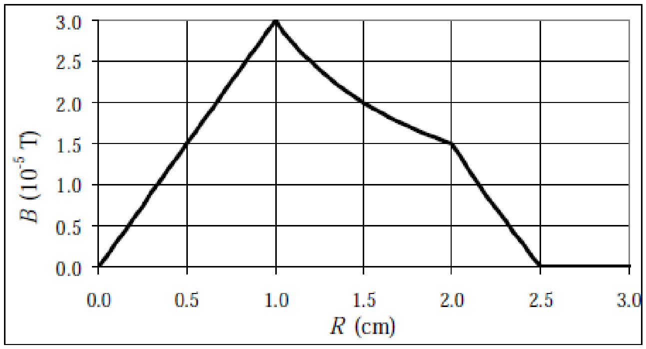
$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{encl} = \mu_0\left[I_0 + J_{outer}\pi\left(R^2 - R_2^2\right)\right]$$

$$B(2\pi r) = \mu_0\left[I_0 - I_0\frac{\pi\left(R^2 - R_2^2\right)}{\pi\left(R_3^2 - R_2^2\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^2 - R^2\right)}{\left(R_3^2 - R_2^2\right)}}$$

(*d*) Outside the outer wire the net current enclosed is zero.

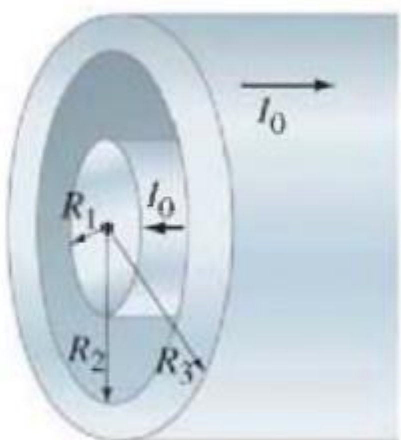
$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \;\rightarrow\; B(2\pi R) = 0 \;\rightarrow\; \boxed{B = 0}$$

(*e*) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH28.XLS," on tab "Problem 28.31e."



Q10

Suppose the current in the coaxial cable of Problem 9 , Fig. is not uniformly distributed, but instead the current density j varies linearly with distance from the center: $j_1 = C_1 R$ for the inner conductor and $j_2 = C_2 R$ for the outer conductor. Each conductor still carries the same total current I_0, in opposite directions. Determine the magnetic field in terms of I_0 in the same four regions of space as in Problem 9 .



Sol 10

We first find the constants C_1 and C_2 by integrating the currents over each cylinder and setting the integral equal to the total current.

$$I_0 = \int_0^{R_1} C_1 R 2\pi R dR = 2\pi C_1 \int_0^{R_1} R^2 dR = \frac{2}{3}\pi R_1^3 C_1 \;\rightarrow\; C_1 = \frac{3I_0}{2\pi R_1^3}$$

$$-I_0 = 2\pi C_2 \int_{R_2}^{R_3} R^2 dR = \frac{2}{3}\pi\left(R_3^3 - R_2^3\right)C_2 \;\rightarrow\; C_2 = \frac{-3I_0}{2\pi\left(R_3^3 - R_2^3\right)}$$

(*a*) Inside the inner wire the enclosed current is determined by integrating the current density inside the radius R.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \int_0^{R} (C_1 R') 2\pi R' dR' = \frac{2}{3}\mu_0 \pi C_1 R^3 = \frac{2}{3}\mu_0 \pi \left(\frac{3I_0}{2\pi R_1^3}\right) R^3$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^3}{\pi R_1^3} \rightarrow \boxed{B = \frac{\mu_0 I_0 R^2}{2\pi R_1^3}}$$

(*b*) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} \;\rightarrow\; B(2\pi R) = \mu_0 I_0 \;\rightarrow\; \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

(*c*) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0\left[I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR\right] = \mu_0\left[I_0 + \int_{R_2}^{R} (C_2 R) 2\pi R dR\right]$$

$$= \mu_0 I_0\left[1 - \frac{2}{3}\pi C_2\left(R^3 - R_2^3\right)\right] = \mu_0\left[I_0 - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)}\right]$$

$$B(2\pi r) = \mu_0 I_0\left[\frac{\left(R_3^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)} - \frac{\left(R^3 - R_2^3\right)}{\left(R_3^3 - R_2^3\right)}\right] \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}\frac{\left(R_3^3 - R^3\right)}{\left(R_3^3 - R_2^3\right)}}$$

(*d*) Outside the outer wire the net current enclosed is zero.

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \;\rightarrow\; B(2\pi R) = 0 \;\rightarrow\; \boxed{B = 0}$$