Electromagnetic Fields – Tutorial 07

Magnetism II

#	Student ID	Student Name	Grade (10)
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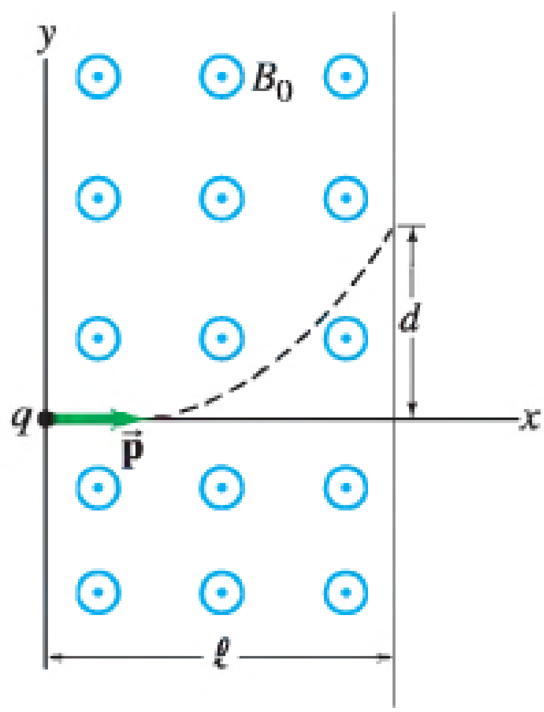
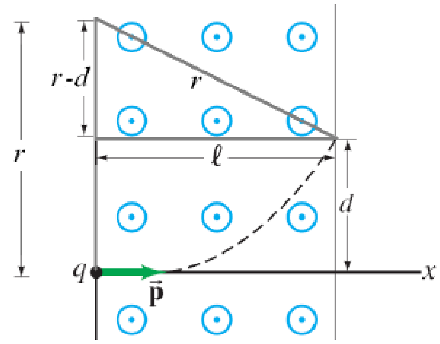
Q1	What is the velocity of a beam of electrons that goes undeflected when passing through perpendicular electric and magnetic fields of magnitude 8.8×10^3 V/m and 7.5×10^{-3} T, respectively? What is the radius of the electron orbit if the electric field is turned off?
Sol 1	The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force. $F_E = F_B \rightarrow qE = qvB \rightarrow v = \frac{E}{B} = \frac{8.8\times10^3\ \text{V/m}}{7.5\times10^{-3}\text{T}} = 1.173\times10^6\ \text{m/s} \approx \boxed{1.2\times10^6\ \text{m/s}}$ If the electric field is turned off, the magnetic force will cause circular motion. $F_B = qvB = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\text{kg})(1.173\times10^6\ \text{m/s})}{(1.60\times10^{-19}\text{C})(7.5\times10^{-3}\text{T})} = \boxed{8.9\times10^{-4}\ \text{m}}$

Q2	For a particle of mass m and charge q moving in a circular path in a magnetic field B, (a) show that its kinetic energy is proportional to r^2, the square of the radius of curvature of its path, and (b) show that its angular momentum is $L = qBr^2$, about the center of the circle.
Sol 2	(a) we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. The kinetic energy is given by $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m}$ and so we see that $\boxed{K \propto r^2}$. (b) The angular momentum of a particle moving in a circular path is given by $L = mvr$. , we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. Combining these relationships gives $L = mvr = m\frac{rqB}{m}r = \boxed{qBr^2}$.

Q 3	An electron experiences the greatest force as it travels 2.8×10^6 m/s in a magnetic field when it is moving northward. The force is vertically upward and of magnitude 8.2×10^{-13} N. What is the magnitude and direction of the magnetic field?
Sol 3	The magnetic field can be found from Eq. , and the direction is found from the right hand rule. Remember that the charge is negative. $$F_{max} = qvB \rightarrow B = \frac{F_{max}}{qv} = \frac{8.2\times10^{-13}\,\text{N}}{(1.60\times10^{-19}\,\text{C})(2.8\times10^{6}\,\text{m/s})} = \boxed{1.8\,\text{T}}$$ The direction would have to be $\boxed{\text{East}}$ for the right hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

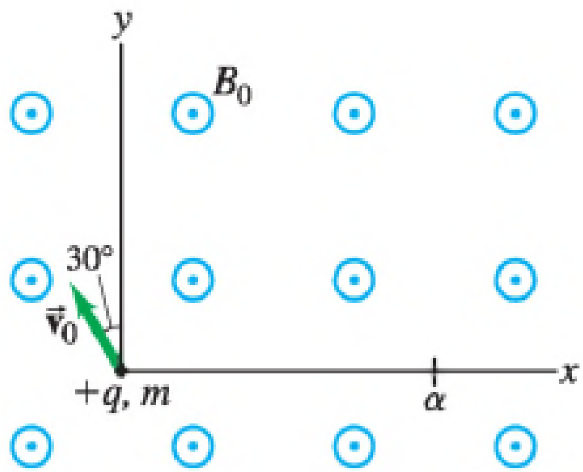
Q4	A particle with charge q and momentum p, initially moving along the x axis, enters a region where a uniform magnetic field $\vec{\mathbf{B}} = B_0\hat{\mathbf{k}}$ extends over a width $x = \ell$ as shown in Fig. The particle is deflected a distance d in the $+y$ direction as it traverses the field. Determine (a) whether q is positive or negative, and (b) the magnitude of its momentum p. 

Sol 4	(*a*) For the particle to move upward the magnetic force must point upward, by the right hand rule we see that the force on a positively charged particle would be downward. Therefore, the charge on the particle must be **negative**. (*b*) In the figure we have created a right triangle to relate the horizontal distance ℓ, the displacement *d*, and the radius of curvature, *r*. Using the Pythagorean theorem we can write an expression for the radius in terms of the other two distances.  $r^2=(r-d)^2+\ell^2 \rightarrow r=\frac{d^2+\ell^2}{2d}$ Since the momentum is perpendicular to the magnetic field, we can solve for the momentum by relating the maximum force (Eq. 27-5b) to the centripetal force on the particle. $F_{\text{max}}=qvB_0=\frac{mv^2}{r} \rightarrow p=mv=qB_0r=\boxed{\frac{qB_0(d^2+\ell^2)}{2d}}$

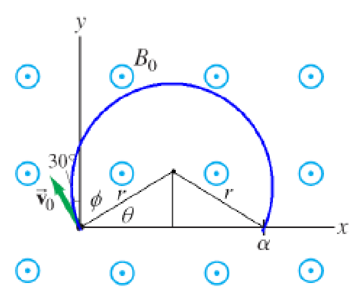
Q5	Suppose the Earth's magnetic field at the equator has magnitude 0.50×10^{-4} T and a northerly direction at all points. Estimate the speed a singly ionized uranium ion ($m = 238$ u, $q = e$) would need to circle the Earth 5.0 km above the equator. Can you ignore gravity? [Ignore relativity.]
Sol 5	The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is 6.38×10^6 km, and the altitude is added to that. $F_B=qvB=m\frac{v^2}{r} \rightarrow v=\frac{qrB}{m}=\frac{(1.60\times10^{-19}\text{C})(6.385\times10^6\text{m})(0.50\times10^{-4}\text{T})}{238(1.66\times10^{-27}\text{kg})}=\boxed{1.3\times10^8\text{ m/s}}$ Compare the size of the magnetic force to the force of gravity on the ion. $\frac{F_B}{F_g}=\frac{qvB}{mg}=\frac{(1.60\times10^{-19}\text{C})(1.3\times10^8\text{ m/s})(0.50\times10^{-4}\text{T})}{238(1.66\times10^{-27}\text{kg})(9.80\text{ m/s}^2)}=2.3\times10^8$ **Yes**, we may ignore gravity. The magnetic force is more than 200 million times larger than gravity.

Q6

A particle with charge $+q$ and mass m travels in a uniform magnetic field $\vec{\mathbf{B}} = B_0\hat{\mathbf{k}}$. At time $t = 0$, the particle's speed is v_0 and its velocity vector lies in the xy plane directed at an angle of 30° with respect to the y axis as shown in Fig. At a later time $t = t_\alpha$, the particle will cross the x axis at $x = \alpha$. In terms of q, m, v_0, and B_0, determine (a) α, and (b) t_α.



Sol 6



(a) Since the velocity is perpendicular to the magnetic field, the particle will follow a circular trajectory in the x-y plane of radius r. The radius is found using the centripetal acceleration.

$$qvB = \frac{mv^2}{r} \quad \to \quad r = \frac{mv}{qB}$$

From the figure we see that the distance α is the chord distance, which is twice the distance $r\cos\theta$. Since the velocity is perpendicular to the radial vector, the initial direction and the angle ϕ are complementary angles. The angles ϕ and θ are also complementary angles, so $\theta = 30°$.

$$\alpha = 2r\cos\theta = \frac{2mv_0}{qB_0}\cos 30° = \boxed{\sqrt{3}\frac{mv_0}{qB_0}}$$

(b) From the diagram, we see that the particle travels a circular path, that is 2ϕ short of a complete circle. Since the angles ϕ and θ are complementary angles, so $\phi = 60°$. The trajectory distance is equal to the circumference of the circular path times the fraction of the complete circle. Dividing the distance by the particle speed gives t_α.

$$t_\alpha = \frac{\ell}{v_0} = \frac{2\pi r}{v_0}\left(\frac{360° - 2(60°)}{360°}\right) = \frac{2\pi}{v_0}\frac{mv_0}{qB_0}\left(\frac{2}{3}\right) = \boxed{\frac{4\pi m}{3qB_0}}$$

Q 7	How much work is required to rotate the current loop in a uniform magnetic field $\vec{\mathbf{B}}$ from (*a*) $\theta = 0°$ $(\vec{\mu} \parallel \vec{\mathbf{B}})$ to $\theta = 180°$, (*b*) $\theta = 90°$ to $\theta = -90°$?
So l7	The work required by an external agent is equal to the change in potential energy. The potential energy is given by Eq. $U = -\vec{\mu}\cdot\vec{\mathbf{B}}$. (*a*) $W = \Delta U = \left(-\vec{\mu}\cdot\vec{\mathbf{B}}\right)_{final} - \left(-\vec{\mu}\cdot\vec{\mathbf{B}}\right)_{initial} = \left(\vec{\mu}\cdot\vec{\mathbf{B}}\right)_{initial} - \left(\vec{\mu}\cdot\vec{\mathbf{B}}\right)_{final} = NIAB\left(\cos\theta_{initial} - \cos\theta_{final}\right)$ $= NIAB\left(\cos 0° - \cos 180°\right) = \boxed{2NIAB}$ (*b*) $W = NIAB\left(\cos\theta_{initial} - \cos\theta_{final}\right) = NIAB\left(\cos 90° - \cos(-90°)\right) = \boxed{0}$

Q 8	A 15-loop circular coil 22 cm in diameter lies in the *xy* plane. The current in each loop of the coil is 7.6 A clockwise, and an external magnetic field $\vec{\mathbf{B}} = (0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}})$ T passes through the coil. Determine (*a*) the magnetic moment of the coil, $\vec{\mu}$; (*b*) the torque on the coil due to the external magnetic field; (*c*) the potential energy *U* of the coil in the field (take the same zero for *U* as we did in our discussion of
So l8	(*a*) The magnetic moment of the coil is given by Eq. . Since the current flows in the clockwise direction, the right hand rule shows that the magnetic moment is down, or in the negative *z*-direction. $\vec{\mu} = NI\vec{\mathbf{A}} = 15(7.6\,\mathrm{A})\pi\left(\frac{0.22\,\mathrm{m}}{2}\right)^2\left(-\hat{\mathbf{k}}\right) = -4.334\,\hat{\mathbf{k}}\,\mathrm{A\cdot m^2} \approx \boxed{-4.3\hat{\mathbf{k}}\,\mathrm{A\cdot m^2}}$ (*b*) We use Eq. 27-11 to find the torque on the coil. $\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}} = \left(-4.334\,\hat{\mathbf{k}}\,\mathrm{A\cdot m^2}\right)\times\left(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}}\right)\mathrm{T} = \boxed{\left(2.6\hat{\mathbf{i}} - 2.4\hat{\mathbf{j}}\right)\mathrm{m\cdot N}}$ (*c*) We use Eq. 27-12 to find the potential energy of the coil. $U = -\vec{\mu}\cdot\vec{\mathbf{B}} = -\left(-4.334\,\hat{\mathbf{k}}\,\mathrm{A\cdot m^2}\right)\left(0.55\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}} - 0.65\hat{\mathbf{k}}\right)\mathrm{T} = -\left(4.334\,\mathrm{A\cdot m^2}\right)\left(0.65\,\mathrm{T}\right)$ $= \boxed{-2.8\,\mathrm{J}}$

Q9	If the current to a motor drops by 12%, by what factor does the output torque change?
Sol 9	, we see that the torque is proportional to the current, so if the current drops by 12%, the output torque will also drop by 12%. Thus the final torque is 0.88 times the initial torque.

Q10	What is the value of q/m for a particle that moves in a circle of radius 8.0 mm in a 0.46-T magnetic field if a crossed 260-V/m electric field will make the path straight?
Sol 10	$\frac{q}{m} = \frac{E}{B^2 r} = \frac{(260\,\text{V/m})}{(0.46\,\text{T})^2 (0.0080\,\text{m})} = \boxed{1.5\times 10^5\,\text{C/kg}}$

Q11	In a probe that uses the Hall effect to measure magnetic fields, a 12.0-A current passes through a 1.50-cm-wide 1.30-mm-thick strip of sodium metal. If the Hall emf is 1.86 μV, what is the magnitude of the magnetic field (take it perpendicular to the flat face of the strip)? Assume one free electron per atom of Na, and take its specific gravity to be 0.971.
Sol 11	We find the magnetic field using Eq. 27-14, with the drift velocity given by Eq. 25-13. To determine the electron density we divide the density of sodium by its atomic weight. This gives the number of moles of sodium per cubic meter. Multiplying the result by Avogadro's number gives the number of sodium atoms per cubic meter. Since there is one free electron per atom, this is also the density of free electrons. $B = \frac{\mathcal{E}_H}{v_d d} = \frac{\mathcal{E}_H}{\left(\frac{I}{ne(td)}\right)d} = \frac{\mathcal{E}_H net}{I} = \frac{\mathcal{E}_H et}{I}\left(\frac{\rho N_A}{m_A}\right)$ $= \frac{(1.86\times10^{-6}\text{V})(1.60\times10^{-19}\text{C})(1.30\times10^{-3}\text{m})}{12.0\text{ A}}\frac{(0.971)(1000\text{ kg/m}^3)(6.022\times10^{23}\text{e/mole})}{0.02299\text{ kg/mole}}$ $= \boxed{0.820\text{T}}$