Lecture (08)
Magnetic Sources

By:

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Magnetic Field Due to a Straight Wire

- We saw that the magnetic field due to the electric current in a long straight wire is such that the field lines are circles with the wire at the center.
- field B due to a long straight wire at a point near it is directly proportional to the current I in the wire and inversely proportional to the distance r from the wire:

$$B \propto \frac{I}{r}$$

- The proportionality constant is written as

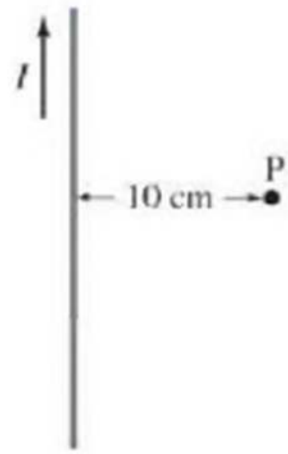
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \qquad \mu_0/2\pi;$$

- μ_0, **which is called the permeability of free space,**

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}.$$

Example 01

- An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P, 10 cm due north of the wire



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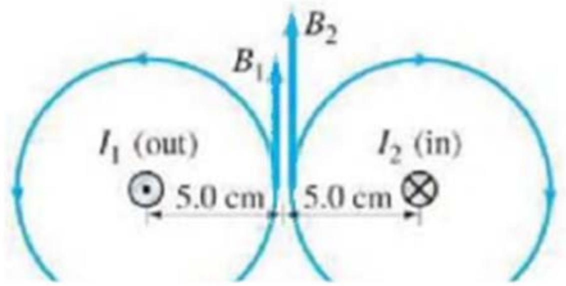
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(25\,\mathrm{A})}{(2\pi)(0.10\,\mathrm{m})} = 5.0 \times 10^{-5}\,\mathrm{T},$$

Example 02

- Two parallel straight wires 10.0 cm apart carry currents in opposite directions
- Current *I1 =* 5.0 A is out of the page, and *I2 =* 7.0 A is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires



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$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(5.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.0 \times 10^{-5}\,\mathrm{T};$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(7.0\,\mathrm{A})}{2\pi(0.050\,\mathrm{m})} = 2.8 \times 10^{-5}\,\mathrm{T}.$$

$$B = B_1 + B_2 = 4.8 \times 10^{-5}\,\mathrm{T}.$$

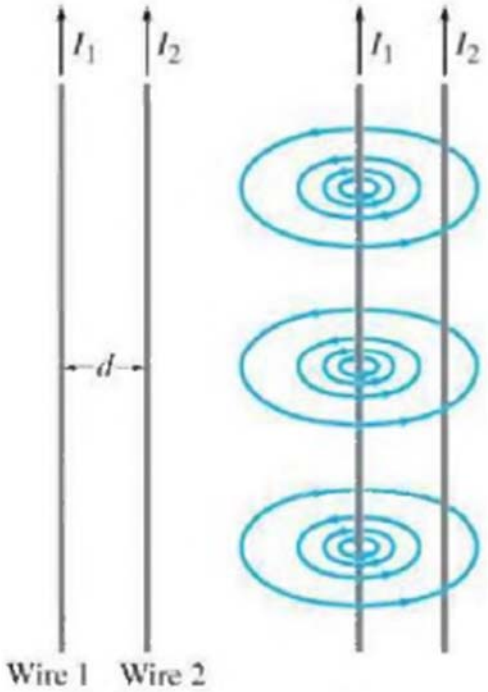
Force between Two Parallel Wires

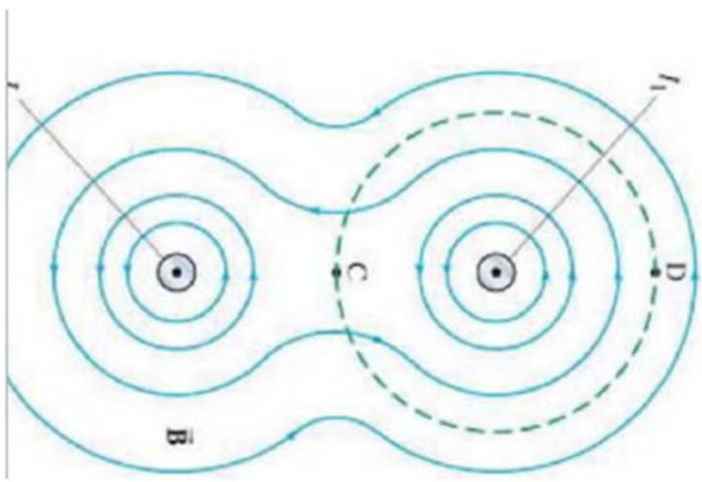
- Consider two long parallel wires separated by a distance *d,* They carry currents *I1* and *I*2, respectively. Each current produces a magnetic field that is "felt" by the other, so each must exert a force on the other.
- magnetic field *B1* produced by I1

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

- force *F2* exerted by *B1* on a length *i2* of wire 2, $F_2 = I_2 B_1 \ell_2$
- substitute *B1*

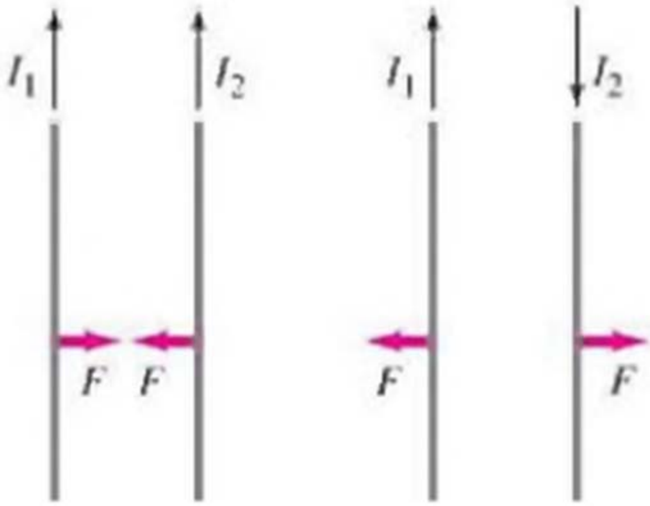
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2$$





Magnetic field lines around two long parallel wires whose equal currents, /1 and *I2,* are coming out of the paper toward the viewer.

- If we use right-hand-rule
- exerts an attractive force, as long as the currents are in the same direction
- force is in the opposite direction currents are in the opposite direction



Example 03

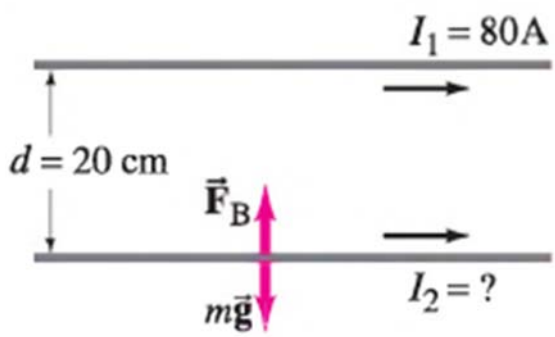
- **The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.**

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2$$

$$F = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(8.0\,\mathrm{A})^2(2.0\,\mathrm{m})}{(2\pi)(3.0 \times 10^{-3}\,\mathrm{m})} = 8.5 \times 10^{-3}\,\mathrm{N}.$$

Example 04

- **Suspending a wire with a current. A horizontal wire**
- **carries a current *I1 =* 80 A dc. A second parallel wire 20 cm below it (must carry how much current *I2* so that it doesn't fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.**



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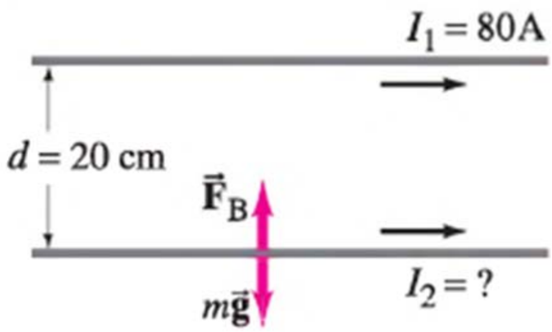


$$F = mg = (0.12 \times 10^{-3}\,\text{kg/m})(1.0\,\text{m})(9.8\,\text{m/s}^2) = 1.18 \times 10^{-3}\,\text{N}.$$

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell$$

$$I_2 = \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right) = \frac{2\pi(0.20\,\text{m})}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(80\,\text{A})}\,\frac{(1.18 \times 10^{-3}\,\text{N/m})}{(1.0\,\text{m})} = 15\,\text{A}.$$

Definitions of the Ampere

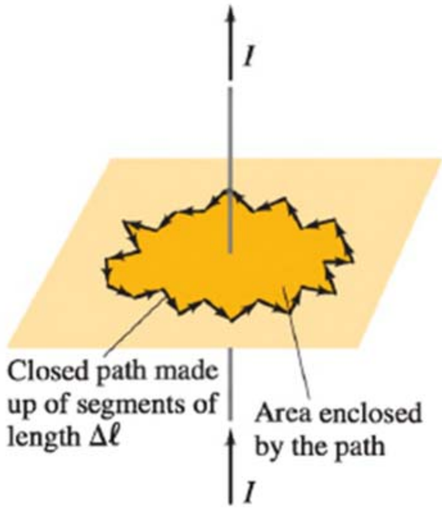
- $\mu_0 = 4\pi \times 10^{-7}$ T.m/A
- The ampere, the unit of current
- Ampere is defined in terms of the magnetic field *B* it produces using the defined value of μ_0
- **we use the force between two parallel current-carrying wires, If *I1 = I2 =* 1 A, and the two wires are exactly 1 m apart**

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})}{(2\pi)}\,\frac{(1\,\text{A})(1\,\text{A})}{(1\,\text{m})} = 2 \times 10^{-7}\,\text{N/m}.$$

- *one* **ampere** *is defined as that current flowing in each o f two long parallel wires 1 m apart, which results in a force o f exactly 2* **X** *10^{-7} N per meter o f length*

Ampere's Law

- general relation between a current in a wire of any shape and the magnetic field around it.
- Consider a closed path around a current as shown in Fig.
- imagine this path as being made up of short segments each of length ΔL
- First, we take the product of the length of each segment times the component of B parallel to that segment (call this component B||).



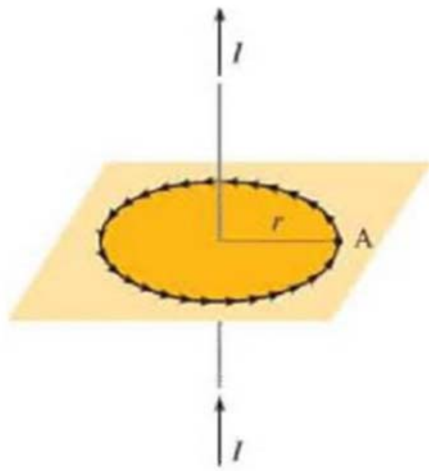
- **Sum all these terms, according to Ampere, the result will be equal to μ_0 times the net current I_{encl}**

$$\sum B_{\|} \Delta \ell = \mu_0 I_{\text{encl}}$$

- **lengths ΔL are chosen so that $B_{||}$ is essentially constant along each length.**
- **sum must be made over a *closed path*; and I_{encl} is the net current passing through the surface bounded by this closed path, In the limit $\Delta l \rightarrow 0$,**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}$$

- let us apply it to the simple case of a single long straight wire carrying a current *I* which we've already examined.
- We know the magnetic field lines are circles with the wire at their center.
- we choose as our path of integration a circle of radius *r*
- B will be tangent to the circle.
- since all points on the path are the same distance from the wire, by symmetry we expect *B* to have the same magnitude at each point.
- $I_{encl} = I$



- $$\mu_0 I = \oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}}$$

$$= \oint B\, d\ell = B \oint d\ell =$$

$$\oint d\ell = 2\pi r,$$

$$= B(2\pi r).$$

$$B = \frac{\mu_0 I}{2\pi r}.$$

notes

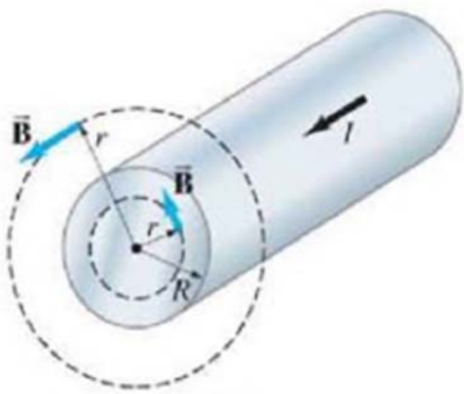
- $F_2 = I_2 B_1 \ell_2$

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2$$

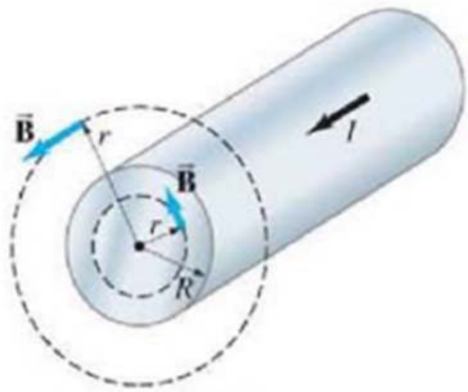
$$B = \frac{\mu_0 I}{2\pi r}.$$

Example 05

- **A long straight cylindrical wire conductor of radius *R* carries a current *I* of uniform current density in the conductor.**
- **Determine the magnetic field due to this current at**
- *(a)* **points outside the conductor** *(r > R),*
- **(b) points inside the conductor** *(r < R).*
- **(c) If *R* = 2.0 mm and *I* = 60 A, what is *B* at *r* = 1.0 mm, *r* = 2.0 mm, and *r* = 3.0 mm?**



- **wire is long, straight, and cylindrical,**
- **we expect from symmetry that the magnetic field must be the same at all points that are the same distance from the center of the conductor.**
- **So *B* must have the same value at all points the same distance from the center.**
- **We also expect B to be tangent to circles around the wire**



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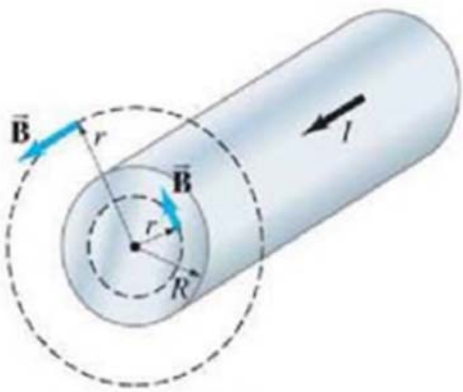

(*a*) We apply Ampère's law, integrating around a circle $(r > R)$ centered on the wire (Fig. 28–11a), and then $I_{encl} = I$:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = B(2\pi r) = \mu_0 I_{encl}$$

or

$$B = \frac{\mu_0 I}{2\pi r}, \qquad [r > R]$$

which is the same result as for a thin wire.

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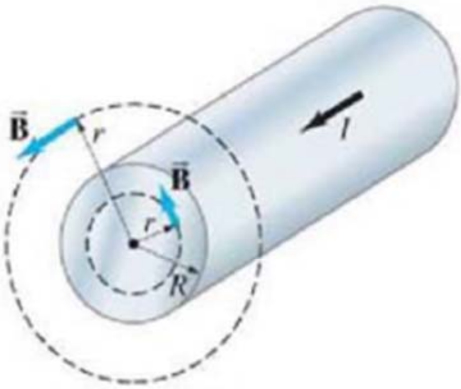


(*b*) Inside the wire ($r < R$), we again choose a circular path concentric with the cylinder; we expect $\vec{\mathbf{B}}$ to be tangential to this path, and again, because of the symmetry, it will have the same magnitude at all points on the circle. The current enclosed in this case is less than I by a factor of the ratio of the areas:

$$I_{\text{encl}} = I\,\frac{\pi r^2}{\pi R^2}.$$

So Ampère's law gives

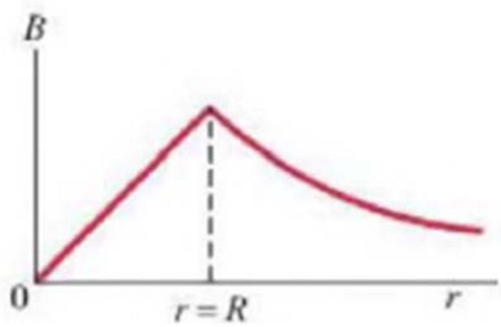
$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

so

$$B = \frac{\mu_0 I r}{2\pi R^2}. \qquad [r < R]$$

- **The field is zero at the center of the conductor and increases linearly with *r* until *r = R;* beyond *r = R, B* decreases as *1/r.***



If R = 2.0 mm and I = 60 A, what is B at r = 1.0 mm, r = 2.0 mm, and r = 3.0 mm?

(*c*) At $r = 2.0\,\text{mm}$, the surface of the wire, $r = R$, so

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(60\,\text{A})}{(2\pi)(2.0 \times 10^{-3}\,\text{m})} = 6.0 \times 10^{-3}\,\text{T}.$$

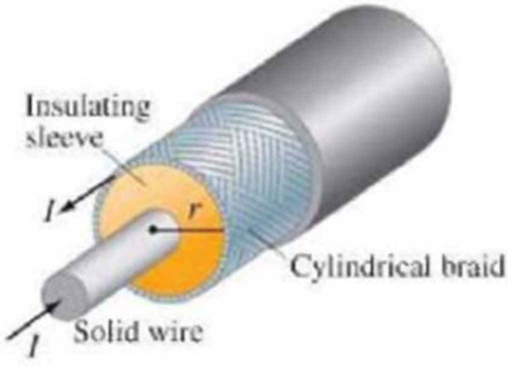
- At r=3

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\, 10^{-7} \times 60}{2\,\pi \times 3 \times 10^{-3}}$$

At r=1

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{4\pi\, 10^{-7} \times 60 \times (1 \times 10^{-3})}{2\,\pi\,(2 \times 10^{-3})^2}$$

- **Coaxial cable. A *coaxial cable* is a single wire surrounded by a cylindrical metallic braid,**
- **The two conductors are separated by an insulator.**
- **The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground.**
- **Describe the magnetic field.**
- ***(a)* in the space between the conductors, and**
- **(*b*) outside the cable**

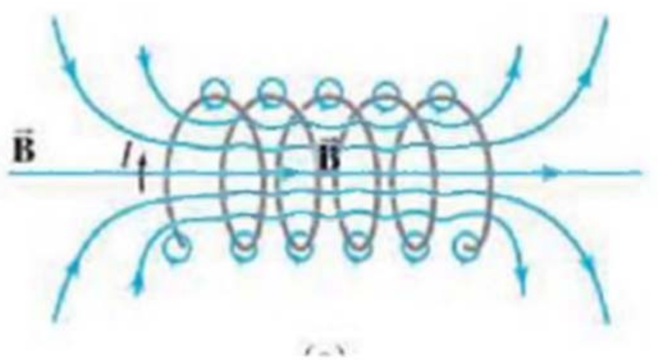


- *(a)* **In the space between the conductors, apply Ampere's law for a circular path around the center wire, magnetic field lines will be concentric circles centered on the center of the wire, and the magnitude** $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

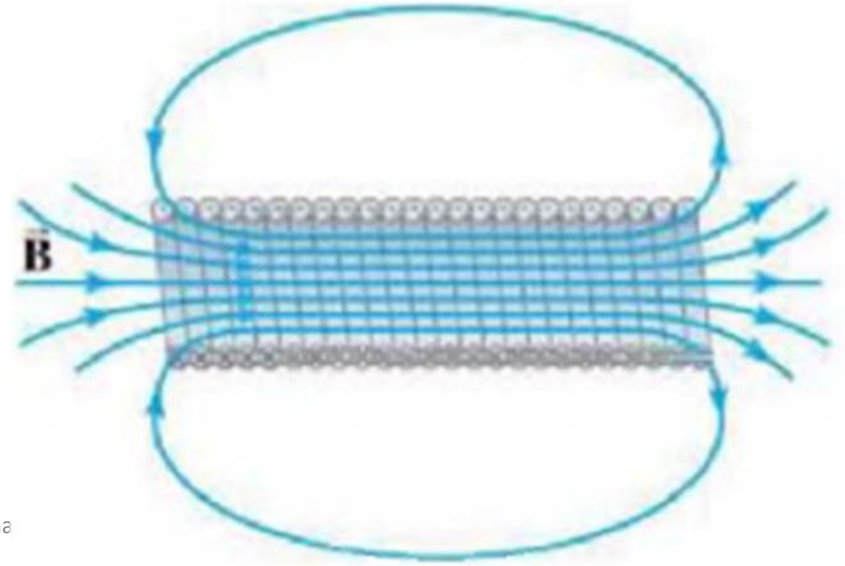
- *(b)* **Outside the cable, we can draw a similar circular path, Now, however, there are two currents enclosed by the path, and they add up to zero. The field outside the cable is zero.**

Magnetic Field of a Solenoid and a Toroid

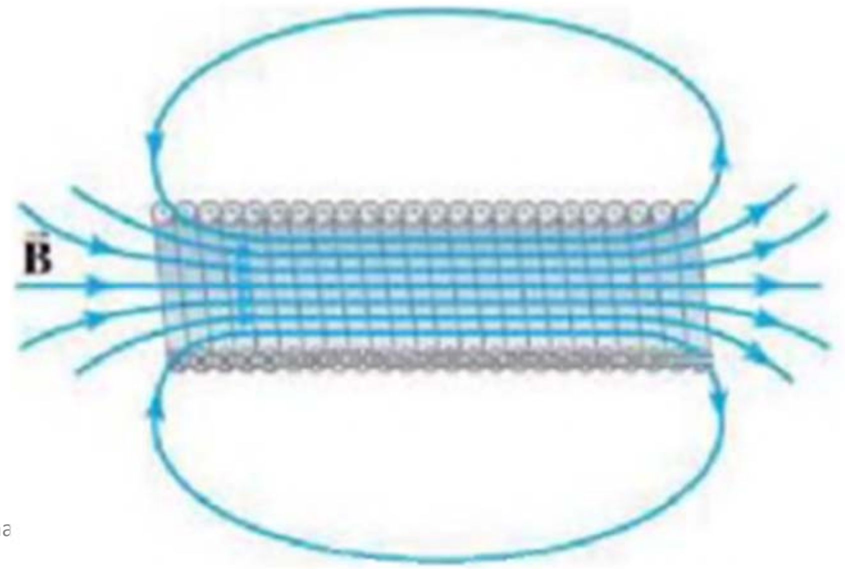
- A long coil of wire consisting of many loops is called a solenoid.
- Each loop produces a magnetic field
- Near each wire, the field lines are very nearly circles as for a straight wire (that is, at distances that are small compared to the curvature of the wire).
- Between any two wires, the fields due to each loop tend to cancel.



- Toward the center of the solenoid, the fields add up to give a field fairly large and fairly uniform.
- For a long solenoid with closely packed coils, the field is nearly uniform and parallel to the solenoid axis within the entire cross section,

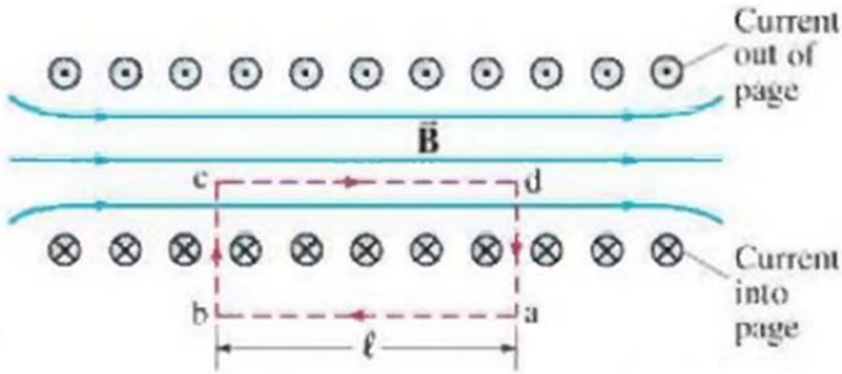


- The field outside the solenoid is very small compared to the field inside, except near the ends.
- Note that the same number of field lines that are concentrated inside the solenoid, spread out into the vast open space outside



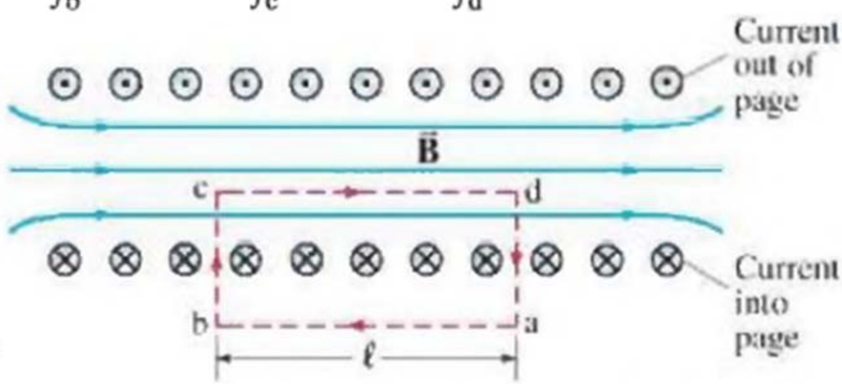
- **use Ampere's law to determine the magnetic field inside a very long (ideally, infinitely long) closely packed solenoid**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \int_a^b \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_b^c \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_d^a \vec{\mathbf{B}} \cdot d\vec{\ell}.$$



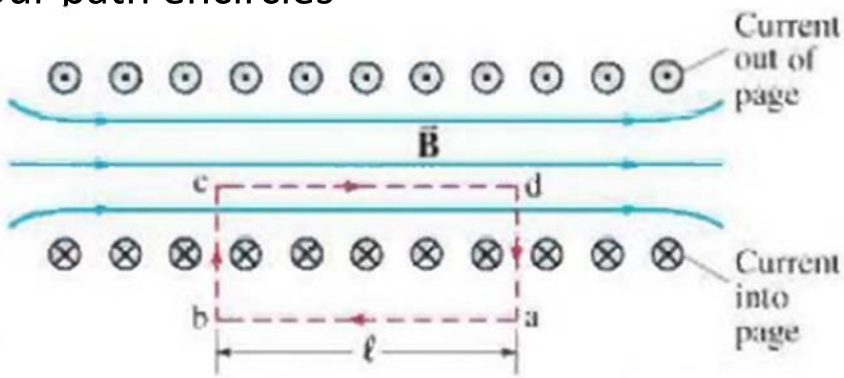
- **The field outside the solenoid is so small as to be negligible compared to the field inside. Thus the first term in this sum will be zero.**
- **B is perpendicular to the segments be and da inside the solenoid, and is nearly zero between and outside the coils,**

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \int_a^b \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_b^c \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_c^d \vec{\mathbf{B}} \cdot d\vec{\ell} + \int_d^a \vec{\mathbf{B}} \cdot d\vec{\ell}.$$



$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \int_c^d \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = B\ell,$$

- where *l* is the length cd. Now we determine the current enclosed by this loop for the right side of Ampere's law.
- If a current *I* flows in the wire of the solenoid,
- the total current enclosed by our path abed is *N I* where *N* is the number of loops our path encircles



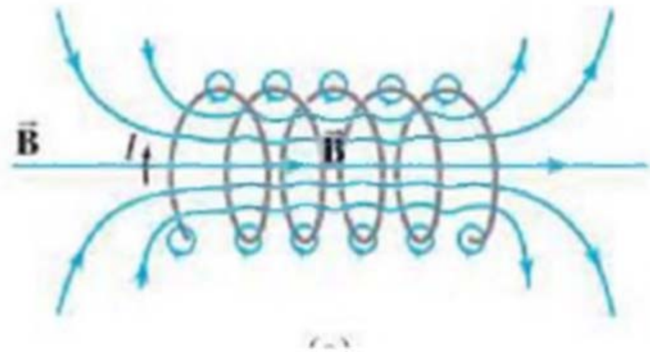
$$B\ell = \mu_0 NI.$$

- **If we let** *n = N / l* **be the** *number of loops per unit length,* **then**

$$B = \mu_0 nI.$$

Example

Field inside a solenoid. A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.



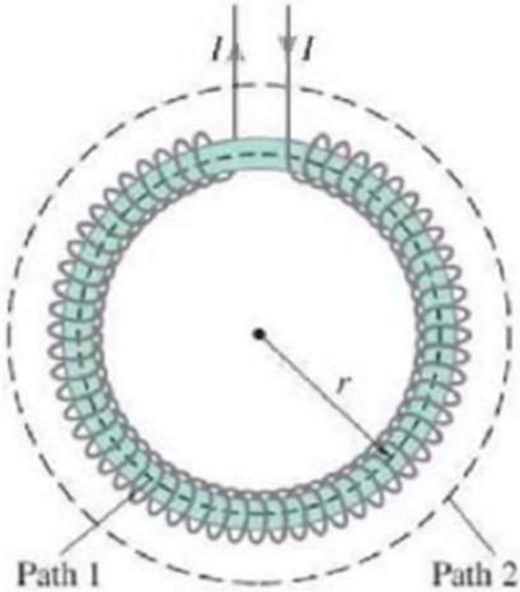
Field inside a solenoid. A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

$$n = 400/0.10\ \text{m} = 4.0 \times 10^3\ \text{m}^{-1}.$$

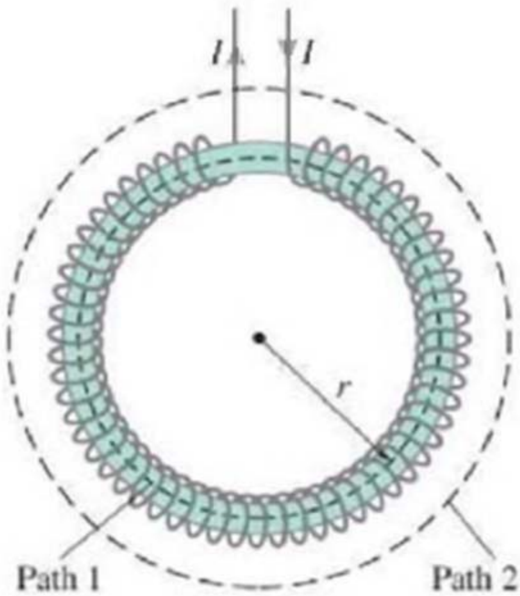
$$B = \mu_0 n I = (4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(4.0 \times 10^3\ \text{m}^{-1})(2.0\ \text{A}) = 1.0 \times 10^{-2}\ \text{T}.$$

Example

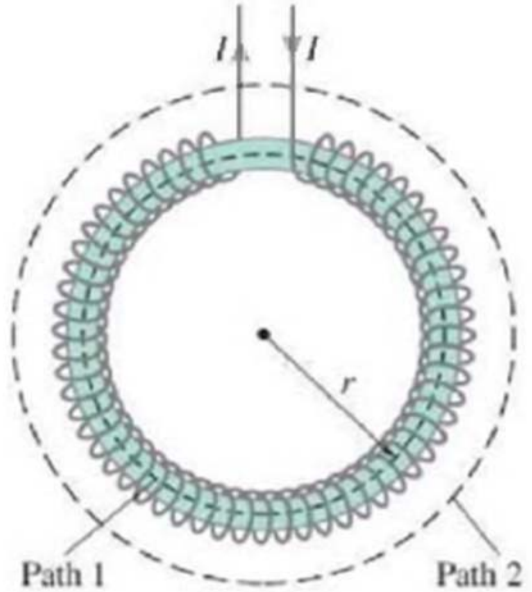
- Toroid. Use Ampere's law to determine the magnetic field
- *(a)* inside and
- (*b*) outside a toroid,
- which is like a solenoid bent into the shape of a circle as shown in Fig.



- **The magnetic field lines inside the toroid will be circles concentric with the toroid. (If you think of the toroid as a solenoid bent into a circle, the field lines bend along with the solenoid.)**
- **The direction of B is clockwise.**
- **We choose as our path of integration one of these field lines of radius *r* inside the toroid as shown by the dashed line labeled "path 1"**



- so B will be tangent to the path and will have the same magnitude at all points along the path
- This chosen path encloses *all* the coils; if there are N coils, each carrying current I, then $I_{encl} = N I$.



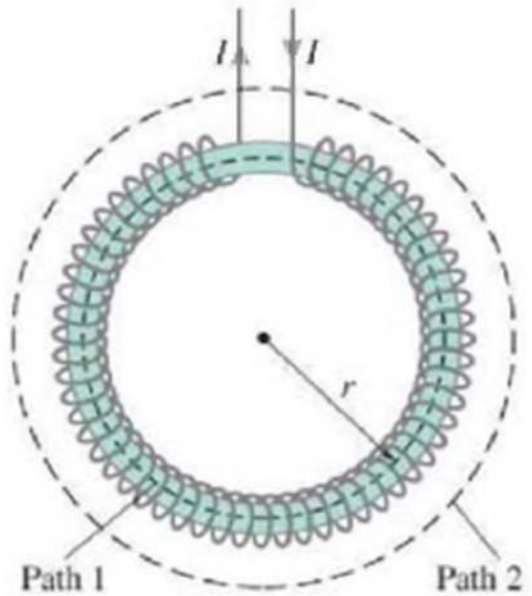
- (a) Ampere's law applied along this path gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{encl}$$

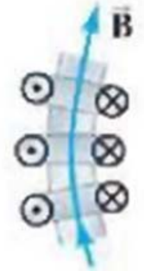
$$B(2\pi r) = \mu_0 N I,$$

$$B = \frac{\mu_0 N I}{2\pi r}.$$

- The magnetic field B is not uniform within the toroid: it is largest along the inner edge (where r is smallest) and smallest at the outer edge



- (*b*) Outside the toroid, we choose as our path of integration a circle concentric with the toroid, "path 2"
- This path encloses N loops carrying current I in one direction and N loops carrying the same current in the opposite direction.
- Figure shows the directions of the current for the parts the loop on the inside and outside of the toroid.)
- Thus the net current enclosed by path 2 is zero.



$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = 0$$

$$B = 0.$$

Thanks,..

See you next week (ISA),…