Lecture (04)

Transient Response of Transmission Lines

By:

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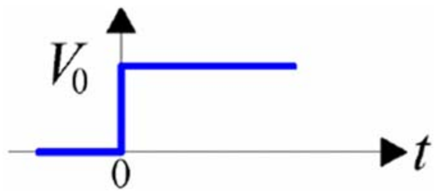
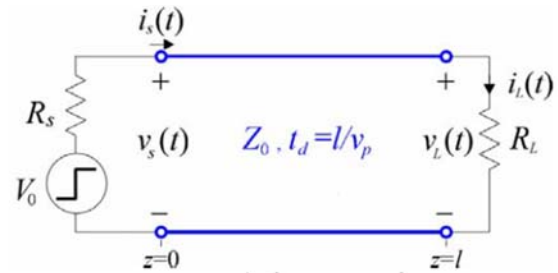
- When there is an interface between different materials (ε, μ), discontinuity exists,
- ⇒ partial reflection & transmission,
- ⇒ total *v*, *i* are determined by superposition
- Goal: Transient response of a terminated transmission line or cascaded lines excited by a step-like voltage source



Proof 01: find reflection coefficient in terms of Z_0, R_L

- A step voltage source of amplitude V_0 and internal resistance R_s
- drives a lossless transmission line of characteristic impedance Z_0, length *l*, phase velocity v_p
- (one-way signal traveling time $t_d = L/V_p$), and is terminated by a load resistance R_L



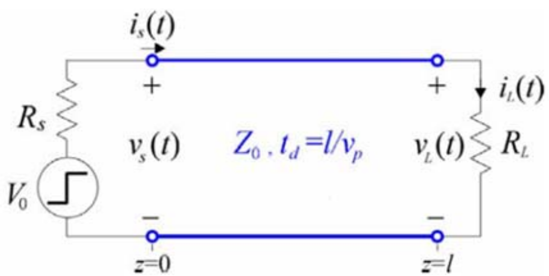
- a voltage signal starts to propagates in the *Z+* direction with velocity V_p
- At $t = 0$

$$v_1^+(z,t) = \begin{cases} V_1^+, \text{if } z < v_p t \\ 0, \text{otherwise} \end{cases}$$

- (valid for all $t > 0$, but is only interested during $z \in [0,L]$),

$$\text{where } V_1^+ = \frac{Z_0}{Z_0 + R_s} V_0$$

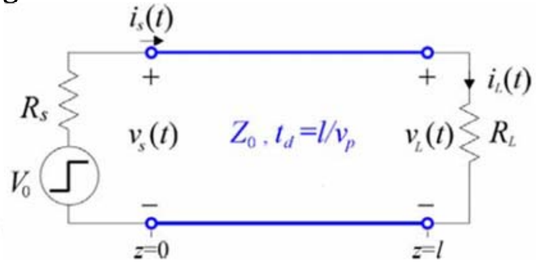
$$I_s = \frac{V_0}{Z_0 + Rs} = \frac{V_1^+}{Z_0}$$



- When the disturbance V+(z ,t) arrives at the load ($z = L$) At $t = t_d$,
- voltage wave will be reflected

$$v_1^-(z,t) = \begin{cases} V_1^-, \text{if } z > l - v_p(t - t_d) \\ 0, \text{otherwise} \end{cases}$$

- (valid for all $t_d > t$)
- will be generated and propagate in the − z direction.



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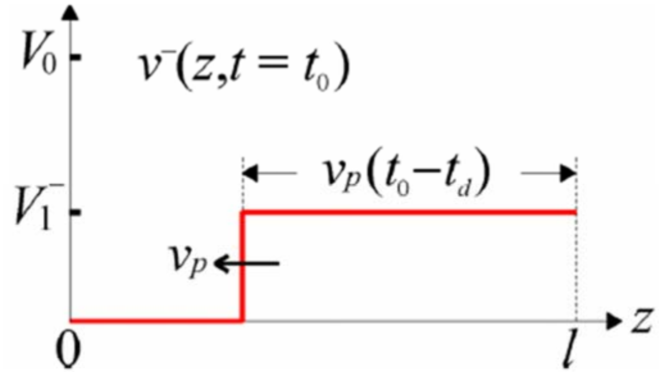


- The total voltage at the load a t = *td+* is equal to their superposition:

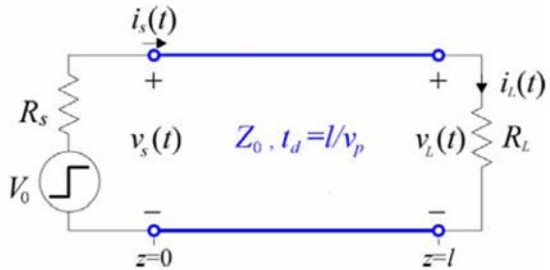
$$v_L(t) = v_1^+(l,t) + v_1^-(l,t)$$

- but

$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$

- the total current at $L = Z$, $t = td^+$ is

$$i_L(t) = \frac{v_1^+(l,t)}{Z_0} - \frac{v_1^-(l,t)}{Z_0}$$



$$i_L(t) = \frac{v_1^+(l,t)}{Z_0} - \frac{v_1^-(l,t)}{Z_0}$$

$$v_L(t) = v_1^+(l,t) + v_1^-(l,t)$$

- Ohm's law

$$v_L(t) = i_L(t) \cdot R_L(t)$$

- Substitute in previous equation

$$v_1^+(l,t) + v_1^-(l,t) = \frac{R_L}{Z_0}\left[v_1^+(l,t) - v_1^-(l,t)\right].$$

- *Divide by $V_1^+(L , t)$*

$$1 + \frac{V_1^-(l,t)}{v_1^+(l,t)} = \frac{R_l}{Z_o}[1 + \frac{V_1^-(l,t)}{v_1^+(l,t)}]$$

- Γ_L (gama) defining the load voltage reflection coefficient

$$\Gamma_L \equiv \frac{v_1^-(l,t)}{v_1^+(l,t)}$$

- substitute with Γ_L

$$1+\Gamma_L = \frac{R_L}{Z_0}(1-\Gamma_L)$$

$$1 + \Gamma = \frac{RL}{Z0}\ (1\text{-}\Gamma)$$

$$1 + \Gamma = \frac{RL}{Z0} - \frac{RL}{Z0}\Gamma$$

$$1 - \frac{RL}{Z0} = -\frac{RL}{Z0}\Gamma\text{-}\Gamma$$

$$\frac{Z0 - RL}{Z0} = -\Gamma(\frac{RL - Z0}{Z0})$$

$$\Gamma = \frac{RL - Z0}{RL + Z0}$$

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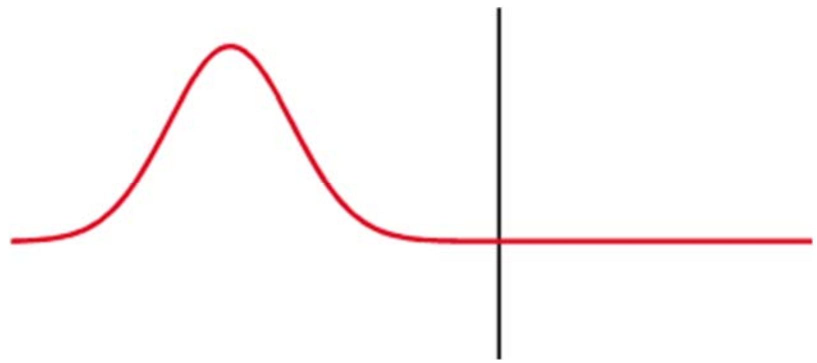
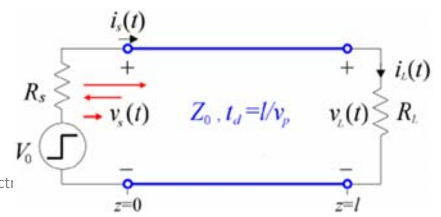


Proof2: Steady state response if TX is as if there were no line

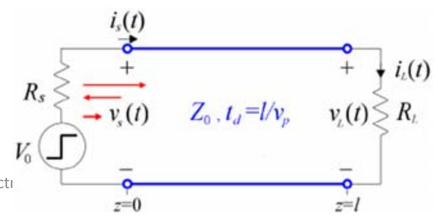
- The reflected voltage wave $V_1^-(z, t)$ will arrive at the source ($z = 0$) at $t_d = 2t$
- generating a reflected voltage

$$v_2^+(z,t) = \begin{cases} V_2^+, \text{if } z < v_p(t-2t_d) \\ 0, \text{otherwise} \end{cases} \quad (\text{valid for all } \ t > 2t_d)$$



- propagating in the + z direction.
- This process can be viewed as a voltage disturbance propagating on a line of characteristic impedance Z_0 and being incident on a resistance of R_s
- The source voltage réflexion coefficient Γ_S is:

$$\Gamma_S \equiv \frac{v_2^+(l,t)}{v_1^-(l,t)} = \frac{R_S - Z_0}{R_S + Z_0}$$



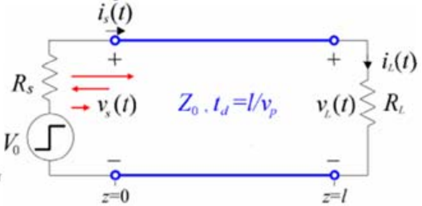
- Note that $V_1^+(z, t)$ is created at $t = 0$, and will continue to exist "forever".
- The total voltage and current at the source at $t = 2\, t_d^+$ are formulated as

$$\frac{v2+}{v1+} = \frac{v2+}{v1-} \cdot \frac{v1-}{v1+}$$

$$v_S(t) = v_1^+(0,t) + v_1^-(0,t) + v_2^+(0,t) = v_1^+(0,t)\left(1 + \Gamma_L + \Gamma_L\Gamma_S\right)$$

$$i_S(2t_d) = \frac{v_1^+(0,2t_d)}{Z_0} - \frac{v_1^-(0,2t_d)}{Z_0} + \frac{v_2^+(0,2t_d)}{Z_0} = \frac{v_1^+(0,2t_d)}{Z_0}\left(1 - \Gamma_L + \Gamma_L\Gamma_S\right)$$

$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$



- This process will continue indefinitely.
- The total voltage at the source will converge to:

$$\lim_{t\to\infty} v_S(t) = v_1^+(0, t\to\infty)\cdot\left(1 + \Gamma_L + \Gamma_L\Gamma_S + \Gamma_L^2\Gamma_S + \Gamma_L^2\Gamma_S^2 + \Gamma_L^3\Gamma_S^2 + \ldots\right)$$

$$= V_1^+ \cdot \left[(1+\Gamma_L) + \Gamma_L\Gamma_S(1+\Gamma_L) + \Gamma_L^2\Gamma_S^2(1+\Gamma_L) + \ldots\right] = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_S}$$

$$I_s = \frac{V_0}{Z_0 + Rs} = \frac{V_1^+}{Z_0}$$

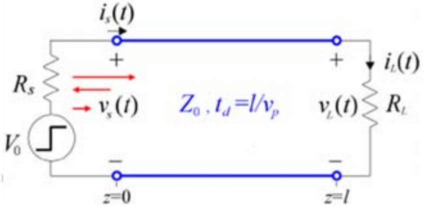
$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0$$

geometric progression

$$\sum_{k=0}^{n-1}(ar^k) = a\left(\frac{1-r^n}{1-r}\right)$$

Power of infinity

$$k^\infty = \begin{cases} \infty & \text{if } K>1 \\ 0 & \text{if } 0<k<1 \end{cases}$$



$$= V_1^+ \cdot \left[(1+\Gamma_L) + \Gamma_L\Gamma_S(1+\Gamma_L) + \Gamma_L^2\Gamma_S^2(1+\Gamma_L) + \ldots\right] = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_S}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_S \equiv \frac{R_S - Z_0}{R_S + Z_0} \qquad V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0$$

$$\lim_{t\to\infty} v_S(t) = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_S}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_S \equiv \frac{R_S - Z_0}{R_S + Z_0} \qquad V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0$$

$$\lim_{t\to\infty} v_S(t) = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_S}$$

$$= v0\,\frac{Z0}{Z0 + Rs}\,\frac{1 + \frac{Rl - Z0}{Rl + Z0}}{1 - \frac{Rl - Z0}{Rl + Z0}\,\frac{Rs - Z0}{Rs + Z0}}$$

$$= v0\,\frac{Z0}{Z0 + Rs}\,\frac{\frac{Rl + Z0 + Rl - Z0}{Rl + Z0}}{\frac{(Rl + Z0)(Rs + Z0) - (Rl - Z0)(Rs - Z0)}{(Rl + Z0)(Rs + Z0)}}$$

$$= v0\,\frac{Z0}{Z0+Rs}\;\frac{\frac{2Rl}{Rl+Z0}((Rl+Z0)(Rs+Z0))}{(Rl+Z0)(Rs+Z0)-(Rl-Z0)(Rs-Z0)}$$

$$= v0\,\frac{Z0}{Z0+Rs}\;\frac{2RL\ (Rs+Z0)}{RlRs+RlZ0+RsZ0+Z02-RlRs-Z02+RlZ0+RsZ0}$$

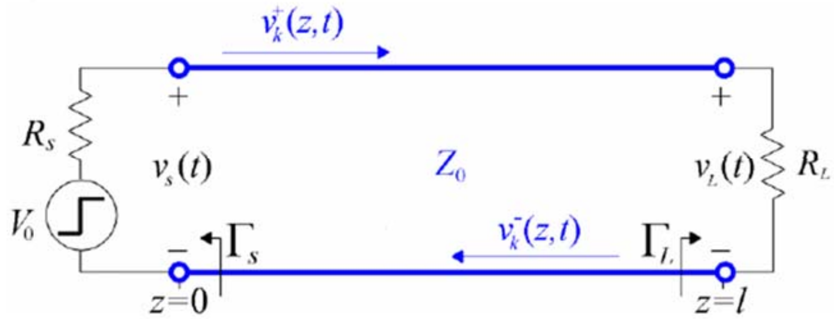
$$= v0\,\frac{2RLZ0}{2RlZ0+2RsZ0}$$

$$\Rightarrow \lim_{t\to\infty} v_S(t) = \frac{R_L}{R_S+R_L}V_0$$

Steady state response is as if there were no line

Bounce diagram

- Bounce diagram is a distance vs. time plot, illustrating successive reflections along a transmission line driven by a “step voltage source.
- Bounce diagram is a convenient tool to solve for transient response of a TX line driven by a step voltage and terminated by a resistor.

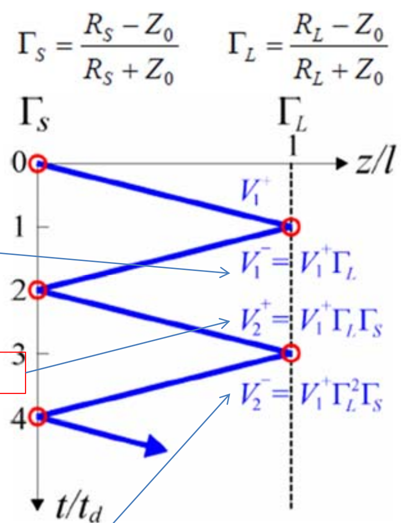


Draw a 2D window with $0< z/l <1$, $t/t_d>0$. Denote the two reflection coefficients Γ_S, Γ_L.

Mark the points $(0,2n)$ and $(1,2n+1)$ for $n = 0, 1, 2, \ldots$

Connecting the points by zigzag lines.

Mark the voltage amplitude of each component wave.



$v_1^- = v_1^+\Gamma_L$

$v_2^+ = v_1^-\Gamma_s = v_1^+\Gamma_L\Gamma_s$

$v_2^- = v_2^+\Gamma_L = v_1^+\Gamma_L^2\Gamma_s$

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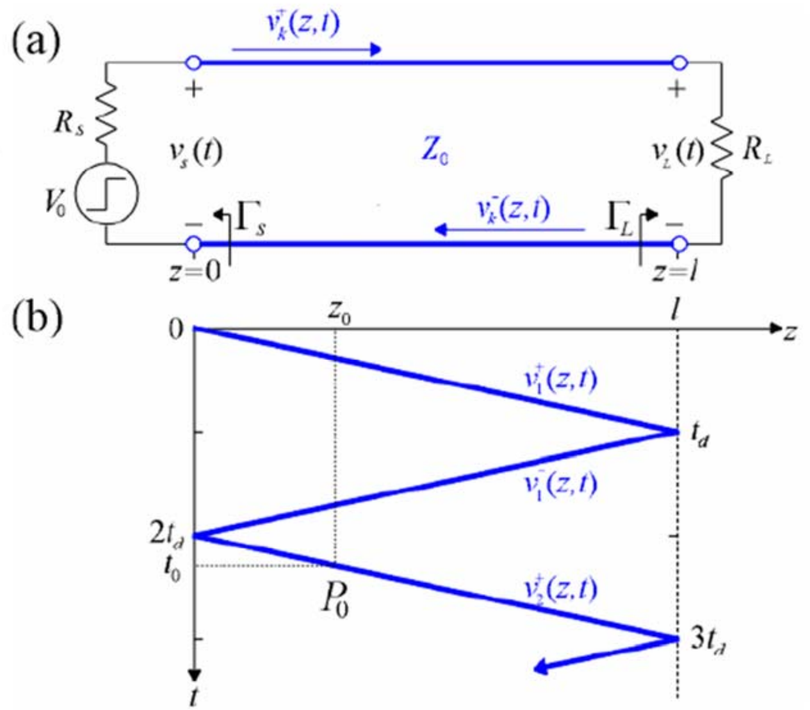
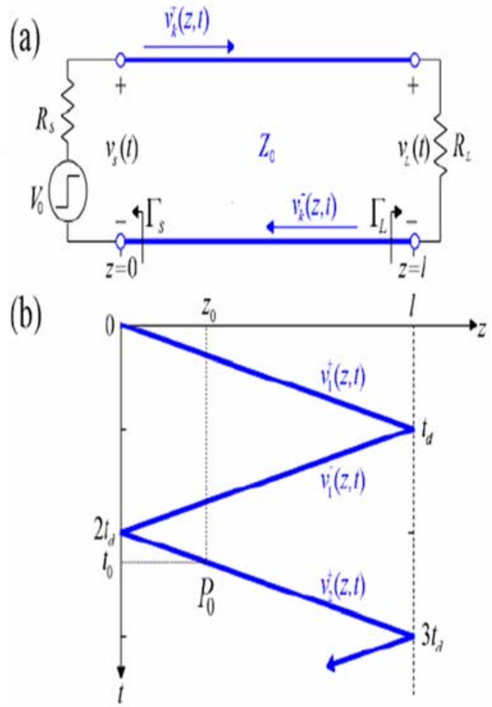


- The temporal voltage distribution at some position $v(za\ ,t)$:
- Draw a vertical line $z_a = z$,
- intersecting with the lines of the plot at successive times t = t_k^+, r_k^- (k = 1, 2, …), , which
- are the instants when the voltage wave $v_k^+(z,t)$, $v_k^-(z,t)$ arrives at the point of interest $z = z_a$.



- The spatial voltage distribution at some instant $V(Z\ ,t_0\ )$: Mark a point $P_0(z\ ,t_0\ )$ on the plot

$$v(z,t_0)=\begin{cases} V_{left}, & \text{for } 0<z<z_0; \\ V_{right}, & \text{for } z_0<z<l \end{cases}$$

- where V_{left} , V_{right} result from the superposition of proper numbers of bouncing voltage waves

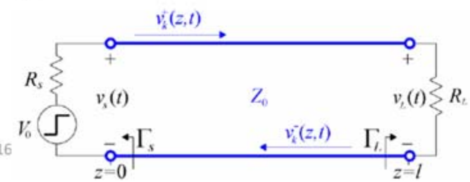
$$V_{left}=v_1^+(z,t)+v_1^-(z,t)+v_2^+(z,t)=V_1^+(1+\Gamma_L+\Gamma_L\Gamma_S),$$

$v_1^- = v_1^+\Gamma_L$

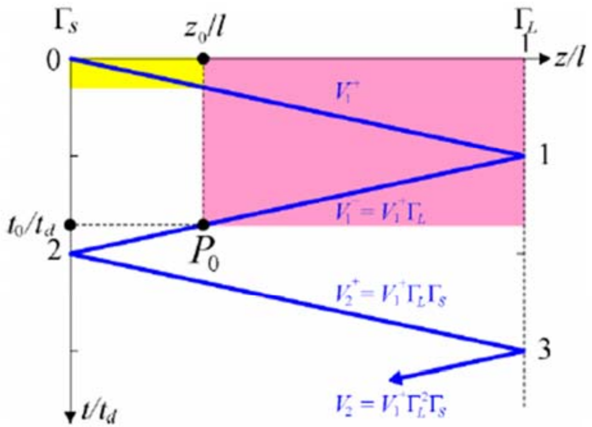
$v_2^+ = v_1^-\Gamma_s = v_1^+\Gamma_L\Gamma_s$

$$V_{right}=v_1^+(z,t)+v_1^-(z,t)=V_1^+(1+\Gamma_L).$$

$v_1^- = v_1^+\Gamma_L$



- The determination of V_{left} and V_{right} has two cases.
- If P_0 falls on a line with positive slope:

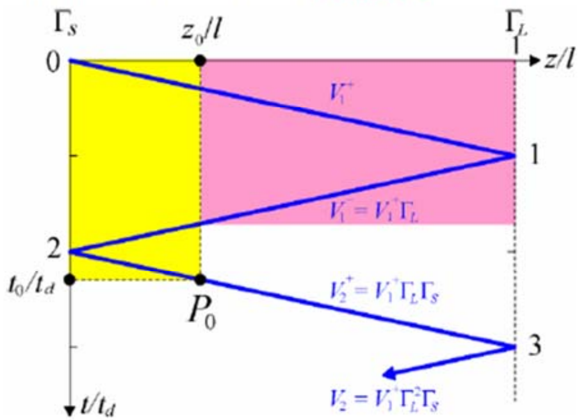


$$V_{left}=V_1^+$$

$$V_{right}=V_1^+ + V_1^- = V_1^+(1+\Gamma_L)$$

V_{right} has one more component.

If P_0 falls on a line with negative slope:



$$V_{left}=V_1^+ + V_1^- + V_2^+ = V_1^+(1+\Gamma_L+\Gamma_L\Gamma_S)$$

$$V_{right}=V_1^+ + V_1^- = V_1^+(1+\Gamma_L)$$

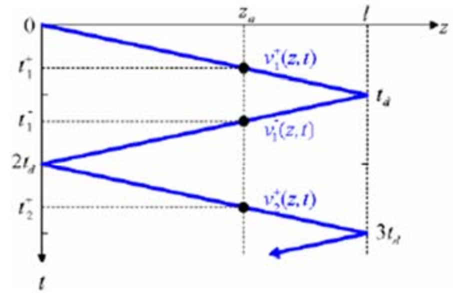
V_{left} has one more component.

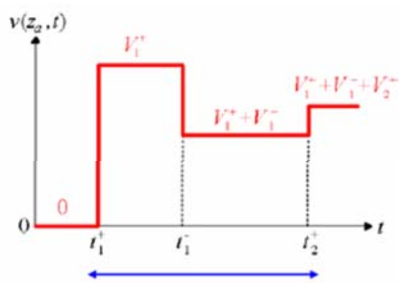
The solution becomes:

$$v(z_a,t)=\begin{cases}0, & \text{if } 0<t<t_1^+\\ V_1^+, & \text{if } t_1^+<t<t_1^-\\ V_1^+(1+\Gamma_L), & \text{if } t_1^-<t<t_2^+\\ V_1^+(1+\Gamma_L+\Gamma_L\Gamma_S), & \text{if } t_2^+<t<t_2^-\\ \dots, & \dots\end{cases}$$

- $v(z_a,t)$ must have constant voltages switched at a series of time instants.

Draw a vertical line $z=z_a$, intersecting with the zigzag lines at $t=t_1^+,t_1^-,t_2^+,\dots$

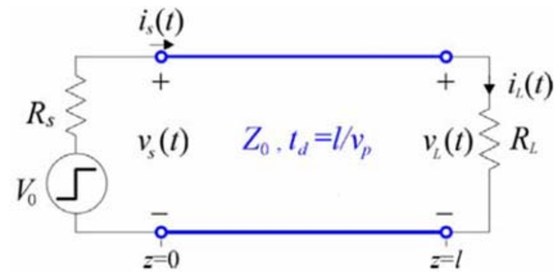




$$t_{i+1}^+ - t_i^+ = t_{i+1}^- - t_i^+ = 2t_d$$

Example 01 (Single TX line with open termination)

Consider a system shown in Fig. where $R_s = 0.25Z_0$, $R_L=\infty$ (open circuited load). Find the terminal voltages $v_S(t)$, $v_L(t)$.



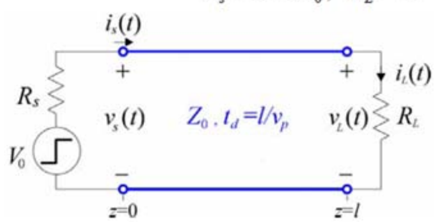
- rules

$$V_1^+ = \frac{Z_0}{Z_0+R_S}V_0$$

$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0}$$

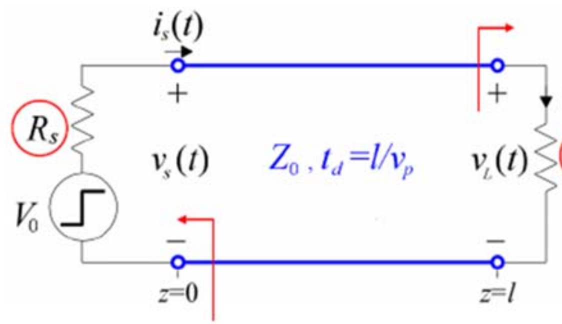
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

$R_s = 0.25Z_0$, $R_L = \infty$



$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 1$$

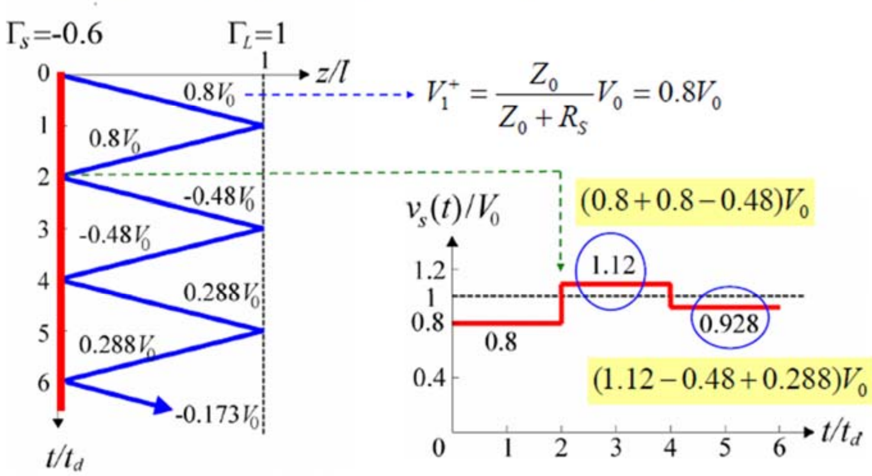
$R_s = 0.25Z_0$



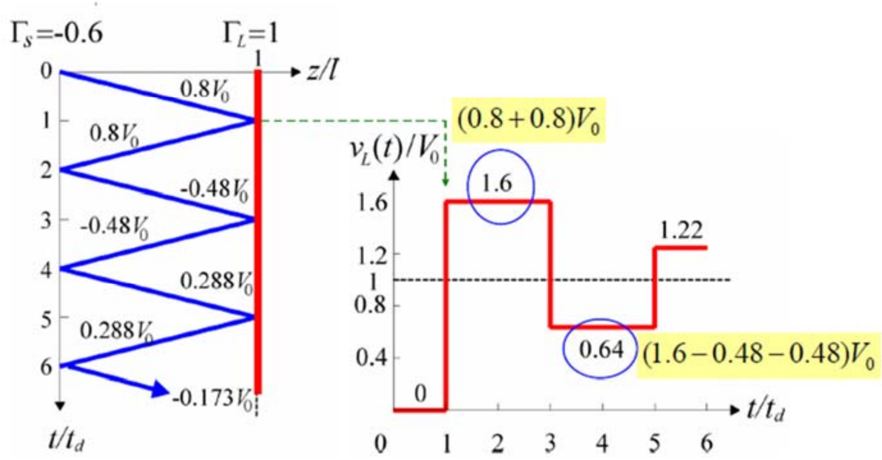
$R_L = \infty$

$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = -0.6$$

(1) $z=0$: $t_1^+ = 0,\ t_1^- = t_2^+ = 2t_d,\ t_2^- = t_3^+ = 4t_d, \ldots$



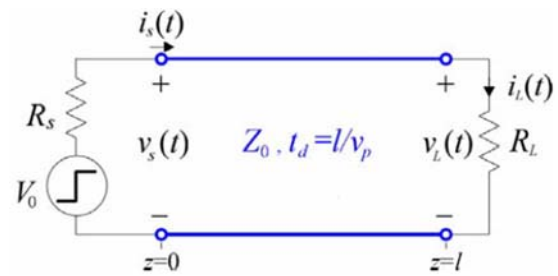
(2) $z=l$: $t_1^+ = t_1^- = t_d,\ t_k^+ = t_k^- = (2k-1)t_d, \ldots$



- Conclusion
- Overshooting and ringing effects during the transient state could be harmful for circuits

Example 02 (Single TX line with matched termination

- Consider a system shown in Fig. where $R_s = 4Z_0$, $R_L = Z_0$ (matched load). Find the terminal voltages $v_S(t)$, $v_L(t)$.



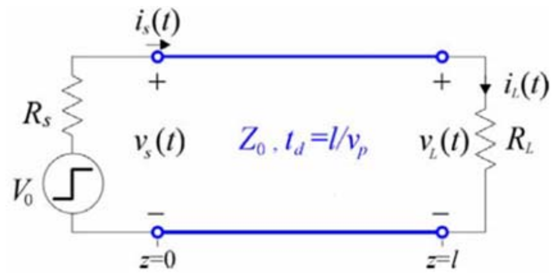
- Rules

$$V_1^+ = \frac{Z_0}{Z_0 + R_s} V_0$$

$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

$$R_s = 4Z_0 , \quad R_L = Z_0$$



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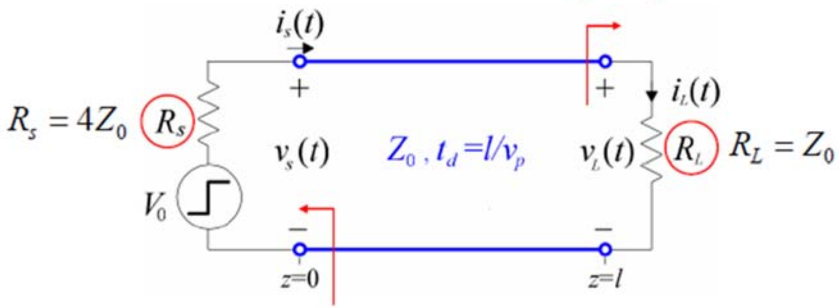
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0$$



$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = 0.6$$

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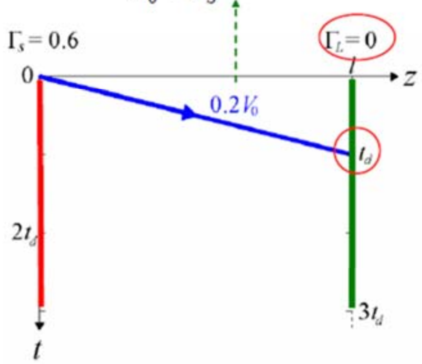
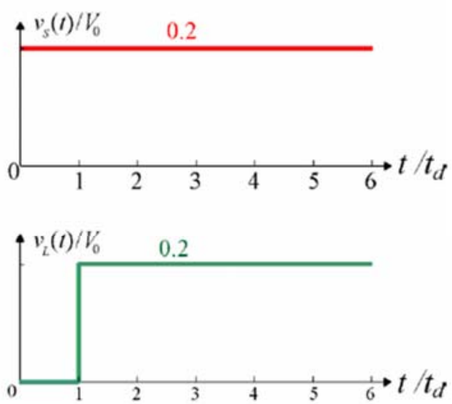
$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0 = 0.2V_0$$

No overshooting & ringing (good!)





- Conclusion
- No overshooting and ringing for the matched load prevents successive reflections

Thanks,..

See you next week (ISA),...