

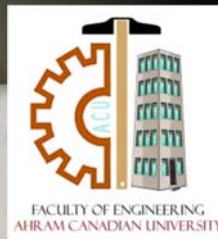


# Lecture (03)

## Transmission Lines Fundamentals (Cont.,..)

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## D'Alembart's solution to wave equation

any function  $f(\cdot)$  of variable  $\tau = t \pm \frac{z}{v_p}$

$f(t - z/v_p)$  ~ distortion-free wave traveling in the  $+z$  direction with velocity  $v_p$

$f(t + z/v_p)$  ~ distortion-free wave traveling in the  $-z$  direction with velocity  $v_p$

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General solution to the voltage (superposition):

$$v(z,t) = f^+(t - z/v_p) + f^-(t + z/v_p)$$

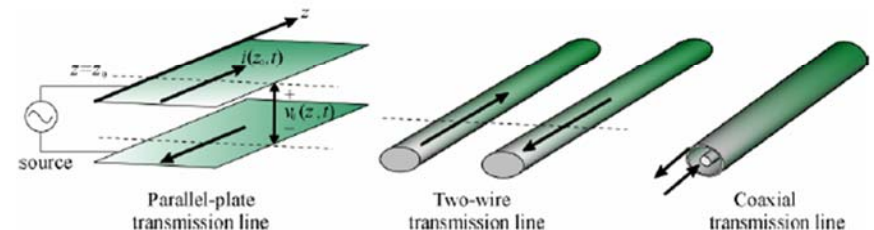
Can be totally different functions  
Their superposition may have "distortion".

- Equation exhibits properties of "wave", a result when the space and time variations of a physical quantity [  $v(z,t)$  in our case] are "coupled" through second-order derivatives.

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- Phase velocity  $v_p = \frac{1}{\sqrt{LC}}$  only depends on the insulating materials, though  $L, C$  also depend on the geometry of the lines



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# Proof 04 : Finding Characteristic impedance

Substitute voltage solution  $v(z,t) = f^+(t-z/v_p) + f^-(t+z/v_p)$

and  $\frac{\partial v}{\partial z} = \frac{\partial f^+}{\partial z} + \frac{\partial f^-}{\partial z}$

Slide 19 Lec 02

$$\frac{\partial f^+}{\partial z} \times \frac{\partial \tau}{\partial t} = \frac{\partial f^+}{\partial \tau} \times \frac{\partial \tau}{\partial z}$$

and  $\frac{\partial \tau}{\partial z} = \frac{-1}{v_p}$

sub  $\frac{\partial f^+}{\partial z} = \frac{\partial f^+}{\partial \tau} \times \frac{-1}{v_p} = \frac{-1}{v_p} \frac{df^+(\tau)}{d\tau}$

into the 1st-order PDE:  $\frac{\partial}{\partial z} v(z,t) = -L \frac{\partial}{\partial t} i(z,t)$  Slide 19 Lec 02

$$\Rightarrow \frac{\partial v}{\partial z} = -\frac{1}{v_p} \frac{df^+(\tau)}{d\tau} + \frac{1}{v_p} \frac{df^-(\tau)}{d\tau} = -L \frac{\partial i}{\partial t}$$

$$\Rightarrow i(z,t) = -\frac{1}{L} \int \frac{\partial v}{\partial z} \partial t = \frac{1}{Lv_p} \int \left[ \frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau} \right] \partial t \quad \because \frac{\partial \tau}{\partial t} = 1,$$

•  $\Rightarrow i(z,t) = -\frac{1}{L} \int \frac{\partial v}{\partial z} \partial t = \frac{1}{Lv_p} \int \left[ \frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau} \right] \partial t$

$$v_p = 1/\sqrt{LC}$$

$$i(z,t) = \sqrt{\frac{C}{L}} \int \left[ \frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau} \right] d\tau$$

$Z_0 = \sqrt{\frac{L}{C}}$  **Characteristic impedance**  
(not the resistance of the conductor or insulator)

$$i(z,t) = \frac{1}{Z_0} [f^+(t-z/v_p) - f^-(t+z/v_p)] = i^+(z,t) + i^-(z,t)$$

$$v(z,t) = f^+(t-z/v_p) + f^-(t+z/v_p)$$

$$i^+(z,t) = \frac{1}{Z_0} [f^+(t-z/v_p)]$$

$$i^-(z,t) = \frac{1}{Z_0} [-f^-(t+z/v_p)]$$

$$v^+(z,t) = f^+(t-z/v_p)$$

$$v^-(z,t) = f^-(t+z/v_p)$$

$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)} = \frac{v(z,t)}{i(z,t)} \quad \text{Physical meaning}$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad (\Omega)$$

- is known as the characterization impedance of the transmission line (not the resistance of conductors or insulator).
- If we denote the voltage component propagating in +z and -z directions as

$$\begin{aligned} v^+(z,t) &= f^+(t - z/v_p) & i^+(z,t) &= \frac{1}{Z_0} f^+(t - z/v_p) \\ v^-(z,t) &= f^-(t + z/v_p) & i^-(z,t) &= -\frac{1}{Z_0} f^-(t + z/v_p), \end{aligned}$$

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$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$

- The characteristic impedance is the ratio of voltage to current for a “single” wave propagating in the +z direction

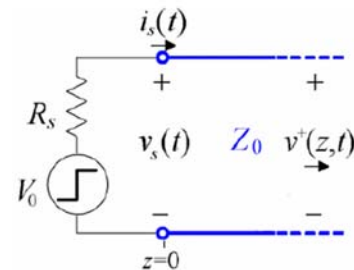
However,  $\frac{v(z,t)}{i(z,t)} \neq \text{constant}$  if two counter-propagating waves coexist.

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## Example

- Consider an infinitely long lossless transmission line of characteristic impedance  $Z_0$  ( $\in R$ ) and phase velocity  $v_p$  connected to a step voltage source of amplitude  $V_0$  and internal resistance  $R_s$ . Find the voltage, current, and power propagating down the line.



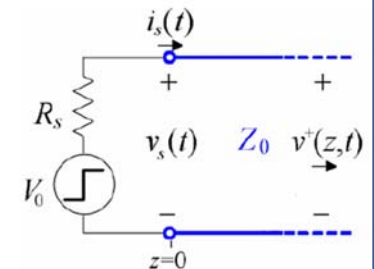
- Assume the line is initially at rest  $v(z,t=0^-) = 0$ ,  $i(z,t=0^-) = 0$ .

- At  $t = 0$ , the source voltage changes from 0 to  $V_0$ .

- In the absence of reflected wave (infinitely long line), the line acts as a load of impedance  $Z_0$  for the source.

- The initial voltage established at the source ends is (VD):

$$v_s(t=0^+) = V_s = \frac{Z_0}{Z_0 + R_s} V_0$$



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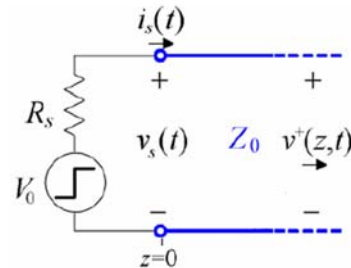
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- The voltage signal of constant amplitude  $V_s$  propagates in the  $+z$  direction with velocity  $v_p$ ,

$$v^+(z,t) = \begin{cases} V_s, & \text{if } z/t < v_p, z > 0, t > 0 \\ 0, & \text{otherwise} \end{cases}$$

- While 
$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$

- then 
$$i^+(z,t) = \frac{v^+(z,t)}{Z_0} = \begin{cases} I_s, & \text{if } z/t < v_p, z > 0, t > 0 \\ 0, & \text{otherwise} \end{cases}$$

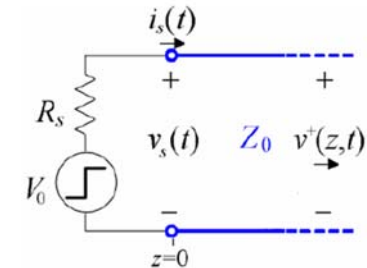


- The total power supplied by the source:

$$P_{tot} = I_s V_0$$

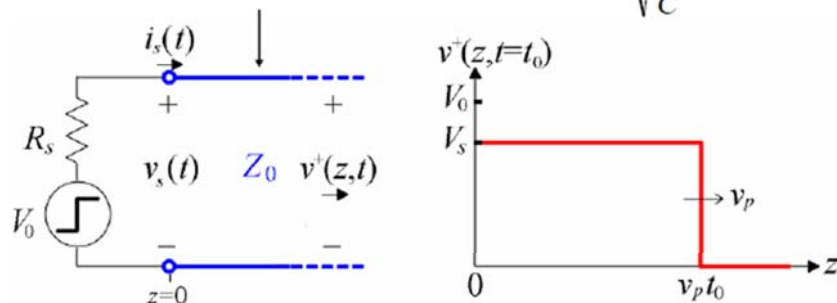
- while only a fraction of it is supplied to (stored in) the line:

$$P_{line}^+ = I_s V_s$$



- Infinitely long line, no reflected wave  $v^-(z,t)$

TX line ~ a load of resistance  $Z_0 = \sqrt{\frac{L}{C}}$



$$v_s(t=0^+) = V_s = \frac{Z_0}{Z_0 + R_s} V_0$$

