Lecture (03)

Transmission Lines Fundamentals (Cont.,..)

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D'Alembart's solution to wave equation

any function $f(\cdot)$ of variable $\tau = t \pm \frac{z}{v_p}$

$f(t - z/v_p)$ ~ distortion-free wave traveling in the $+z$ direction with velocity v_p

$f(t + z/v_p)$ ~ distortion-free wave traveling in the $-z$ direction with velocity v_p

General solution to the voltage (superposition):

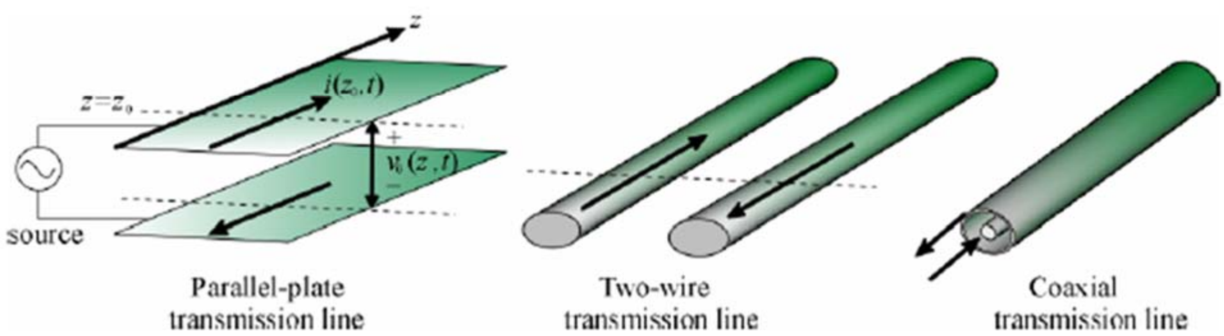
$$v(z,t) = f^{+}(t - z/v_p) + f^{-}(t + z/v_p)$$

Can be totally different functions

Their superposition may have "distortion".

- Equation exhibits properties of "wave", a result when the space and time variations of a physical quantity [$v(z,t)$ in our case] are "coupled" through second-order derivatives.

- Phase velocity $v_p = \frac{1}{\sqrt{LC}}$ only depends on the insulating materials, though *L*, *C* also depend on the geometry of the lines



Proof 04 : Finding Characteristic impedance

Substitute voltage solution $v(z,t) = f^+(t - z/v_p) + f^-(t + z/v_p)$

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and $\dfrac{\partial v}{\partial z} = \dfrac{\partial f^+}{\partial z} + \dfrac{\partial f^-}{\partial z}$

$$\frac{\partial f^+}{\partial z} \times \frac{\partial \tau}{\partial \tau} = \frac{\partial f^+}{\partial \tau} \times \frac{\partial \tau}{\partial z}$$

and $\dfrac{\partial \tau}{\partial z} = \dfrac{-1}{Vp}$

sub $\dfrac{\partial f^+}{\partial z} = \dfrac{\partial f^+}{\partial \tau} \times \dfrac{-1}{vp} = \dfrac{-1}{vp}\dfrac{df^+(\tau)}{d\tau}$

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into the 1st-order PDE: $\dfrac{\partial}{\partial z} v(z,t) = -L \dfrac{\partial}{\partial t} i(z,t)$

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$$\Rightarrow \frac{\partial v}{\partial z} = -\frac{1}{v_p}\frac{df^+(\tau)}{d\tau} + \frac{1}{v_p}\frac{df^-(\tau)}{d\tau} = -L\frac{\partial i}{\partial t}$$

$$\Rightarrow i(z,t) = -\frac{1}{L}\int \frac{\partial v}{\partial z}\partial t = \frac{1}{Lv_p}\int \left[\frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau}\right]\partial t \qquad \because \frac{\partial \tau}{\partial t} = 1,$$

- $$\Rightarrow i(z,t) = -\frac{1}{L}\int \frac{\partial v}{\partial z}\partial t = \frac{1}{L v_p}\int \left[\frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau}\right]\partial t$$

$$v_p = 1/\sqrt{LC}$$

$$i(z,t) = \sqrt{\frac{C}{L}}\int \left[\frac{df^+(\tau)}{d\tau} - \frac{df^-(\tau)}{d\tau}\right] d\tau$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

Characteristic impedance

(not the resistance of the conductor or insulator)

$$i(z,t) = \frac{1}{Z_0}\left[f^+(t - z/v_p) - f^-(t + z/v_p)\right] = i^+(z,t) + i^-(z,t)$$

$$v(z,t) = f^+(t - z/v_p) + f^-(t + z/v_p)$$

$$i^+(z,t) = \frac{1}{Z_0}\left[f^+(t - z/v_p)\right]$$

$$i^-(z,t) = \frac{1}{Z_0}\left[-f^-(t + z/v_p)\right]$$

$$v^+(z,t) = f^+(t - z/v_p)$$

$$v^-(z,t) = f^-(t + z/v_p)$$

$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)} \neq \frac{v(z,t)}{i(z,t)}$$

Physical meaning

$$Z_0 = \sqrt{\frac{L}{C}} \quad (\Omega)$$

- is known as the characterization impedance of the transmission line (not the resistance of conductors or insulator).
- If we denote the voltage component propagating in +*z* and –*z* directions as

$$v^+(z,t) = f^+(t - z/v_p) \qquad i^+(z,t) = \frac{1}{Z_0} f^+(t - z/v_p)$$

$$v^-(z,t) = f^-(t + z/v_p) \qquad i^-(z,t) = -\frac{1}{Z_0} f^-(t + z/v_p),$$

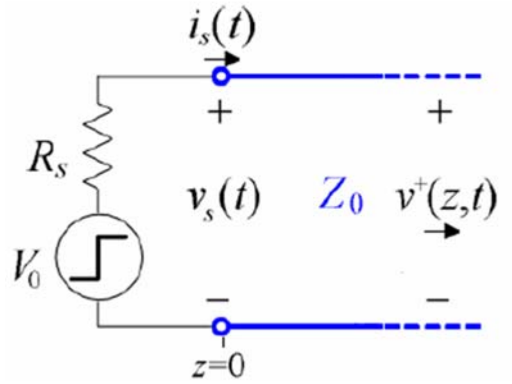
$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$

- The characteristic impedance is the ratio of voltage to current for a “single” wave propagating in the +*z* direction

However, $\frac{v(z,t)}{i(z,t)} \neq$ constant if two counter-propagating waves coexist.

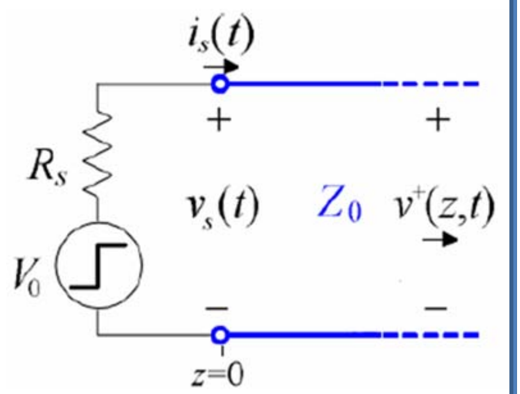
Example

- Consider an infinitely long lossless transmission line of characteristic impedance *Z0* (∈*R*) and phase velocity *vp* connected to a step voltage source of amplitude V0 and internal resistance *Rs* . Find the voltage, current, and power propagating down the line.



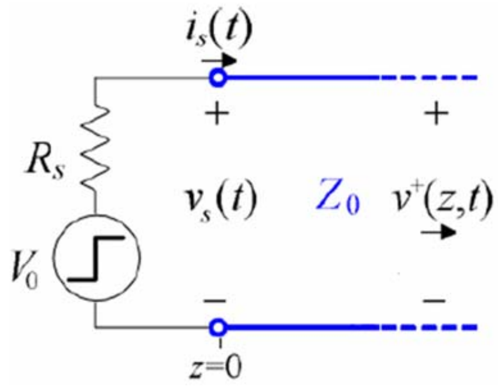
- Assume the line is initially at rest $v(z,t=0^-)=0$, $i(z,t=0^-)=0$.
- At *t* = 0 , the source voltage changes from 0 to *V0.*
- In the absence of reflected wave (infinitely long line), the line acts as a load of impedance *Z0* for the source.
- The initial voltage established at the source ends is (VD):

$$v_s(t=0^+)=V_s=\frac{Z_0}{Z_0+R_s}V_0$$



- The voltage signal of constant amplitude *Vs* propagates in the *+z* direction with velocity *vp* ,

$$v^+(z,t) = \begin{cases} V_s, \text{if } z/t < v_p, z > 0, t > 0 \\ 0, \text{otherwise} \end{cases}$$



- While

$$Z_0 = \frac{v^+(z,t)}{i^+(z,t)} = -\frac{v^-(z,t)}{i^-(z,t)}$$

- then

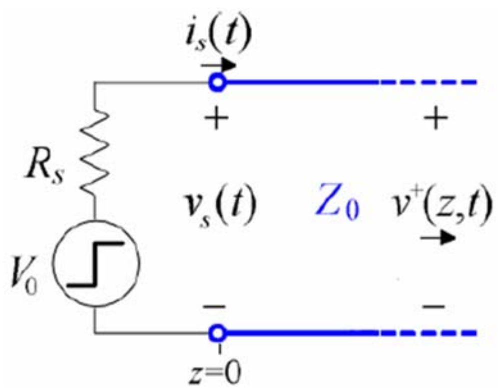
$$i^+(z,t) = \frac{v^+(z,t)}{Z_0} = \begin{cases} I_s, \text{if } z/t < v_p, z > 0, t > 0 \\ 0, \text{otherwise} \end{cases}$$

- The total power supplied by the source:

$$P_{tot} = I_s V_0$$

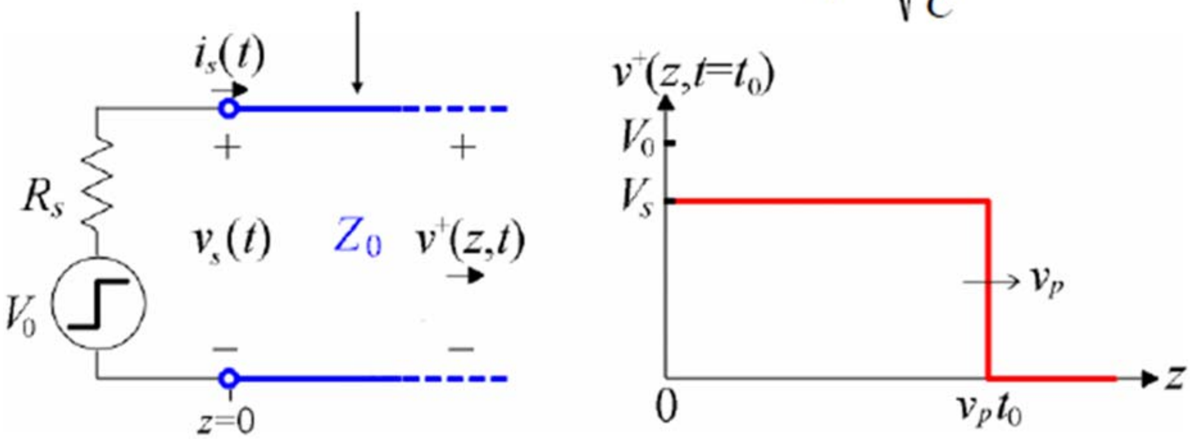
- while only a fraction of it is supplied to (stored in) the line:

$$P^+_{line} = I_s V_s$$



- Infinitely long line, no reflected wave $v^{-}(z,t)$

  TX line ~ a load of resistance $Z_0 = \sqrt{\frac{L}{C}}$



$$v_s(t=0^+) = V_s = \frac{Z_0}{Z_0 + R_s} V_0$$