Electromagnetic Fields – Assignment 07

Magnetism II

#	Student ID	Student Name	Grade (10)
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Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

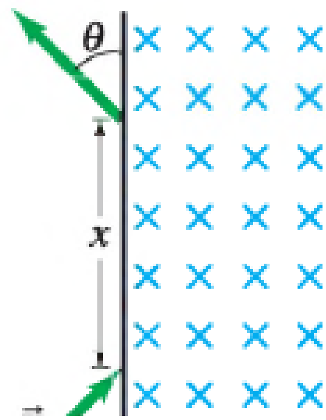
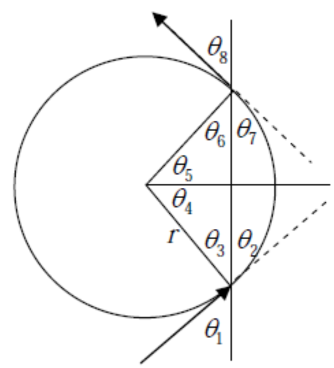
Q1	A doubly charged helium atom whose mass is 6.6×10^{-27} kg is accelerated by a voltage of 2700 V. (*a*) What will be its radius of curvature if it moves in a plane perpendicular to a uniform 0.340-T field? (*b*) What is its period of revolution?
Sol 1	(*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path. $$E_{initial} = E_{final} \rightarrow qV = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2qV}{m}}$$ $$F_{max} = qvB = m\frac{v^2}{r} \rightarrow$$ $$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,T}\sqrt{\frac{2(6.6\times10^{-27}\,kg)(2700\,V)}{2(1.60\times10^{-19}\,C)}} = \boxed{3.1\times10^{-2}\,m}$$ The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above. $$v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi(6.6\times10^{-27}\,kg)}{2(1.60\times10^{-19}\,C)(0.340\,T)} = \boxed{3.8\times10^{-7}\,s}$$

Q2	An electron moves with velocity $\vec{\mathbf{v}} = (7.0\hat{\mathbf{i}} - 6.0\hat{\mathbf{j}}) \times 10^4\,\mathrm{m/s}$ in a magnetic field $\vec{\mathbf{B}} = (-0.80\hat{\mathbf{i}} + 0.60\hat{\mathbf{j}})\,\mathrm{T}$. Determine the magnitude and direction of the force on the electron.
Sol 2	.... The force on the electron is given by Eq $$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = -e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 7.0\times10^4\,\mathrm{m/s} & -6.0\times10^4\,\mathrm{m/s} & 0 \\ -0.80\,\mathrm{T} & 0.60\,\mathrm{T} & 0 \end{vmatrix} = -e(4.2-4.8)\times10^4\,\mathrm{T\cdot m/s}\,\hat{\mathbf{k}}$$ $$= -(1.60\times10^{-19}\,\mathrm{C})(-0.6\times10^4\,\mathrm{T\cdot m/s}\,\hat{\mathbf{k}}) = 9.6\times10^{-16}\,\mathrm{N}\,\hat{\mathbf{k}} \approx \boxed{1\times10^{-15}\,\mathrm{N}\,\hat{\mathbf{k}}}$$ ....

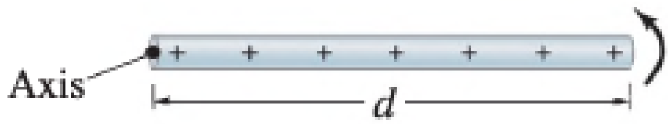
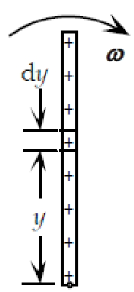
Q3	A proton moves through a region of space where there is a magnetic field $\vec{\mathbf{B}} = (0.45\hat{\mathbf{i}} + 0.38\hat{\mathbf{j}})\,\mathrm{T}$ and an electric field $\vec{\mathbf{E}} = (3.0\hat{\mathbf{i}} - 4.2\hat{\mathbf{j}}) \times 10^3\,\mathrm{V/m}$. At a given instant, the proton's velocity is $\vec{\mathbf{v}} = (6.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}}) \times 10^3\,\mathrm{m/s}$. Determine the components of the total force on the proton.
Sol 3	The total force on the proton is given by the Lorentz equation, Eq. $\vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B}) = e\left[(3.0\hat{i} - 4.2\hat{j}) \times 10^3\,\mathrm{V/m} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6.0\times10^3\,\mathrm{m/s} & 3.0\times10^3\,\mathrm{m/s} & -5.0\times10^3\,\mathrm{m/s} \\ 0.45\mathrm{T} & 0.38\mathrm{T} & 0 \end{vmatrix}\right]$ $= (1.60\times10^{-19}\,\mathrm{C})\left[(3.0\hat{i} - 4.2\hat{j}) + (1.9\hat{i} - 2.25\hat{j} + 0.93\hat{k})\right]\times10^3\,\mathrm{N/C}$ $= (1.60\times10^{-19}\,\mathrm{C})\left[(4.9\hat{i} - 6.45\hat{j} + 0.93\hat{k})\right]\times10^3\,\mathrm{N/C}$ $= (7.84\times10^{-16}\,\hat{i} - 1.03\times10^{-15}\,\hat{j} + 1.49\times10^{-16}\,\hat{k})\,\mathrm{N/C}$ $= \boxed{\left[(0.78\hat{i} - 1.0\hat{j} + 0.15\hat{k})\right]\times10^{-15}\,\mathrm{N}}$

Q4	The path of protons emerging from an accelerator must be bent by 90° by a "bending magnet" so as not to strike a barrier in their path a distance d from their exit hole in the accelerator. Show that the field $\vec{\mathbf{B}}$ in the bending magnet, which we assume is uniform and can extend over an area $d \times d$, must have magnitude $B \geq (2mK/e^2d^2)^{\frac{1}{2}}$, where m is the mass of a proton and K is its kinetic energy.
Sol 4	.... In order for the path to be bent by 90° within a distance d, the radius of curvature must be less than or equal to d. The kinetic energy of the protons can be used to find their velocity. The magnetic force produces centripetal acceleration, and from this, the magnetic field can be determined. $K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow B = \frac{mv}{qr}$ $B \geq \frac{mv}{ed} = \frac{m\sqrt{\frac{2K}{m}}}{ed} = \boxed{\left(\frac{2Km}{e^2d^2}\right)^{1/2}}$ ....

Q5	A 3.40-g bullet moves with a speed of 155 m/s perpendicular to the Earth's magnetic field of 5.00×10^{-5} T. If the bullet possesses a net charge of 18.5×10^{-9} C, by what distance will it be deflected from its path due to the Earth's magnetic field after it has traveled 1.00 km?
Sol 5	The magnetic force produces an acceleration that is perpendicular to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity v and the distance traveled ℓ. The starting speed in the perpendicular direction will be zero. $F_\perp = ma_\perp = qvB \rightarrow a_\perp = \frac{qvB}{m}$ $d_\perp = v_{0\perp}t + \frac{1}{2}a_\perp t^2 = \frac{1}{2}\frac{qvB}{m}\left(\frac{\ell}{v}\right)^2 = \frac{qB\ell^2}{2mv} = \frac{(18.5\times10^{-9}\text{C})(5.00\times10^{-5}\text{T})(1.00\times10^{3}\text{m})^2}{2(3.40\times10^{-3}\text{kg})(155\text{m/s})}$ $= \boxed{8.8\times10^{-7}\text{m}}$ This small distance justifies the assumption of constant acceleration.

Q6	A proton moving with speed $v = 1.3 \times 10^5$ m/s in a field-free region abruptly enters an essentially uniform magnetic field $B = 0.850$ T $(\vec{\mathbf{B}} \perp \vec{\mathbf{v}})$. If the proton enters the magnetic field region at a 45° angle as shown in Fig. 27–46, (*a*) at what angle does it leave, and (*b*) at what distance x does it exit the field? 
Sol 6	(*a*) In the magnetic field, the proton will move along an arc of a circle. The distance x in the diagram is a chord of that circle, and so the center of the circular path lies on the perpendicular bisector of the chord. That perpendicular bisector bisects the central angle of the circle which subtends the chord. Also recall that a radius is perpendicular to a tangent. In the diagram, $\theta_1 = \theta_2$ because they are vertical angles. Then $\theta_2 = \theta_4$, because they are both complements of θ_3, so $\theta_1 = \theta_4$. We have $\theta_4 = \theta_5$ since the central angle is bisected by the perpendicular bisector of the chord. $\theta_5 = \theta_7$ because they are both complements of θ_6, and $\theta_7 = \theta_8$ because they are vertical angles. Thus $\theta_1 = \theta_2 = \theta_4 = \theta_5 = \theta_7 = \theta_8$, and so in the textbook diagram, the angle at which the proton leaves is $\boxed{\theta = 45^\circ}$.  (*b*) The radius of curvature is given by $r = \frac{mv}{qB}$, and the distance x is twice the value of $r\cos\theta$. $$x = 2r\cos\theta = 2\frac{mv}{qB}\cos\theta = 2\frac{(1.67\times10^{-27}\text{ kg})(1.3\times10^5\text{ m/s})}{(1.60\times10^{-19}\text{C})(0.850\text{T})}\cos 45^\circ = \boxed{2.3\times10^{-3}\text{m}}$$

Q7	A 13.0-cm-diameter circular loop of wire is placed with the plane of the loop parallel to the uniform magnetic field between the pole pieces of a large magnet. When 4.20 A flows in the coil, the torque on it is 0.185 m·N. What is the magnetic field strength?
Sol 7	With the plane of the loop parallel to the magnetic field, the torque will be a maximum. We use Eq. $\tau = NIAB\sin\theta \rightarrow B = \frac{\tau}{NIA\sin\theta} = \frac{0.185\,\text{m}\cdot\text{N}}{(1)(4.20\,\text{A})\pi(0.0650\,\text{m})^2\sin 90^\circ} = \boxed{3.32\,\text{T}}$

Q8	Suppose a nonconducting rod of length d carries a uniformly distributed charge Q. It is rotated with angular velocity ω about an axis perpendicular to the rod at one end, Fig. . Show that the magnetic dipole moment of this rod is $\frac{1}{6}Q\omega d^2$. [*Hint*: Consider the motion of each infinitesimal length of the rod.] 
Sol 8	.... To find the total magnetic moment, we divide the rod into infinitesimal pieces of thickness dy. As the rod rotates on its axis the charge in each piece, $(Q/d)\,dy$, creates a current loop around the axis of rotation. The magnitude of the current is the charge times the frequency of rotation, $\omega/2\pi$. By integrating the infinitesimal magnetic moments from each piece, we find the total magnetic moment.  $\bar{\mu} = \int d\bar{\mu} = \int \bar{A}\,dI = \int_0^d (\pi y^2)\left(\frac{\omega}{2\pi}\frac{Q}{d}dy\right) = \frac{Q\omega}{2d}\int_0^d y^2\,dy = \boxed{\frac{Q\omega d^2}{6}}.$ ....

Q9	A galvanometer needle deflects full scale for a 63.0-μA current. What current will give full-scale deflection if the magnetic field weakens to 0.800 of its original value?
Sol 9	....

the deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.860 times its original value, the current must be increased by dividing the original value by 0.860 to obtain the same deflection.

$$(IB)_{\text{initial}} = (IB)_{\text{final}} \quad \rightarrow \quad I_{\text{final}} = \frac{I_{\text{initial}} B_{\text{initial}}}{B_{\text{final}}} = \frac{(63.0\,\mu\text{A})\, B_{\text{initial}}}{0.800 B_{\text{initial}}} = \boxed{78.8\,\mu\text{A}}$$

Q10	An oil drop whose mass is determined to be 3.3×10^{-15} kg is held at rest between two large plates separated by 1.0 cm . If the potential difference between the plates is 340 V, how many excess electrons does this drop have?
Sol 10	………………………………………………………………………………………… The force from the electric field must be equal to the weight. $\lvert qE\rvert = (ne)\left(\frac{V}{d}\right) = mg \rightarrow n = \frac{mgd}{eV} = \frac{(3.3\times10^{-15}\,\text{kg})(9.80\,\text{m/s}^2)(0.010\,\text{m})}{(1.60\times10^{-19})(340\,\text{V})} = 5.94 \approx \boxed{6\ \text{electrons}}$ …………………………………………………………………………………………

Q11	The Hall effect can be used to measure blood flow rate because the blood contains ions that constitute an electric current. (*a*) Does the sign of the ions influence the emf? (*b*) Determine the flow velocity in an artery 3.3 mm in diameter if the measured emf is 0.13 mV and B is 0.070 T. (In actual practice, an alternating magnetic field is used.)
Sol 11	(*a*) The sign of the ions will not change the magnitude of the Hall emf, but will determine the polarity of the emf. (*b*) The flow velocity corresponds to the drift velocity $\mathcal{E}_H = vBd \rightarrow \rightarrow v = \frac{\mathcal{E}_H}{Bd} = \frac{(0.13\times10^{-3}\,V)}{(0.070\,T)(0.0033\,m)} = \boxed{0.56\,m/s}$