Electromagnetic Fields – Assignment 08

#	Student ID	Student Name	Grade (10)
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Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

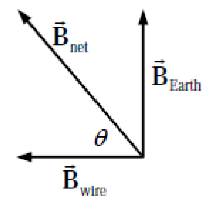
Q1	A vertical straight wire carrying an upward 28-A current exerts an attractive force per unit length of 7.8×10^{-4} N/m on a second parallel wire 7.0 cm away. What current (magnitude and direction) flows in the second wire?
Sol 1	Since the force is attractive, the currents must be in the same direction, so the current in the second wire must also be upward. Use Eq. 28-2 to calculate the magnitude of the second current. $F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2 \rightarrow$ $I_2 = \frac{2\pi}{\mu_0}\frac{F_2}{\ell_2}\frac{d}{I_1} = \frac{2\pi}{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}\left(7.8\times10^{-4}\ \text{N/m}\right)\frac{0.070\ \text{m}}{28\ \text{A}} = 9.75\ \text{A} \approx \boxed{9.8\ \text{A upward}}$

Q2

A horizontal compass is placed 18 cm due south from a straight vertical wire carrying a 43-A current downward. In what direction does the compass needle point at this location? Assume the horizontal component of the Earth's field at this point is 0.45×10^{-4} T and the magnetic declination is 0°.

Sol 2

At the location of the compass, the magnetic field caused by the wire will point to the west, and the Earth's magnetic field points due North. The compass needle will point in the direction of the NET magnetic field.

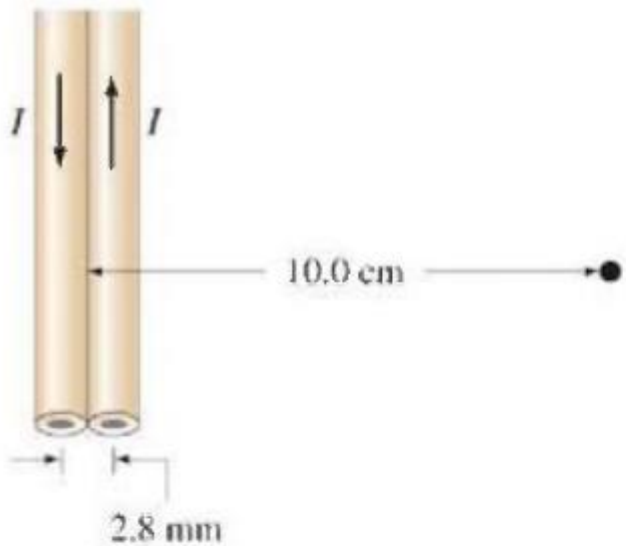


$$B_{wire} = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(43\,\text{A})}{2\pi(0.18\,\text{m})} = 4.78 \times 10^{-5}\,\text{T}$$

$$\theta = \tan^{-1}\frac{B_{Earth}}{B_{wire}} = \tan^{-1}\frac{4.5 \times 10^{-5}\,\text{T}}{4.78 \times 10^{-5}\,\text{T}} = \boxed{43°\text{N of W}}$$

Q3

A third wire is placed in the plane of the two wires shown in Fig. parallel and just to the right. If it carries 25.0 A upward, what force per meter of length does it exert on each of the other two wires? Assume it is 2.8 mm from the nearest wire, center to center.



Sol 3

The center of the third wire is 5.6 mm from the left wire, and 2.8 mm from the right wire. The force on the near (right) wire will attract the near wire, since the currents are in the same direction. The force on the far (left) wire will repel the far wire, since the currents oppose each other. Use Eq. to calculate the force per unit length.

$$F_{near} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{near}}\ell_{near} \rightarrow$$

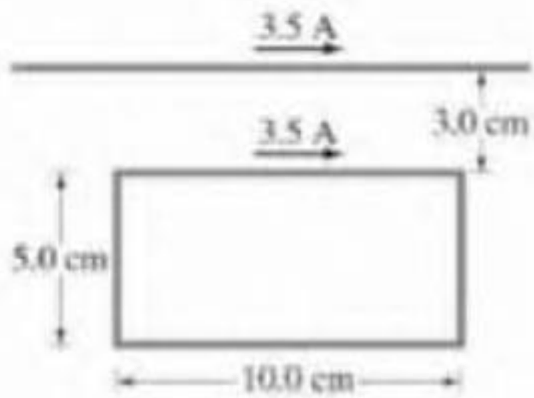
$$\frac{F_{near}}{\ell_{near}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{near}} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\frac{(25.0\,\text{A})(28.0\,\text{A})}{2.8\times10^{-3}\,\text{m}} = \boxed{0.050\,\text{N/m, attractive}}$$

$$F_{far} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{far}}\ell_{far} \rightarrow$$

$$\frac{F_{far}}{\ell_{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{far}} = \frac{4\pi\times10^{-7}\,\text{T}\bullet\text{m/A}}{2\pi}\frac{(25.0\,\text{A})(28.0\,\text{A})}{5.6\times10^{-3}\,\text{m}} = \boxed{0.025\,\text{N/m, repelling}}$$

Q4

A rectangular loop of wire is placed next to a straight wire, as shown in Fig. There is a current of 3.5 A in both wires. Determine the magnitude and direction of the net force on the loop.



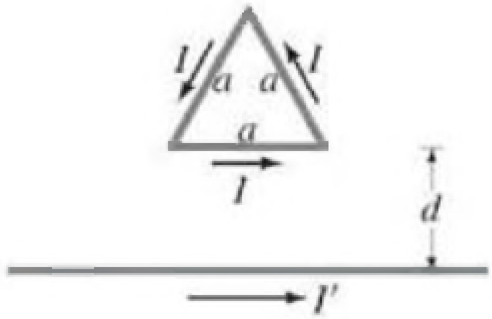
Sol 4

The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq.

$$F_{\text{net}} = F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{near}}}\ell_{\text{near}} - \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d_{\text{far}}}\ell_{\text{far}} = \frac{\mu_0}{2\pi} I_1 I_2 \ell \left(\frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}} \right)$$

$$= \frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi} (3.5\,\text{A})^2 (0.100\,\text{m}) \left(\frac{1}{0.030\,\text{m}} - \frac{1}{0.080\,\text{m}} \right) = \boxed{5.1 \times 10^{-6}\ \text{N, towards wire}}$$

Q5	A triangular loop of side length a carries a current I. If this loop is placed a distance d away from a very long straight wire carrying a current I', determine the force on the loop. 
Sol 5	.... We break the current loop into the three branches of the triangle and add the forces from each of the three branches. The current in the parallel branch flows in the same direction as the long straight wire, so the force is attractive with magnitude given by Eq. $F_1 = \frac{\mu_0 I I'}{2\pi d} a$ By symmetry the magnetic force for the other two segments will be equal. These two wires can be broken down into infinitesimal segments, each with horizontal length dx. The net force is found by integrating Eq. 28-2 over the side of the triangle. We set x=0 at the left end of the left leg. The distance of a line segment to the wire is then given by $r = d + \sqrt{3}x$. Since the current in these segments flows opposite the direction of the current in the long wire, the force will be repulsive. $F_2 = \int_0^{a/2} \frac{\mu_0 I I'}{2\pi\left(d+\sqrt{3}x\right)} dx = \frac{\mu_0 I I'}{2\pi\sqrt{3}} \ln\left(d+\sqrt{3}x\right)\Big\|_0^{a/2} = \frac{\mu_0 I I'}{2\pi\sqrt{3}} \ln\left(1+\frac{\sqrt{3}a}{2d}\right)$ We calculate the net force by summing the forces from the three segments. $F = F_1 - 2F_2 = \frac{\mu_0 I I'}{2\pi d} a - 2\frac{\mu_0 I I'}{2\pi\sqrt{3}} \ln\left(1+\frac{\sqrt{3}a}{2d}\right) = \boxed{\frac{\mu_0 I I'}{\pi}\left[\frac{a}{2d} - \frac{\sqrt{3}}{3}\ln\left(1+\frac{\sqrt{3}a}{2d}\right)\right]}$ ....

Q6

A 2.5-mm-diameter copper wire carries a 33-A current (uniform across its cross section). Determine the magnetic field: (*a*) at the surface of the wire; (*b*) inside the wire, 0.50 mm below the surface; (*c*) outside the wire 2.5 mm from the surface.

Sol 6

(*a*) We use Eq. , with *r* equal to the radius of the wire.

$$B=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(33\ \text{A})}{2\pi\left(1.25\times10^{-3}\ \text{m}\right)}=\boxed{5.3\ \text{mT}}$$

(*b*) We use the results of Example 28-6, for points inside the wire. Note that $r=(1.25-0.50)\,\text{mm}$ $=0.75\,\text{mm}$.

$$B=\frac{\mu_0 I r}{2\pi R^2}=\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(33\,\text{A})\left(0.75\times10^{-3}\,\text{m}\right)}{2\pi\left(1.25\times10^{-3}\,\text{m}\right)^2}=\boxed{3.2\,\text{mT}}$$

(*c*) We use Eq. 28-1, with *r* equal to the distance from the center of the wire.

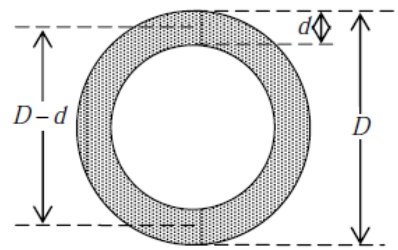
$$B=\frac{\mu_0 I}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(33\ \text{A})}{2\pi\left(1.25\times10^{-3}\ \text{m}+2.5\times10^{-3}\ \text{m}\right)}=\boxed{1.8\ \text{mT}}$$

Q7

A 20.0-m-long copper wire, 2.00 mm in diameter including insulation, is tightly wrapped in a single layer with adjacent coils touching, to form a solenoid of diameter 2.50 cm (outer edge). What is (*a*) the length of the solenoid and (*b*) the field at the center when the current in the wire is 16.7 A?

Sol 7

(*a*) The copper wire is being wound about an average diameter that is approximately equal to the outside diameter of the solenoid minus the diameter of the wire, or $D-d$. See the (not to scale) end-view diagram. The length of each wrapping is $\pi(D-d)$. We divide the length of the wire L by the length of a single winding to determine the number of loops. The length of the solenoid is the number of loops multiplied by the outer diameter of the wire, *d*.



$$\ell = d\frac{L}{\pi(D-d)} = (2.00\times10^{-3}\,\text{m})\frac{20.0\,\text{m}}{\pi\left[2.50\times10^{-2}\,\text{m}-(2.00\times10^{-3}\,\text{m})\right]} = \boxed{0.554\,\text{m}}$$

(*b*) The field inside the solenoid is found using Eq. 28-4. Since the coils are wound closely together, the number of turns per unit length is equal to the reciprocal of the wire diameter.

$$n = \frac{\#\ \text{turns}}{\ell} = \frac{\frac{L}{\pi(D-d)}}{\ell} = \frac{\ell/d}{\ell} = \frac{1}{d}$$

$$B = \mu_0 nI = \frac{\mu_0 I}{d} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(16.7\,\text{A})}{2.00\times10^{-3}\,\text{m}} = \boxed{10.5\,\text{mT}}$$