Electromagnetic Fields – Tutorial 02 & Assignment 02

Transmission Lines Fundamentals

#	Student ID	Student Name	Grade (10)
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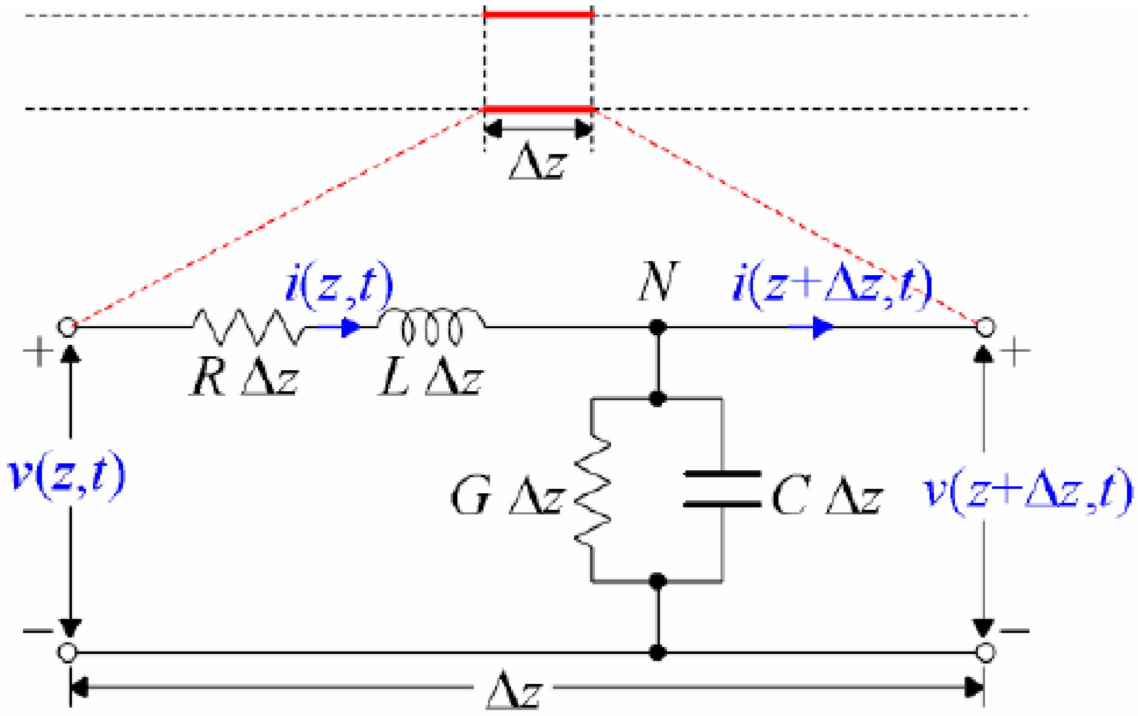
Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

Q 1

For a lossless transmission line we drives the following equation



Equivalent circuit of a real transmission line.

$$\frac{\partial}{\partial z} v(z,t) = -L \frac{\partial}{\partial t} i(z,t)$$

$$\frac{\partial}{\partial z} i(z,t) = -C \frac{\partial}{\partial t} v(z,t)$$

assuming that $R = 0$, $G = 0$

Please modify them if the line is lossy ($R \neq 0$, $G \neq 0$).

Sol 1

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$$v(z,t)-v(z+\Delta z,t)=i(z,t)\cdot R\Delta z+L\Delta z\cdot\frac{\partial}{\partial t}i(z,t)$$

When $\Delta z\to 0$,

$$\to\frac{\partial}{\partial z}v(z,t)=-i(z,t)\cdot R-L\cdot\frac{\partial}{\partial t}i(z,t)$$

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$$i(z,t)=i(z+\Delta z,t)+C\Delta z\cdot\frac{\partial}{\partial t}v(z+\Delta z,t)+v(z+\Delta,t)\cdot G\Delta z$$

When $\Delta z\to 0$, and assume $v(z,t)\approx v(z+\Delta z,t)$

$$\to\frac{\partial}{\partial z}i(z,t)=-v(z,t)\cdot G-C\cdot\frac{\partial}{\partial t}v(z,t)$$

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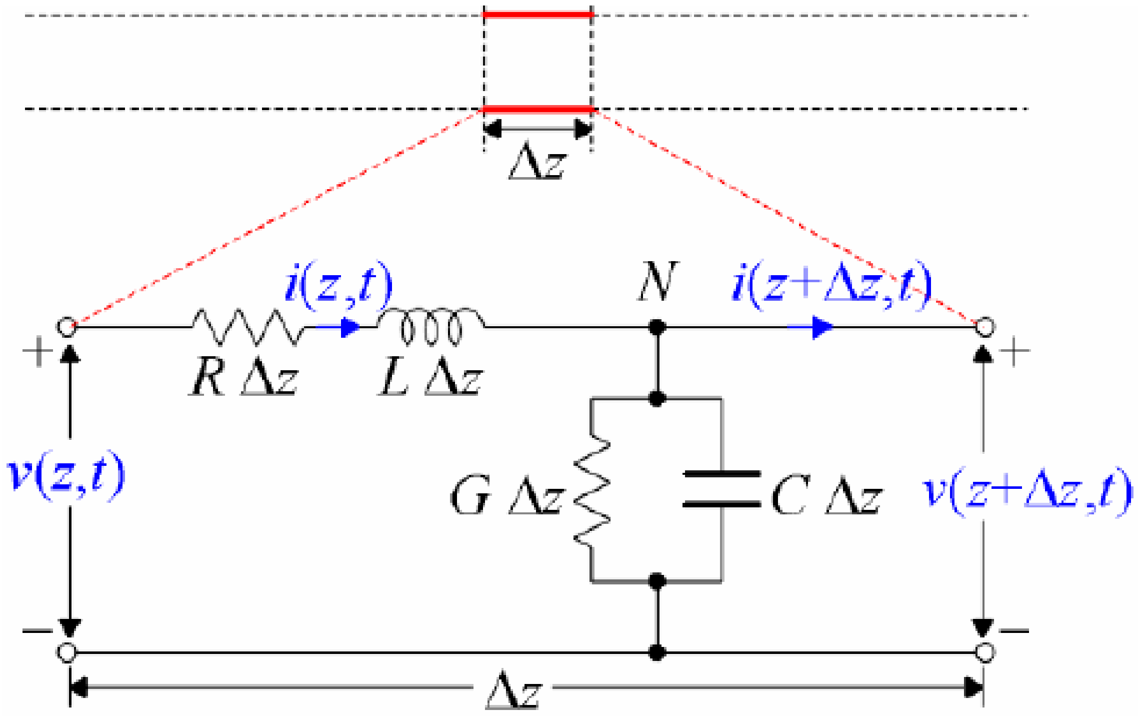
Q2

For a lossless transmission line we drives the following equation



Equivalent circuit of a real transmission line.

$$\frac{\partial^2}{\partial z^2} v(z,t) = LC \frac{\partial^2}{\partial t^2} v(z,t)$$

$$\frac{\partial^2}{\partial z^2} i(z,t) = LC \frac{\partial^2}{\partial t^2} i(z,t)$$

assuming that $R = 0$, $G = 0$

Please modify them if the line is lossy ($R \neq 0, G \neq 0$).

Sol 2	……………………………………………………………………………… ……………………………………………………………………………… …. By taking $\frac{\partial}{\partial z}$ for both sides of $\frac{\partial}{\partial z}v(z,t)=-i(z,t)\cdot R-L\cdot\frac{\partial}{\partial t}i(z,t)$ …. $\rightarrow\frac{\partial^2}{\partial z^2}v(z,t)=-\frac{\partial}{\partial z}i(z,t)\cdot R-L\cdot\frac{\partial}{\partial t}\frac{\partial}{\partial z}i(z,t).$ …. …. Then replace $i(z,t)$ by $\cdot\frac{\partial}{\partial z}i(z,t)=-v(z,t)\cdot G-C\cdot\frac{\partial}{\partial t}v(z,t)$ …. $\rightarrow\frac{\partial^2}{\partial z^2}v(z,t)=\left[v(z,t)\cdot G+\frac{\partial}{\partial t}v(z,t)\cdot C\right]\cdot R+L\cdot\left[\frac{\partial}{\partial t}v(z,t)\cdot G+\frac{\partial^2}{\partial t^2}v(z,t)\cdot C\right]$ …. …. $\rightarrow\frac{\partial^2}{\partial z^2}v(z,t)=GR\cdot v(z,t)+(CR+LG)\cdot\frac{\partial}{\partial t}v(z,t)+LC\cdot\frac{\partial^2}{\partial t^2}v(z,t)$ …. ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………

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By taking $\frac{\partial}{\partial z}$ for both sides of $\frac{\partial}{\partial z} i(z,t) = -v(z,t)\cdot G - C\cdot\frac{\partial}{\partial t} v(z,t)$

$$\rightarrow \frac{\partial^2}{\partial z^2} i(z,t) = -\frac{\partial}{\partial z} v(z,t)\cdot G - C\cdot\frac{\partial}{\partial t}\frac{\partial}{\partial z} v(z,t)$$

Then replace $v(z,t)$ by $\frac{\partial}{\partial z} v(z,t) = -i(z,t)\cdot R - L\cdot\frac{\partial}{\partial t} i(z,t)$

$$\rightarrow \frac{\partial^2}{\partial z^2} i(z,t) = \left[i(z,t)\cdot R + \frac{\partial}{\partial t} i(z,t)\cdot L \right]\cdot G + C\cdot\left[\frac{\partial}{\partial t} i(z,t)\cdot R + \frac{\partial}{\partial t^2} i(z,t)\cdot L \right]$$

$$\rightarrow \frac{\partial^2}{\partial z^2} i(z,t) = RG\cdot i(z,t) + (LG + RC)\cdot\frac{\partial}{\partial t} i(z,t) + LC\cdot\frac{\partial^2}{\partial t^2} i(z,t)$$

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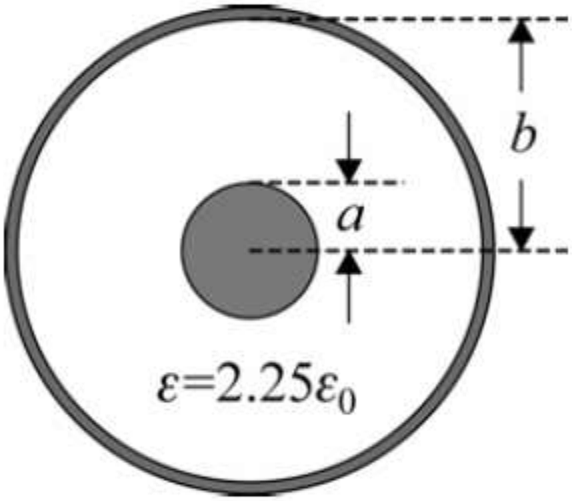
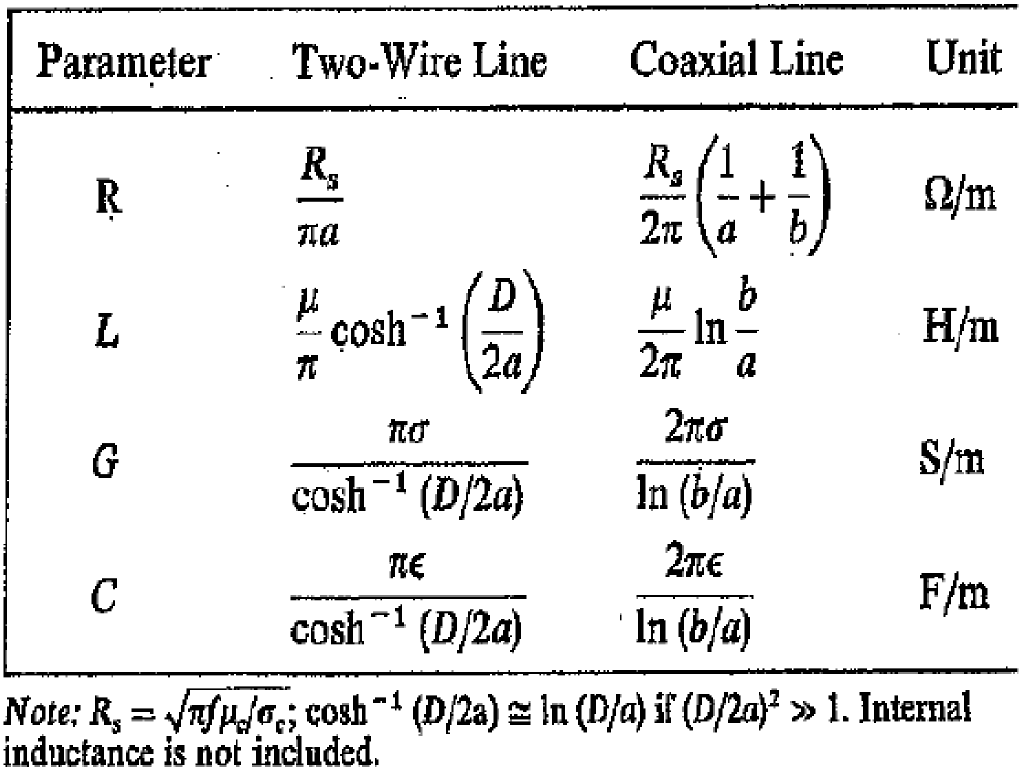
Q3

(10%) The cross-sectional geometry of a widely used coaxial able RG58/U is shown in Fig. (below), where the parameters are: $a = 0.45$ mm, $b = 1.47$ mm, permittivity of vacuum $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ (F/m), permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ (H/m). By Table of the textbook (p447), the inductance and capacitance per unit length of a coaxial cable are formulated as: $L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$, $C = \frac{2\pi\varepsilon}{\ln(b/a)}$. Evaluate L, C, v_p, Z_0 of a RG58/U cable, respectively. (The numbers will remind you of the typical ranges of these parameters in the real world.)



Cross-section of RG58/U coaxial cable.



Parameter	Two-Wire Line	Coaxial Line	Unit
R	$\frac{R_s}{\pi a}$	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	Ω/m
L	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	H/m
G	$\frac{\pi\sigma}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\sigma}{\ln(b/a)}$	S/m
C	$\frac{\pi\epsilon}{\cosh^{-1}(D/2a)}$	$\frac{2\pi\epsilon}{\ln(b/a)}$	F/m

Note: $R_s = \sqrt{\pi f \mu_c/\sigma_c}$; $\cosh^{-1}(D/2a) \cong \ln(D/a)$ if $(D/2a)^2 \gg 1$. Internal inductance is not included.

Sol 3

$$L=\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)=\frac{4\pi\times10^{-7}}{2\pi}\ln\left(\frac{1.47}{0.45}\right)=2.37\times10^{-7}\ (\mathrm{H/m})=0.237\ (\mu\mathrm{H/m})$$

$$C=\frac{2\pi\varepsilon}{\ln(b/a)}=\frac{2\pi\times2.25\times\frac{1}{36\pi}\times10^{-9}}{\ln\left(\frac{1.47}{0.45}\right)}=1.06\times10^{-10}\ (\mathrm{F/m})=106\ (\mathrm{pF/m})$$

$$v_p=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2.37\times10^{-7}\times1.06\times10^{-10}}}=1.995\times10^{8}\ (\mathrm{m/s})\approx 20\ (\mathrm{cm/ns})$$

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{2.37\times10^{-7}}{1.06\times10^{-10}}}\approx 47.3\ (\Omega)$$