Electric Circuits II – Tutorial 07

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#	Student ID	Student Name	Grade (10)
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Q1	Given the sinusoidal voltage $v(t) = 50\cos(30t + 10°)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t = 10$ ms.
Sol 1	(a) V_m = **50 V**. (b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = \underline{0.2094s}$ = **209.4ms** (c) Frequency f = ω/(2π) = 30/(2π) = **4.775 Hz**. (d) At t=1ms, v(0.01) = 50cos(30x0.01rad + 10°) = 50cos(1.72° + 10°) = **44.48 V** and ωt = **0.3 rad**.

Q2	For the following pairs of sinusoids, determine which one leads and by how much. (a) $v(t) = 10\cos(4t - 60°)$ and $i(t) = 4\sin(4t + 50°)$ (b) $v_1(t) = 4\cos(377t + 10°)$ and $v_2(t) = -20\cos 377t$ (c) $x(t) = 13\cos 2t + 5\sin 2t$ and $y(t) = 15\cos(2t - 11.8°)$

Sol 2	(a) $v(t) = 10\cos(4t - 60°)$ $i(t) = 4\sin(4t + 50°) = 4\cos(4t + 50° - 90°) = 4\cos(4t - 40°)$ Thus, **i(t) leads v(t) by 20°**. (b) $v_1(t) = 4\cos(377t + 10°)$ $v_2(t) = -20\cos(377t) = 20\cos(377t + 180°)$ Thus, **$v_2(t)$ leads $v_1(t)$ by 170°**. (c) $x(t) = 13\cos(2t) + 5\sin(2t) = 13\cos(2t) + 5\cos(2t - 90°)$ $\mathbf{X} = 13\angle 0° + 5\angle{-90°} = 13 - j5 = 13.928\angle{-21.04°}$ $x(t) = 13.928\cos(2t - 21.04°)$ $y(t) = 15\cos(2t - 11.8°)$ phase difference $= -11.8° + 21.04° = 9.24°$ Thus, **y(t) leads x(t) by 9.24°**.

Q3	Calculate these complex numbers and express your results in rectangular form: (a) $\dfrac{60\angle 45°}{7.5 - j10} + j2$ (b) $\dfrac{32\angle -20°}{(6 - j8)(4 + j2)} + \dfrac{20}{-10 + j24}$

Sol 3	(a) $\frac{60\angle 45^\circ}{7.5-j10}+j2 = \frac{60\angle 45^\circ}{12.5\angle -53.13^\circ}+j2$ $= 4.8\angle 98.13^\circ + j2 = -0.6788+j4.752+j2$ $= \mathbf{-0.6788 + j6.752}$ (b) $(6-j8)(4+j2) = 24-j32+j12+16 = 40-j20 = 44.72\angle -26.57^\circ$ $\frac{32\angle -20^\circ}{(6-j8)(4+j2)}+\frac{20}{-10+j24} = \frac{32\angle -20^\circ}{44.72\angle -26.57^\circ}+\frac{20}{26\angle 112.62^\circ}$ $= 0.7156\angle 6.57^\circ + 0.7692\angle -112.62^\circ = 0.7109+j0.08188-0.2958-j0.71$ $= \mathbf{0.4151-j0.6281}$

Q4	Evaluate the following complex numbers and leave your results in polar form: (a) $5\angle 30^\circ\left(6 - j8 + \frac{3\angle 60^\circ}{2 + j}\right)$
Sol 4	$(5\angle 30^\circ)(6-j8+1.1197+j0.7392) = (5\angle 30^\circ)(7.13-j7.261)$ $= (5\angle 30^\circ)(10.176\angle -45.52^\circ) =$ $\mathbf{50.88\angle -15.52^\circ}$.

Q5	Find the phasors corresponding to the following signals: (a) $v(t) = 21\cos(4t - 15^\circ)$ V (b) $i(t) = -8\sin(10t + 70^\circ)$ mA

Sol 5	(a) $V = \underline{21 < -15^o \text{ V}}$ (b) $i(t) = 8\sin(10t + 70^o + 180^o) = 8\cos(10t + 70^o + 180^o - 90^o) = 8\cos(10t + 160^o)$ $\mathbf{I = 8\angle 160° \text{ mA}}$

Q6	Simplify the following expressions: (a) $\dfrac{(5 - j6) - (2 + j8)}{(-3 + j4)(5 - j) + (4 - j6)}$ (b) $\dfrac{(240\underline{/75°} + 160\underline{/-30°})(60 - j80)}{(67 + j84)(20\underline{/32°})}$
Sol 6	(a) $\dfrac{3 - j14}{-7 + j17} = \dfrac{14.318\angle -77.91°}{18.385\angle 112.38°} = 0.7788\angle 169.71° = \underline{-0.7663 + j0.13912}$ (b) $\dfrac{(62.116 + j231.82 + 138.56 - j80)(60 - j80)}{(67 + j84)(16.96 + j10.5983)} = \dfrac{24186 - 6944.9}{246.06 + j2134.7} = \underline{-1.922 - j11.55}$

Q7	Using phasors, find: (a) $3\cos(20t + 10°) - 5\cos(20t - 30°)$ (b) $40\sin 50t + 30\cos(50t - 45°)$ (c) $20\sin 400t + 10\cos(400t + 60°)$ $-5\sin(400t - 20°)$

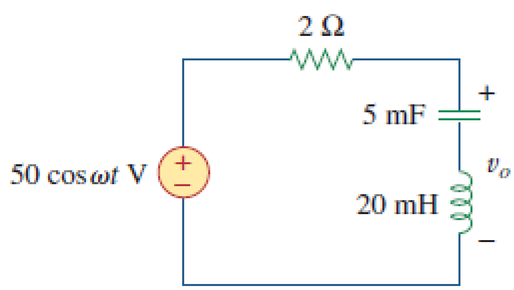
Sol 7	(a) $3\angle 10° - 5\angle -30° = 2.954 + j0.5209 - 4.33 + j2.5$ $= -1.376 + j3.021$ $= 3.32\angle 114.49°$ Therefore, $3\cos(20t + 10°) - 5\cos(20t - 30°)$ $=$ **$3.32\cos(20t + 114.49°)$** (b) $40\angle -90° + 30\angle -45° = -j40 + 21.21 - j21.21$ $= 21.21 - j61.21$ $= 64.78\angle -70.89°$ Therefore, $40\sin(50t) + 30\cos(50t - 45°) =$ **$64.78\cos(50t - 70.89°)$** (c) Using $\sin\alpha = \cos(\alpha - 90°)$, $20\angle -90° + 10\angle 60° - 5\angle -110° = -j20 + 5 + j8.66 + 1.7101 + j4.699$ $= 6.7101 - j6.641$ $= 9.44\angle -44.7°$ Therefore, $20\sin(400t) + 10\cos(400t + 60°) - 5\sin(400t - 20°)$ $=$ **$9.44\cos(400t - 44.7°)$**

Q8	An alternating voltage is given by $v(t) = 55\cos(5t + 45°)$ V. Use phasors to find $$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)dt$$ Assume that the value of the integral is zero at $t = -\infty$.
Sol 8	Let $f(t) = 10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)dt$ $F = 10V + j\omega 4V - \frac{2V}{j\omega}, \quad \omega = 5, \quad V = 55\angle 45°$ $F = 10V + j20V + j0.4V = (10 + j20.4)V = 22.72\angle 63.89°(55\angle 45°) = 1249.6\angle 108.89°$ f(t) = **$1249.6\cos(5t + 108.89°)$**

Q9	Find $v(t)$ in the following integrodifferential equations using the phasor approach: (a) $v(t) + \int v\,dt = 10\cos t$ (b) $\frac{dv}{dt} + 5v(t) + 4\int v\,dt = 20\sin(4t + 10°)$
Sol 9	(a) $\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0°, \quad \omega = 1$ $\mathbf{V}(1 - j) = 10$ $\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45°$ Therefore, $v(t) = \mathbf{7.071cos(t + 45°)\ V}$ (b) $j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$ $\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle -80°$ $\mathbf{V} = \frac{20\angle -80°}{5 + j3} = 3.43\angle -110.96°$ Therefore, $v(t) = \mathbf{3.43cos(4t - 110.96°)\ V}$

Q10	What is the instantaneous voltage across a 2-μF capacitor when the current through it is $i = 4\sin(10^6 t + 25°)$ A?

Sol 10	$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2\times10^{-6})} = -j0.5$$ $$\mathbf{V} = \mathbf{IZ} = (4\angle 25°)(0.5\angle -90°) = 2\angle -65°$$ Therefore $v(t) = \mathbf{2\sin(10^6 t - 65°)\ V}$.

Q11	What value of ω will cause the forced response, v_o, in Fig. 9.41 to be zero? 
Sol 11	$v_o = 0$ when $jX_L - jX_C = 0$ so $X_L = X_C$ or $\omega L = \frac{1}{\omega C} \longrightarrow \omega = \frac{1}{\sqrt{LC}}$. $$\omega = \frac{1}{\sqrt{(5\times10^{-3})(20\times10^{-3})}} = \mathbf{100\ rad/s}$$