Electric Circuits II – Tutorial 04

#	Student ID	Student Name	Grade (10)
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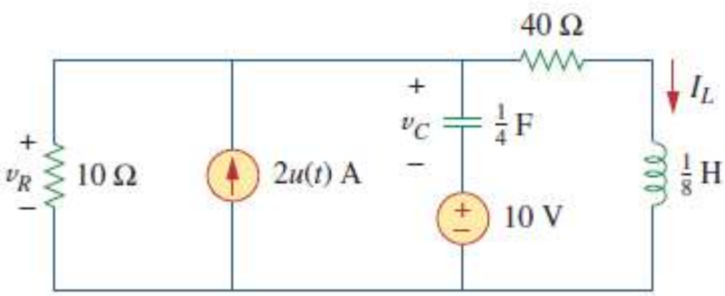
Q1

Refer to the circuit shown in Fig. Calculate:

(a) $i_L(0^+)$, $v_C(0^+)$, and $v_R(0^+)$,

(b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, and $dv_R(0^+)/dt$,

(c) $i_L(\infty)$, $v_C(\infty)$, and $v_R(\infty)$.

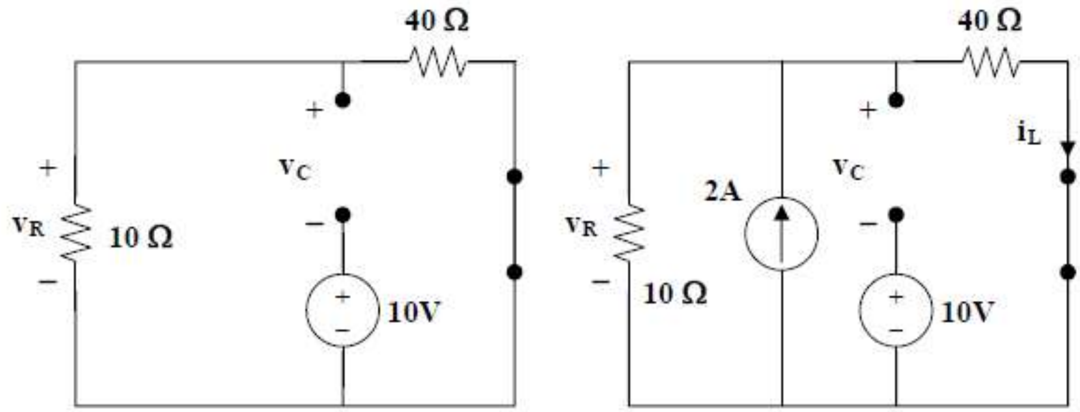


Sol 1

At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10$V.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **0A**, the capacitor has a voltage equal to **–10V**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) = \mathbf{0\ V}$.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $L di_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt = \mathbf{0 A/s}$, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C = \mathbf{8\ V/s}$. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 = \mathbf{8\ V/s}$.



(a) (b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \mathbf{400\ mA}$$

$$v_C(\infty) = 2[10||40] - 10 = 16 - 10 = \mathbf{6V}$$

$$v_R(\infty) = 2[10||40] = \mathbf{16\ V}$$

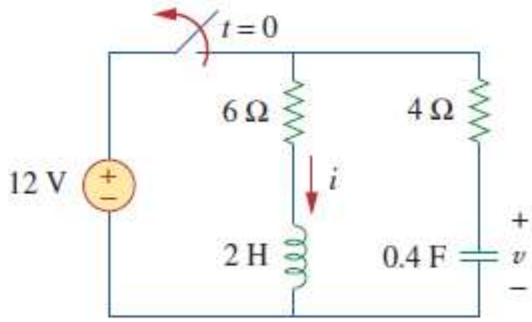
Q2

For the circuit in Fig. , find:

(a) $i(0^+)$ and $v(0^+)$,

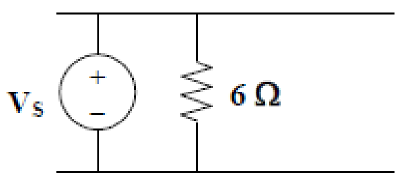
(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

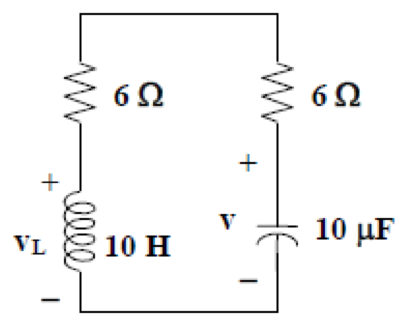


Sol 2

(a) At t = 0-, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

i(0-) = 12/6 = 2A, v(0-) = 12V

At t = 0+, i(0+) = i(0-) = **2A**, v(0+) = v(0-) = **12V**

(b) For $t > 0$, we have the equivalent circuit shown in Figure (b).

$$v_L = L di/dt \text{ or } di/dt = v_L/L$$

Applying KVL at $t = 0+$, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 = \mathbf{-4\ A/s}$

Similarly, $i_C = C dv/dt, \text{ or } dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

$$dv(0+)/dt = -2/0.4 = \mathbf{-5\ V/s}$$

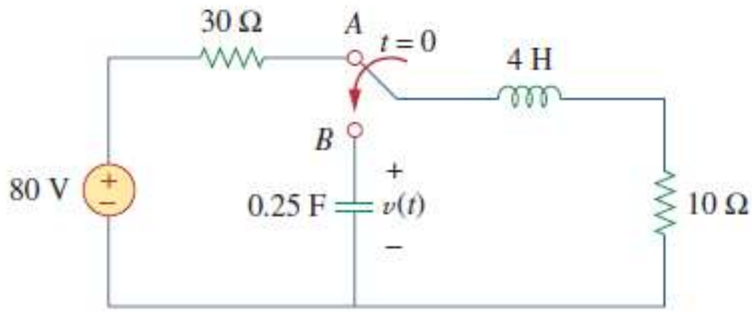
(c) As t approaches infinity, the circuit reaches steady state.

$$i(\infty) = \mathbf{0\ A},\ v(\infty) = \mathbf{0\ V}$$

Q3	The differential equation that describes the voltage in an *RLC* network is $$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$ Given that $v(0) = 0$, $dv(0)/dt = 10$ V/s, obtain $v(t)$.
Sol 3	$s^2 + 5s + 4 = 0$, thus $s_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = -4, -1$. $$v(t) = (Ae^{-4t} + Be^{-t}),\ v(0) = 0 = A + B, \text{ or } B = -A$$ $$dv/dt = (-4Ae^{-4t} - Be^{-t})$$ $$dv(0)/dt = 10 = -4A - B = -3A \text{ or } A = -10/3 \text{ and } B = 10/3.$$ Therefore, $v(t) = \mathbf{(-(10/3)e^{-4t} + (10/3)e^{-t})\ V}$

Q4

The switch in Fig.   moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0$, find $v(t)$ for $t > 0$.



Sol 4

When the switch is in position A, $v(0^-) = 0$ and $i_L(0) = 80/40 = 2$ A. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

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Since $\alpha > \omega_o$, we have overdamped case.

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.56}$ –0.5 and –2   ).9336

$$v(t) = Ae^{-2t} + Be^{-0.5t} \quad (1)$$

$$v(0) = 0 = A + B \quad (2)$$

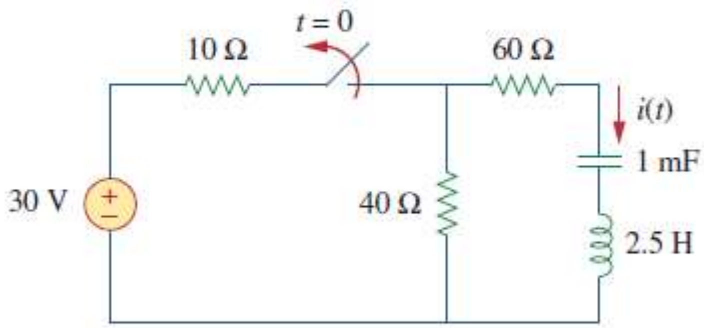
$$i_C(0) = C(dv(0)/dt) = -2 \text{ or } dv(0)/dt = -2/C = -8.$$

But $\dfrac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$

$$\frac{dv(0)}{dt} = -2A - 0.5B = -8 \quad (3)$$

Solving (2) and (3) gives A= 1.3333 and B = −1.3333

$$v(t) = \mathbf{5.333e^{-2t} - 5.333e^{-0.5t}}\ \text{V}.$$

<table>
<tr><td>Q5</td><td>Find $i(t)$ for $t > 0$ in the circuit of Fig.
</td></tr>
<tr><td>Sol 5</td><td>At $t = 0$, $i(0) = 0$, $v_C(0) = 40x30/50 = 24V$
For $t > 0$, we have a source-free RLC circuit.
$$\alpha = R/(2L) = (40+60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x2.5}} = 20$$
$\omega_o = \alpha$ leads to critical damping
$i(t) = [(A+Bt)e^{-20t}]$, $i(0) = 0 = A$
$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\}$,
but $di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0+24]$
Hence, $B = -9.6$ or $i(t) = \mathbf{[-9.6te^{-20t}]\ A}$</td></tr>
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