كلية الهندسة

Faculty of Engineering

جامعة الأهرام الكندية

AHRAM CANADIAN UNIVERSITY

Electric Circuits II – Tutorial 03

Step Response of RC/RL Circuits

#	Student ID	Student Name	Grade (10)
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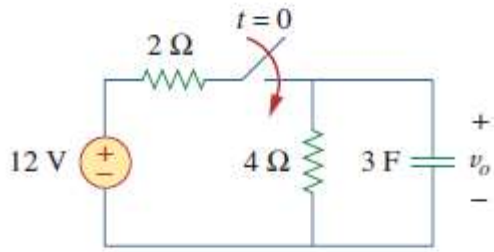
Q1

(a) If the switch in Fig. has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.



Sol 1

(a)

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$$

$$v_o(0) = 0, \qquad v_o(\infty) = \frac{4}{4+2}(12) = 8$$

$$\tau = R_{eq}C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \frac{4}{3}$$

$$\tau = \frac{4}{3}(3) = 4$$

$$v_o(t) = 8 - 8e^{-t/4}$$

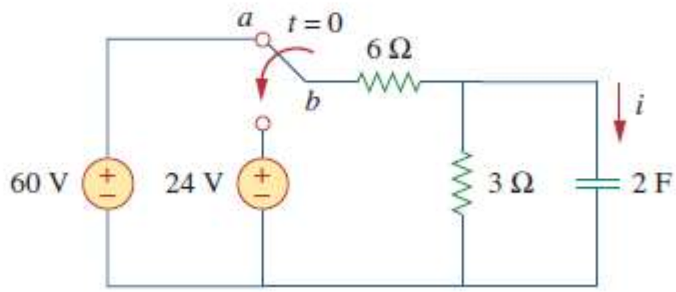
$$v_o(t) = \mathbf{8(1 - e^{-0.25t})\ V}$$

(b) For this case, $v_o(\infty) = 0$ so that

$$v_o(t) = v_o(0)e^{-t/\tau}$$

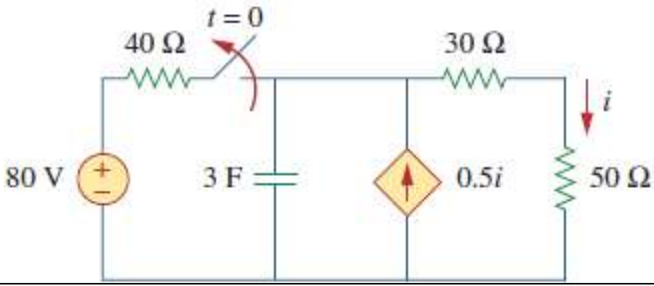
$$v_o(0) = \frac{4}{4+2}(12) = 8, \qquad \tau = RC = (4)(3) = 12$$

$$v_o(t) = \mathbf{8e^{-t/12}\ V}$$

Q2	The switch in Fig. 7.111 has been in position a for a long time. At $t = 0$, it moves to position b. Calculate $i(t)$ for all $t > 0$. 
Sol 2	$R_{eq} = 6 \parallel 3 = 2\,\Omega, \qquad \tau = RC = 4$ $v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$ Using voltage division, $v(0) = \frac{3}{3+6}(60) = 20\,V, \qquad v(\infty) = \frac{3}{3+6}(24) = 8\,V$ Thus, $v(t) = 8 + (20-8)\,e^{-t/4} = 8 + 12\,e^{-t/4}$ $i(t) = C\frac{dv}{dt} = (2)(12)\left(\frac{-1}{4}\right)e^{-t/4} = -6\,e^{-0.25t}\ A$

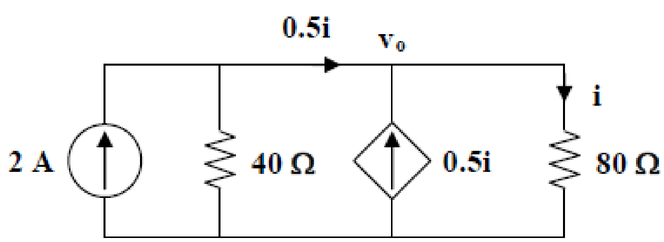
Q3

Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.



Sol 3

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



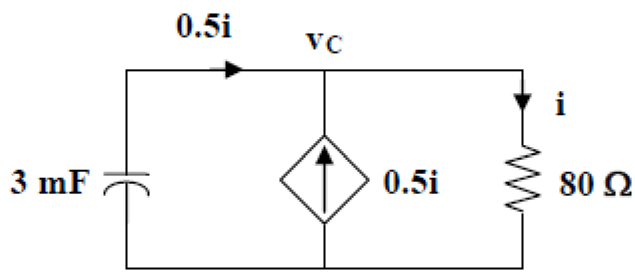
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $$\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

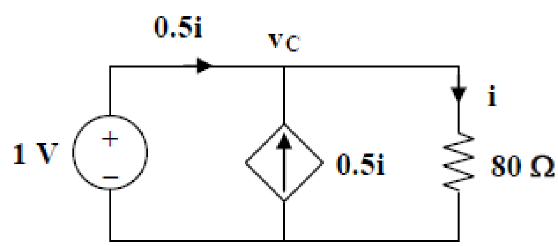
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)\, e^{-t/\tau}, \qquad \tau = R_{th} C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5\, i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th} C = 480$$

$$v_C(0) = 64 \text{ V}$$

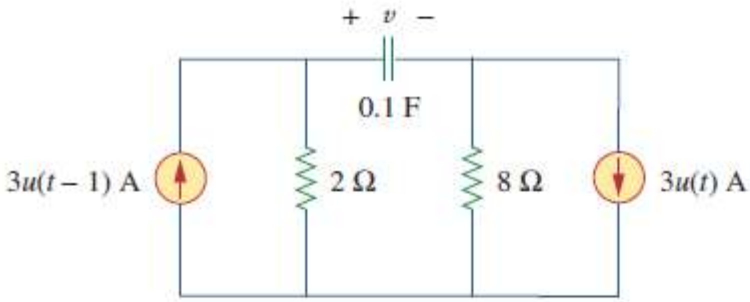
$$v_C(t) = 64\, e^{-t/480}$$

$$0.5\, i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right) 64\, e^{-t/480}$$

$$i(t) = \mathbf{800}\, e^{-t/480} u(t) \text{ } mA$$

Q4

Determine $v(t)$ for $t > 0$ in the circuit of Fig. if $v(0) = 0$.



Sol 4

For $t < 0$, $u(t) = 0$, $u(t-1) = 0$, $v(0) = 0$

For $0 < t < 1$, $\tau = RC = (2+8)(0.1) = 1$

$$v(0) = 0, \qquad v(\infty) = (8)(3) = 24$$
$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$
$$v(t) = 24\left(1 - e^{-t}\right)$$

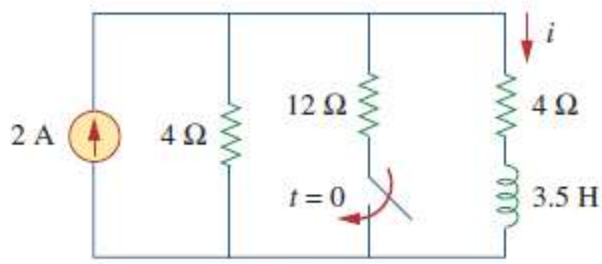
For $t > 1$, $v(1) = 24\left(1 - e^{-1}\right) = 15.17$

$$-6 + v(\infty) - 24 = 0 \longrightarrow v(\infty) = 30$$
$$v(t) = 30 + (15.17 - 30)\, e^{-(t-1)}$$
$$v(t) = 30 - 14.83\, e^{-(t-1)}$$

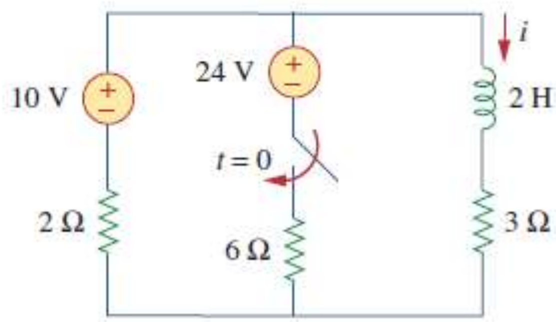
Thus,

$$v(t) = \begin{cases} \mathbf{24\left(1 - e^{-t}\right) V}, & \mathbf{0 < t < 1} \\ \mathbf{30 - 14.83\, e^{-(t-1)} V}, & \mathbf{t > 1} \end{cases}$$

Q5	Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.



(a)



(b)

Sol 5

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \mathbf{1\ A}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + (4 \,||\, 12) = 7\ \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{(4\,||\,12)}{4 + (4\,||\,12)}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-2t}$$

$$i(t) = \frac{\mathbf{1}}{\mathbf{7}}\left(\mathbf{6 - e^{-2t}}\right)\mathbf{A}$$

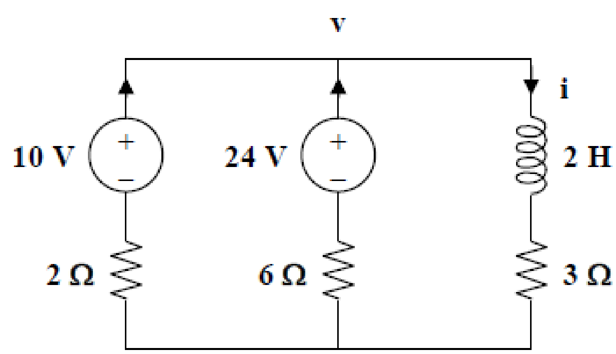
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \mathbf{2\ A}$

After t = 0, $R_{eq} = 3 + (6 \,||\, 2) = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



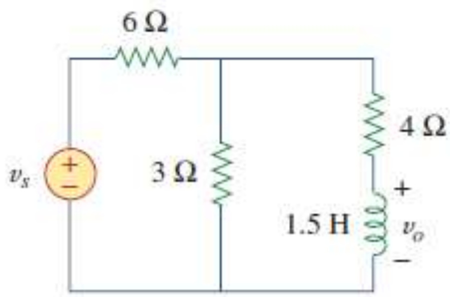
$$\frac{10-v}{2} + \frac{24-v}{6} = \frac{v}{3} \longrightarrow v = 9 \quad i(\infty) = \frac{v}{3} = 3\text{ A and}$$

$$i(t) = 3 + (2-3)e^{-9t/4}$$

$$i(t) = \mathbf{3 - e^{-9t/4}\ A}$$

Q6

Determine the step response $v_o(t)$ to $v_s = 18u(t)$ in the circuit of Fig.



Sol 6

Let i(t) be the current through the inductor.

For $t < 0$, $\quad v_s = 0, \quad i(0) = 0$

For $t > 0$, $\quad R_{eq} = 4 + (6 \,||\, 3) = 6\ \Omega$ and $\tau = \dfrac{L}{R_{eq}} = \dfrac{1.5}{6} = 0.25\,\text{sec.}$

At $t = \infty$, the inductor becomes a short and the current delivered by the 18 volts source is $I_s = 18/[6+(3||4)] = 18/7.714 = 2.333$ amps. The voltage across the 4-ohm resistor is equal to 18–6(2.333) = 18–14 = 4 volts. Therefore the current through the inductor is equal to $i(\infty) = 4/4 = 1$ amp.

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 1(1 - e^{-4t})\ \text{amps.}$$

$$v_o(t) = L\frac{di}{dt} = (1.5)(1)(-4)(-e^{-4t})$$

$$v_o(t) = \mathbf{[6e^{-4t}]u(t)\ volts.}$$