Electric Circuits II – Tutorial 01

Source Free RC Circuits

Q1

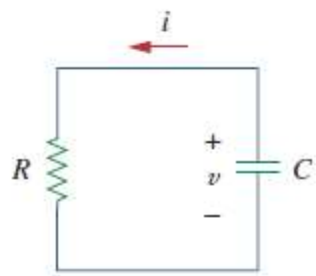
7.1 In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t}\ \text{V}, \quad t > 0$$
$$i(t) = 8e^{-200t}\ \text{mA}, \quad t > 0$$

(a) Find the values of R and C.

(b) Calculate the time constant τ.

(c) Determine the time required for the voltage to decay half its initial value at $t = 0$.



Sol 1

(a) $\tau = RC = 1/200$

For the resistor, $V = iR = 56e^{-200t} = 8Re^{-200t} \times 10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ \text{k}\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200 X 7 X 10^{3}} = \underline{0.7143 \mu F}$$

(b) $\tau = 1/200 =$ **5 ms**

(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2} x 56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ \text{ms}}$$

<table>
<tr><td>Q2</td><td>

7.3 Determine the time constant for the circuit in Fig. 7.83

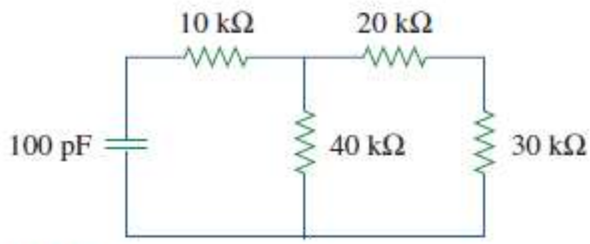


Figure 7.83
For Prob. 7.3.

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<tr><td>Sol 2</td><td>

R = 10 +20//(20+30) =10 + 40x50/(40 + 50)=32.22 kΩ

$$\tau = RC = 32.22X10^3 X100X10^{-12} = \underline{3.222\ \mu S}$$

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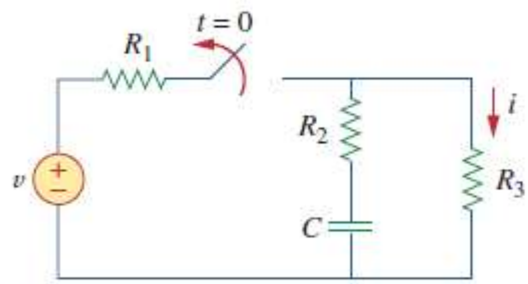
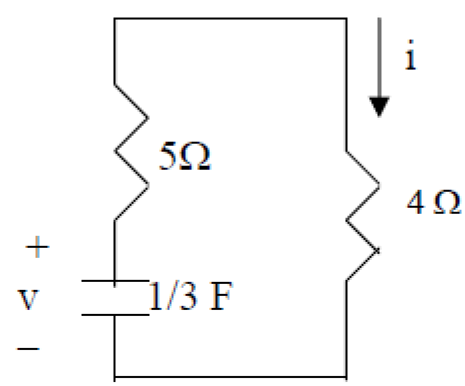
Q3	**7.5** Using Fig. 7.85, design a problem to help other students better understand source-free *RC* circuits.  **Figure 7.85** For Prob. 7.5.
Sol 3	Let v be the voltage across the capacitor. For t <0, $$v(0^-) = \frac{4}{2+4}(24) = 16 \text{ V}$$ For t >0, we have a source-free RC circuit as shown below.  $$\tau = RC = (4+5)\frac{1}{3} = 3s$$

$$v(t) = v(0)e^{-t/\tau} = 16e^{-t/3}\ \text{V}$$

$$i(t) = -C\frac{dv}{dt} = -\frac{1}{3}\left(-\frac{1}{3}\right)16e^{-t/3} = \underline{1.778e^{-t/3}\ \text{A}}$$

Q4

7.7 Assuming that the switch in Fig. 7.87 has been in position *A* for a long time and is moved to position *B* at $t = 0$. Then at $t = 1$ second, the switch moves from *B* to *C*. Find $v_C(t)$ for $t \geq 0$.

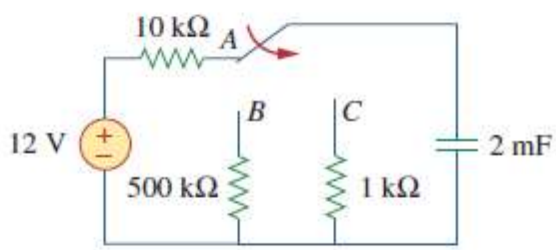


Figure 7.87
For Prob. 7.7.

Sol 4

Step 1. Determine the initial voltage on the capacitor. Clearly it charges to 12 volts when the switch is at position A because the circuit has reached steady state.

This then leaves us with two simple circuits, the first a 500 Ω resistor in series with a 2 mF capacitor and an initial charge on the capacitor of 12 volts. The second circuit which exists from t = 1 sec to infinity. The initial condition for the second circuit will be $v_C(1)$ from the first circuit. The time constant for the first circuit is (500)(0.002) = 1 sec and the time constant for the second circuit is (1,000)(0.002) = 2 sec. $v_C(\infty) = 0$ for both circuits.

Step 1.

$v_C(t) =$ **$12e^{-t}$ volts** for $0 < t < 1$ sec and $= 12e^{-1}e^{-2(t-1)}$ at t = 1 sec, and

$=$ **$4.415e^{-2(t-1)}$ volts** for $1 \text{ sec} < t < \infty$.

$12e^{-t}$ volts for $0 < t < 1$ sec, **$4.415e^{-2(t-1)}$ volts** for $1 \text{ sec} < t < \infty$.

Q5

7.9 The switch in Fig. 7.89 opens at $t = 0$. Find v_o for $t > 0$.

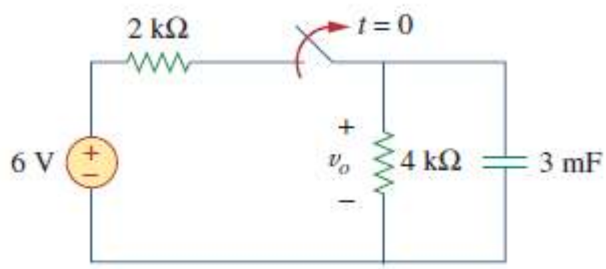


Figure 7.89
For Prob. 7.9.

Sol 5

For t < 0, the switch is closed so that

$$v_o(0) = \frac{4}{2+4}(6) = 4\ \text{V}$$

For t >0, we have a source-free RC circuit.

$$\tau = RC = 3x10^{-3}x4x10^{3} = 12s$$

$$v_o(t) = v_o(0)e^{-t/\tau} = \mathbf{4e^{-t/12}\ V}.$$