



# Lecture (01) First Order Circuits

By:

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## Revision

### Capacitors

$$i = \frac{dq}{dt} \quad q = Cv \quad C = \frac{\epsilon A}{d}$$

$$i = C \frac{dv}{dt}$$

$$v(t) = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$$

where  $v(t_0) = q(t_0)/C$  is the voltage across the capacitor at time  $t_0$ .

$$w = \frac{1}{2} Cv^2 = \frac{q^2}{2C}$$

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### Notes:

$i = C \frac{dv}{dt}$  when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus: A capacitor is an open circuit to dc

- The voltage on a capacitor cannot change suddenly, Why? huge change  $\Delta V$  in  $t=0 \rightarrow I=\infty$ , which is physically impossible

### Inductors

$$L = \frac{N^2 \mu A}{\ell}$$

$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$$

$$w = \frac{1}{2} Li^2$$

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## Notes

$$v = L \frac{di}{dt}$$

- the voltage across an inductor is zero when the current is constant
- An inductor acts like a short circuit to dc.
- The current through an inductor cannot change instantaneously
- The current on a inductor cannot change suddenly,

Why? huge change  $\Delta i$  in  $t=0 \rightarrow v=\infty$  , which is physically impossible

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## Introduction

- we have considered the three passive elements (resistors, capacitors, and inductors)
- we are prepared to consider circuits that contain various combinations of two or three of the passive elements.
- We shall examine two types of simple circuits: a circuit comprising a resistor and capacitor and a circuit comprising a resistor and an inductor.
- These are called *RC* and *RL* circuits, respectively.
- the analysis of *RC* and *RL* circuits by applying Kirchhoff's laws, as we did for resistive circuits.
- The only difference is that applying Kirchhoff's laws to purely resistive circuits results in algebraic equations, while applying the laws to *RC* and *RL* circuits produces differential equations

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- The differential equations resulting from analyzing *RC* and *RL* circuits are of the first order.

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## The Source-Free *RC* Circuit

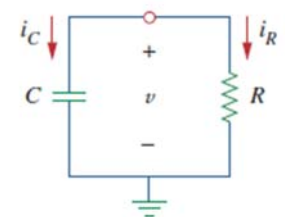
- A source-free *RC* circuit occurs when its dc source is suddenly disconnected.
- The energy already stored in the capacitor is released to the resistors.
- Our objective is to determine the circuit response,
- we assume to be the voltage  $V(t)$ , across the capacitor

$$v(0) = V_0 \rightarrow t = 0$$

$$w(0) = \frac{1}{2} CV_0^2 \rightarrow t = 0$$

- KCL :

$$i_C + i_R = 0$$



- Ohm:

$$i_R = v/R$$

$$i_c = C \frac{dv}{dt}$$

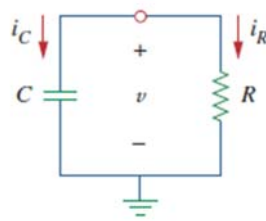
- Substitute:

$$C \frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

- Rearrange

$$\frac{dv}{v} = -\frac{1}{RC} dt$$



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- From integration table:  $\int \frac{1}{x} dx = \ln|x|$       $\frac{dv}{v} = -\frac{1}{RC} dt$

- Solving differential equation by integrating both sides

$$\ln v = -\frac{t}{RC} + k$$

- Taking power e of both sides

$$V(t) = e^{-\frac{t}{RC}} e^k$$

- Let  $K=e^k$

$$V(t) = K e^{-\frac{t}{RC}}$$

- At  $t=0 \rightarrow V(0) = e^0 K = K = V_0$

$$V(t) = V_0 e^{-t/RC}$$

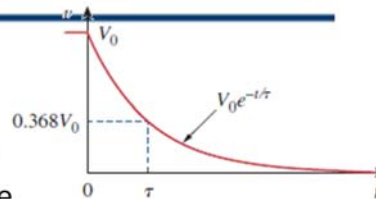
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$$V(t) = V_0 e^{-t/RC}$$

- voltage response of the RC circuit is an exponential decay of the initial voltage.
- natural response: response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source.
- the voltage decreases is expressed in terms of the *time constant*, denoted by  $\tau$  "tau"

$$\tau = RC$$



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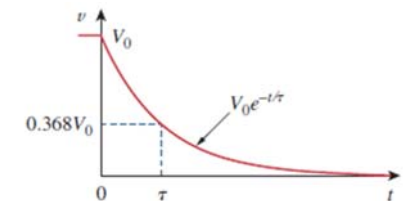
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$$v(t) = V_0 e^{-t/\tau}$$

- The time constant of a circuit is the time required for the response to decay to a factor of  $1/e$  or 36.8 % percent of its initial value.1

at  $t = \tau$ ,

$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368V_0$$

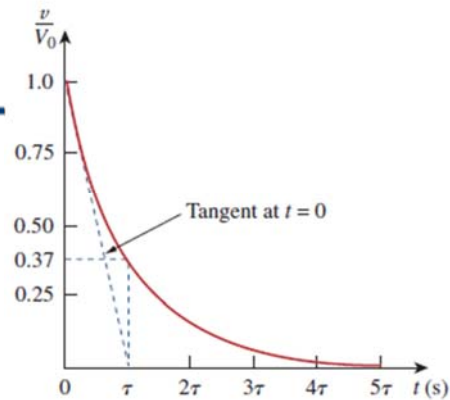


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Values of  $v(t)/V_0 = e^{-t/\tau}$ .

$t$	$v(t)/V_0$
$\tau$	0.36788
$2\tau$	0.13534
$3\tau$	0.04979
$4\tau$	0.01832
$5\tau$	0.00674



- the voltage  $V(t)$  is less than 1 percent of after (five time  $\tau$  constants)
- it takes ( $5\tau$ ) for the circuit to reach its final state or steady state when no changes take place with time.

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- smaller the time constant, the more rapidly the voltage decreases, the faster the response
- whereas a circuit with a large time constant gives a slow response because it takes longer to reach steady state.

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- Calculating current

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

- power dissipated in the resistor is  $p(t) = vi_R = \frac{V_0^2}{R} e^{-2t/\tau}$

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- The energy absorbed by the resistor up to time  $t$  is

$$w_R(t) = \int_0^t p(\lambda) d\lambda = \int_0^t \frac{V_0^2}{R} e^{-2\lambda/\tau} d\lambda$$

- From integration table

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

- $w_R(t) \equiv -\frac{\tau V_0^2}{2R} e^{-2\lambda/\tau} \Big|_0^t = \frac{1}{2} CV_0^2 (1 - e^{-2t/\tau}), \quad \tau = RC$

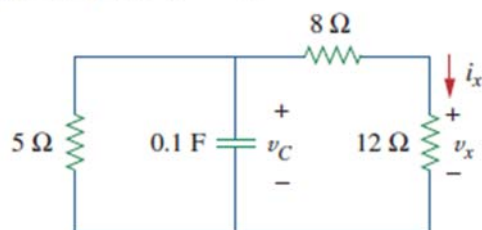
- Notice that as  $t \rightarrow \infty, w_R(\infty) \rightarrow \frac{1}{2} CV_0^2$ ,
- The energy that was initially stored in the capacitor is eventually dissipated in the resistor

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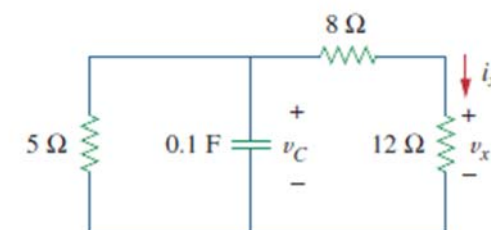
# Example 01

let  $v_C(0) = 15 \text{ V}$ . Find  $v_C$ ,  $v_x$ , and  $i_x$  for  $t > 0$ .



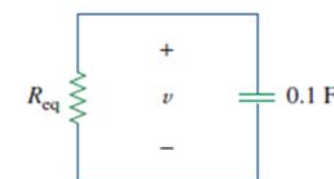
1. Our objective is always to first obtain capacitor voltage

- find the equivalent resistance or the Thevenin resistance at the capacitor terminals.



- 8 in series 12  $\rightarrow$  20 ohm
- 20 in parallel 5  $\rightarrow$

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4 \Omega$$



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- The time constant is

$$\tau = R_{eq}C = 4(0.1) = 0.4 \text{ s}$$

- Voltage response with time

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4} \text{ V},$$

$$v_C = v = 15e^{-2.5t} \text{ V}$$

- Return to original circuit, we can use voltage division to get

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t} \text{ V}$$

- Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t} \text{ A}$$

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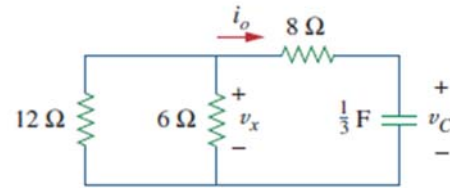
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## Example 02

Let  $v_C(0) = 60$  V. Determine  $v_C$ ,  $v_x$ , and  $i_o$  for  $t \geq 0$ .



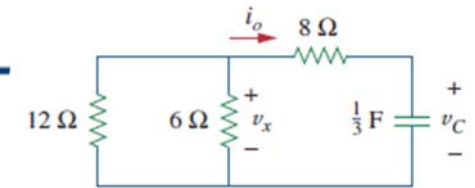
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- Find Req:  
12 parallel with 6  $\rightarrow$  4 ohm  
4 series with 8  $\rightarrow$  12 ohm
- Find time constant ( $\tau$ )  
 $\tau = 12/3 = 4$  sec

- Find  $V_c$   
 $v = v(0)e^{-t/\tau} = 60 e^{-0.25t}$
- Return to original circuit and use VD

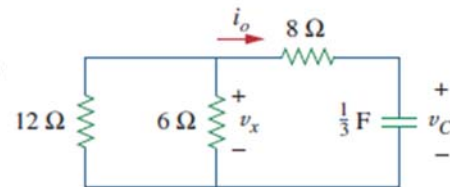
$$v_x = \frac{4}{4 + 8} V_c$$



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$$v_x = \frac{1}{3} \times 60e^{-0.25t} = 20 e^{-0.25t}$$



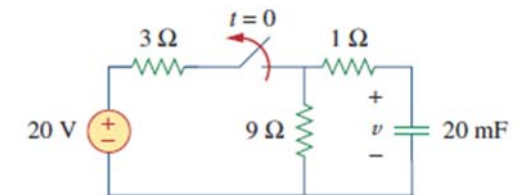
$$I_o = -\frac{v_x}{4} = -5e^{-0.25t}$$

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## Example 03

- The switch in the circuit in Fig. has been closed for a long time, and it is opened at  $t = 0$ . Find  $v(t)$  for  $t \geq 0$ . Calculate the initial energy stored in the capacitor.



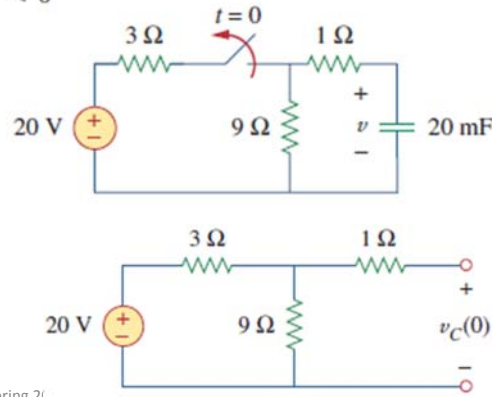
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- For  $t < 0$ , the switch is closed; the capacitor is an open circuit to dc,
- Using voltage division

$$v_C(t) = \frac{9}{9+3}(20) = 15 \text{ V}, \quad t < 0$$

$$v_C(0) = V_0 = 15 \text{ V}$$



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- For  $t > 0$  the switch is opened, and we have the RC circuit shown

$$R_{eq} = 1 + 9 = 10 \Omega$$

- The time constant is

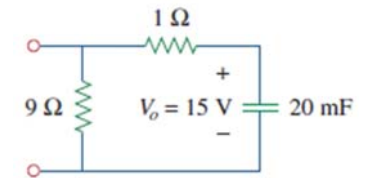
$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2 \text{ s}$$

- the voltage across the capacitor for  $t > 0$

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2} \text{ V}$$

$$v(t) = 15e^{-5t} \text{ V}$$

- Initial energy  $w_C(0) = \frac{1}{2}Cv_C^2 = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25 \text{ J}$

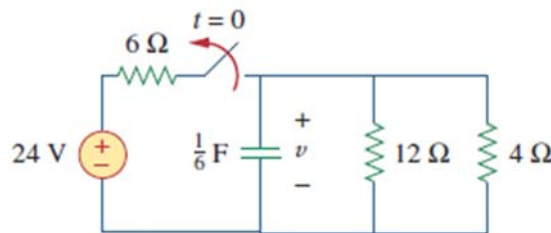


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## Example 04

If the switch in Fig. opens at  $t = 0$ , find  $v(t)$  for  $t \geq 0$  and  $w_C(0)$ .



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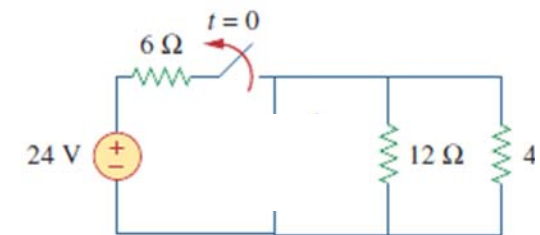
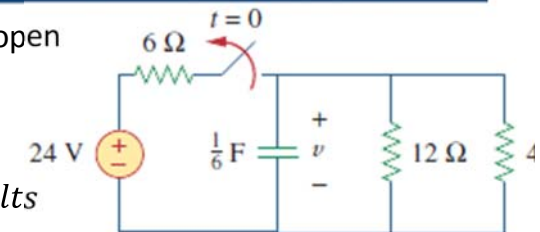
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- At  $t < 0$  switch is close cap is open

12 in parallel with 4  $\rightarrow$  3 ohm

- Using V.D.

$$V_C(0) = 24 \times \frac{3}{6+3} = 8 \text{ Volts}$$



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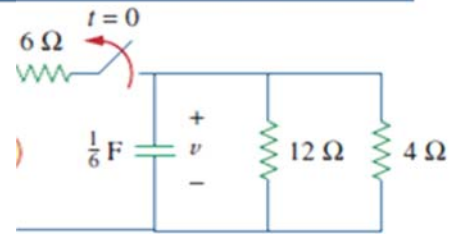


- For  $t > 0$ , switch is open
- $R_{eq} = 3 \text{ ohm}$
- Time constant

$$\tau = R_{eq} \times C = 3 \times \frac{1}{6} = 0.5 \text{ s}$$

$$v(t) = Vc(0)e^{-t/\tau} = 8e^{-2t}$$

$$Wc(0) = \frac{1}{2} C Vc^2 = \frac{1}{2} \frac{1}{6} 8^2 = 5.333 \text{ w}$$



## Summery

RC  
 $V = V_0 e^{-t/RC}$   
 $\tau = RC$   
 $V = V_0 e^{-t/\tau}$

$$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$

$$w_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}),$$

RL  
 $I = I_0 e^{-tR/L}$   
 $\tau = \frac{L}{R}$   
 $I = I_0 e^{-t/\tau}$

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$



**Thanks,..  
 See you next week (ISA),...**