Electric Circuits II – Assignment 07

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#	Student ID	Student Name	Grade (10)
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Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

Q1

A current source in a linear circuit has

$$i_s = 15\cos(25\pi t + 25°)\ \text{A}$$

(a) What is the amplitude of the current?
(b) What is the angular frequency?
(c) Find the frequency of the current.
(d) Calculate i_s at $t = 2$ ms.

Sol 1

(a) amplitude = **15 A**

(b) $\omega = 25\pi = \mathbf{78.54\ rad/s}$

(c) $f = \dfrac{\omega}{2\pi} = \mathbf{12.5Hz}$

(d) $I_s = 15\angle 25°\ \text{A}$

$$I_s(2\ \text{ms}) = 15\cos((500\pi)(2\times10^{-3}) + 25°)$$
$$= 15\cos(\pi + 25°) = 15\cos(205°)$$
$$= \mathbf{-13.595\ A}$$

Q2	Given $v_1 = 45\sin(\omega t + 30°)$ V and $v_2 = 50\cos(\omega t - 30°)$ V, determine the phase angle between the two sinusoids and which one lags the other.
Sol 2	$v_1 = 45\sin(\omega t + 30°)$ V $= 45\cos(\omega t + 30° - 90°) = 45\cos(\omega t - 60°)$ V $v_2 = 50\cos(\omega t - 30°)$ V This indicates that the phase angle between the two signals is **30°** and that v_1 **lags** v_2.

Q3	Calculate these complex numbers and express your results in rectangular form: (c) $20 + (16\angle -50°)(5 + j12)$
Sol 3	…… …… $20 + (16\angle -50°)(13\angle 67.38°) = 20+208\angle 17.38° = 20 + 198.5+j62.13$ $= \mathbf{218.5+j62.13}$ ……

Q4	Evaluate the following complex numbers and leave your results in polar form: (b) $\dfrac{(10\angle 60°)(35\angle -50°)}{(2 + j6) - (5 + j)}$
Sol 4	$\dfrac{(10\angle 60°)(35\angle -50°)}{(-3 + j5) = (5.83\angle 120.96°)} = \mathbf{60.02\angle -110.96^{\circ}}.$

Q5	Find the phasors corresponding to the following signals: ((c) $v(t) = 120 \sin(10t - 50°)$ V ((d) $i(t) = -60 \cos(30t + 10°)$ mA
Sol 5	(c) $v(t) = 120\sin(10^3 t - 50^o) = 120\cos(10^3 t - 50^o - 90^o)$ $\mathbf{V = 120\angle -140° \text{ V}}$ (d) $i(t) = -60\cos(30t + 10^o) = 60\cos(30t + 10^o + 180^o)$ $\mathbf{I = 60\angle -170° \text{ mA}}$

Q6	Simplify the following expressions: (c) $\left(\frac{10 + j20}{3 + j4}\right)^2 \sqrt{(10 + j5)(16 - j20)}$
Sol 6	…… …… (c) $\left[\frac{10 + j20}{3 + j4}\right]^2 \sqrt{(10 + j5)(16 - j20)}$ $= [(22.36\angle 63.43^\circ)/(5\angle 53.13^\circ)]^2[(11.18\angle 26.57^\circ)(25.61\angle -51.34^\circ)]^{0.5}$ $= [4.472\angle 10.3^\circ]^2[286.3\angle -24.77^\circ]^{0.5} = (19.999\angle 20.6^\circ)(16.921\angle -12.38^\circ) = 338.4\angle 8.22^\circ$ or **334.9+j48.38** …… …… …… …… …… …… …… …… …… …… …… …… …… …… …… ……

Q7	Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$ V and $v_2 = 12\cos(50t + 30°)$ V, find v.
Sol 7	$V = V_1 + V_2 = 10 < -60^o + 12 < 30^o = 5 - j8.66 + 10.392 + j6 = 15.62 < -9.805^o$ $v(t) = \mathbf{15.62\cos(50t - 9.8^\circ)\ V}$

Q8

Simplify the following:

(a) $f(t) = 5\cos(2t + 15°) - 4\sin(2t - 30°)$

(b) $g(t) = 8\sin t + 4\cos(t + 50°)$

(c) $h(t) = \int_0^t (10\cos 40t + 50\sin 40t)\,dt$

Sol 8

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(a) $F = 5\angle 15^o - 4\angle -30^o - 90^o = 6.8296 + j4.758 = 8.3236\angle 34.86^o$

$$\underline{f(t) = 8.324\cos(30t + 34.86^o)}$$

(b) $G = 8\angle -90^o + 4\angle 50^o = 2.571 - j4.9358 = 5.565\angle -62.49^o$

$$\underline{g(t) = 5.565\cos(t - 62.49^o)}$$

(c) $H = \frac{1}{j\omega}\left(10\angle 0^o + 50\angle -90^o\right)$ $\omega = 40$

i.e. $H = 0.25\angle -90^o + 1.25\angle -180^o = -j0.25 - 1.25 = 1.2748\angle -168.69^o$

$$h(t) = 1.2748\cos(40t - 168.69°)$$

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Q9

Using phasors, determine $i(t)$ in the following equations:

(a) $2\frac{di}{dt} + 3i(t) = 4\cos(2t - 45°)$

(b) $10\int i\,dt + \frac{di}{dt} + 6i(t) = 5\cos(5t + 22°)$ A

Sol 9

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle 45°, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle 45°$$

$$I = \frac{4\angle 45°}{3 + j4} = \frac{4\angle 45°}{5\angle 53.13°} = 0.8\angle -8.13°$$

Therefore, $i(t) = \mathbf{800\cos(2t - 8.13°)\ mA}$

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22°, \quad \omega = 5$$

$$(-j2 + j5 + 6)\mathbf{I} = 5\angle 22°$$

$$\mathbf{I} = \frac{5\angle 22°}{6 + j3} = \frac{5\angle 22°}{6.708\angle 26.56°} = 0.745\angle -4.56°$$

Therefore, $i(t) = \mathbf{745\cos(5t - 4.56°)\ mA}$

Q10	A voltage $v(t) = 100\cos(60t + 20°)$ V is applied to a parallel combination of a 40-kΩ resistor and a 50-μF capacitor. Find the steady-state currents through the resistor and the capacitor.
Sol 10	Since R and C are in parallel, they have the same voltage across them. For the resistor, $V = I_R R \longrightarrow I_R = V/R = \frac{100 < 20^o}{40k} = 2.5 < 20^o$ mA $i_R = \underline{2.5\cos(60t + 20^o)\ \text{mA}}$ For the capacitor, $i_C = C\frac{dv}{dt} = 50x10^{-6}(-60)x100\sin(60t + 20^o) = \underline{-300\sin(60t + 20^o)\ \text{mA}}$

Q11	A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.
Sol 11	$110 = \sqrt{v_R^2 + v_L^2}$ $v_L = \sqrt{110^2 - v_R^2}$ $v_L = \sqrt{110^2 - 85^2} = \mathbf{69.82\ V}$