Electric Circuits II – Assignment 06

#	Student ID	Student Name	Grade (10)
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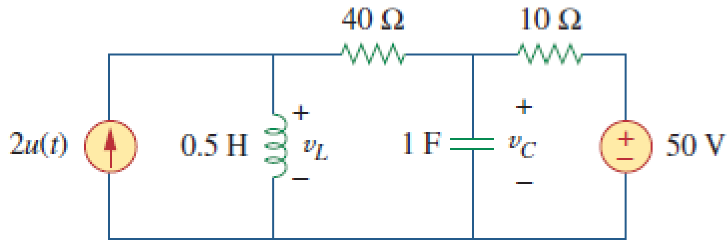
Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

Q1

Consider the circuit in Fig. Find $v_L(0^+)$ and $v_C(0^+)$.



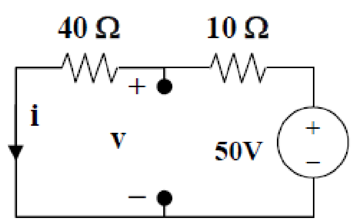
Sol 1

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

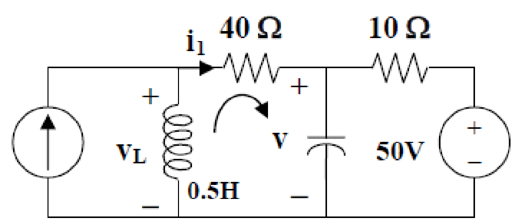
$$v(0+) = v(0-) = 40, \ i(0+) = i(0-) = 1$$

By KCL, $2 = i(0+) + i_1 = 1 + i_1$ which leads to $i_1 = 1$. By KVL, $-v_L + 40i_1 + v(0+) = 0$ which leads to $v_L(0+) = 40x1 + 40 = 80$

$$v_L(0+) = \mathbf{80\ V}, \quad v_C(0+) = \mathbf{40\ V}$$



(a)



(b)

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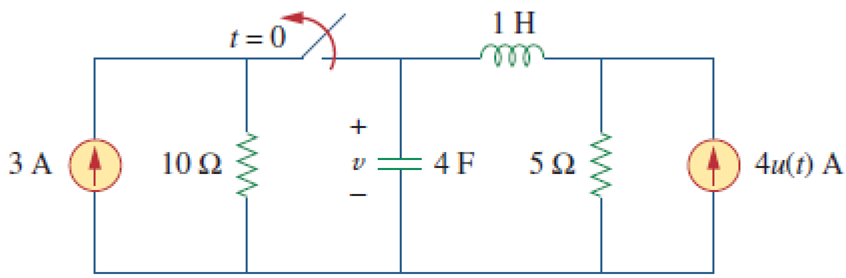
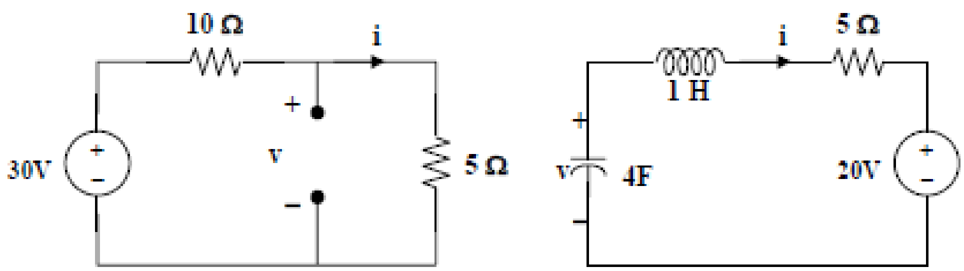
Q2

Find $v(t)$ for $t > 0$ in the circuit of Fig.



Sol 2

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



(a) (b)

$$i(0) = 30/15 = 2\text{ A},\ \ v(0) = 5\text{x}30/15 = 10\text{ V}$$

For $t > 0$, we have a series RLC circuit, shown in (b).

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5,\ \text{clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505$$

$$v(t) = V_s + [A_1 e^{-4.949t} + A_2 e^{-0.0505t}],\ V_s = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \text{ or}$$
$$A_2 = -10 - A_1$$
(1)

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = -2/4 = -1/2$$

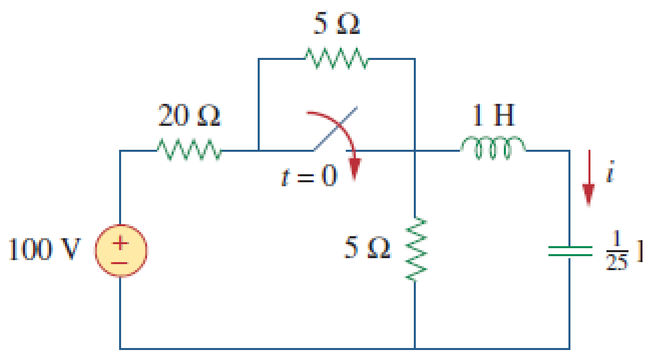
Hence, $-0.5 = -4.949A_1 - 0.0505A_2$ (2)

From (1) and (2), $-0.5 = -4.949A_1 + 0.0505(10 + A_1)$ or
$-4.898A_1 = -0.5 - 0.505 = -1.005$

$A_1 = 0.2052, \; A_2 = -10.205$

Q3

For the network in Fig , find $i(t)$ for $t > 0$.

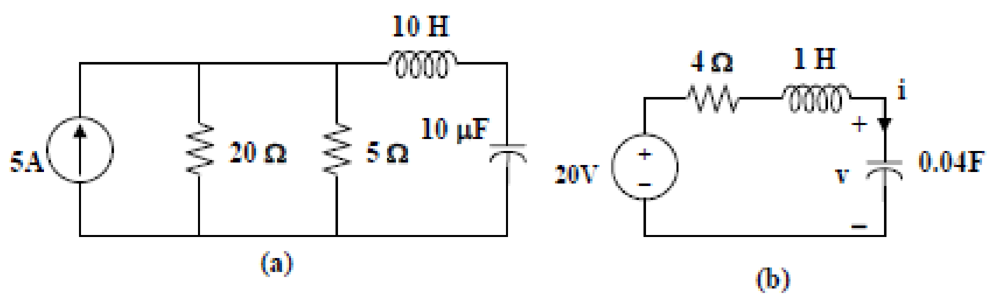


Sol 3

At t = 0^-, the switch is open. i(0) = 0, and

$$v(0) = 5x100/(20+5+5) = 50/3$$

For t > 0, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).



$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$

$\alpha = R/(2L) = (4)/(2x1) = 2$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$

Thus, $v(t) = V_s + [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}]$,

where $\omega_d = 4.583$ and $V_s = 20$

$v(0) = 50/3 = 20 + A$ or $A = -10/3$

$i(t) = Cdv/dt$

$= C(-2)\,[(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}] + C\omega_d[(-A\sin(\omega_d t) + B\cos(\omega_d t))e^{-2t}]$

$$i(0) = 0 = -2A + \omega_d B$$

$$B = 2A/\omega_d = -20/(3x4.583) = -1.455$$

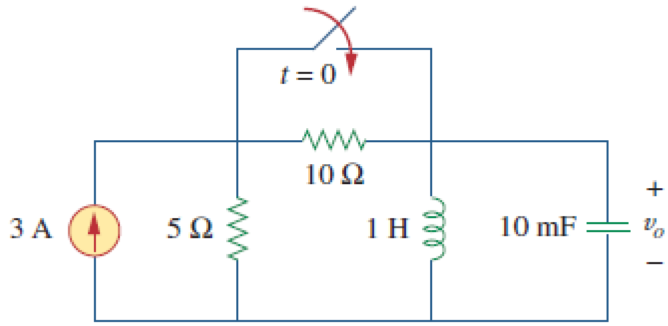
$$i(t) = C\{[(0\cos(\omega_d t) + (-2B - \omega_d A)\sin(\omega_d t))]e^{-2t}\}$$

$$= (1/25)\{[(2.91 + 15.2767)\sin(\omega_d t))]e^{-2t}\}$$

$$i(t) = \mathbf{727.5\sin(4.583t)e^{-2t}\ mA}$$

Q4

Find the output voltage $v_o(t)$ in the circuit of Fig.



Sol 4

At t = 0-, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$

and $v_o(0) = 0$.

For t > 0, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$

$$i(0) = 1 = 3 + A \text{ or } A = -2$$

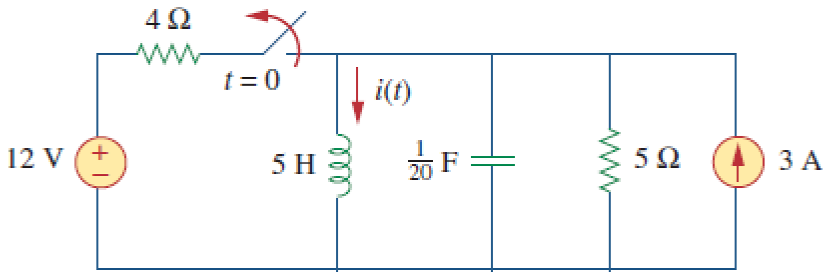
$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

Thus, $v_o(t) = \mathbf{(200te^{-10t})\ V}$

Q5

Determine $i(t)$ for $t > 0$ in the circuit of Fig.



Sol 5

For $t = 0^-$, $i(0) = 3 + 12/4 = 6$ and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = I_s + [(A + Bt)e^{-2t}], \quad I_s = 3$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

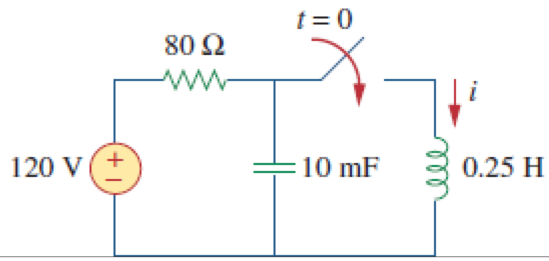
$$v = L\,di/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t) = \{3 + [(3 + 6t)e^{-2t}]\}$ A

Q6

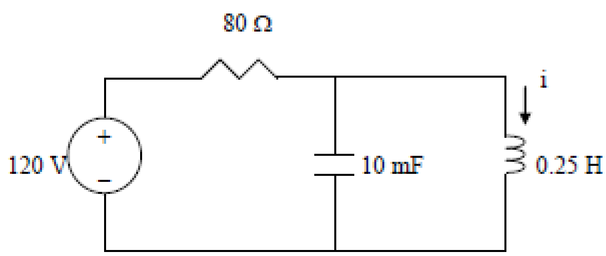
After being open for a day, the switch in the circuit of Fig. is closed at $t = 0$. Find the differential equation describing $i(t)$, $t > 0$.



Sol 6

At $t<0$, $i(0^-) = 0, v_c(0^-) = 120V$

For $t>0$, we have the circuit as shown below.



$$\frac{120 - V}{R} = C\frac{dv}{dt} + i \longrightarrow 120 = V + RC\frac{dv}{dt} + iR \qquad (1)$$

But $\qquad v_L = v = L\frac{di}{dt} \qquad (2)$

Substituting (2) into (1) yields

$$120 = L\frac{di}{dt} + RCL\frac{d^2i}{dt^2} + iR \longrightarrow 120 = \frac{1}{4}\frac{di}{dt} + 80x\frac{1}{4}x10x10^{-3}\frac{d^2i}{dt^2} + 80i$$

or

$$(d^2i/dt^2) + 0.125(di/dt) + 400i = 600$$

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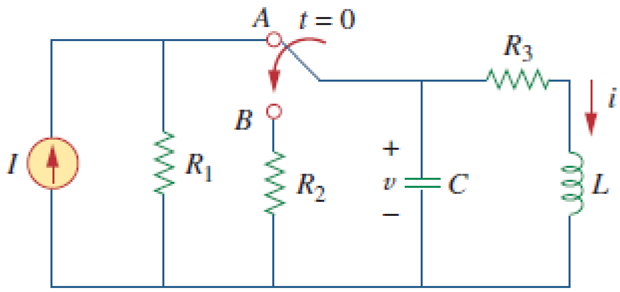
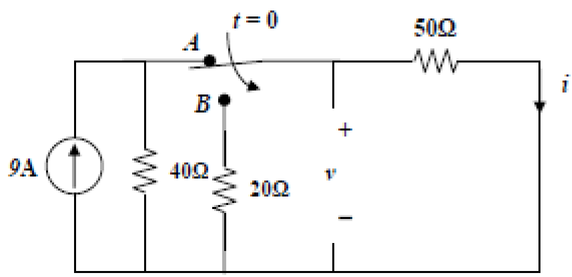
Q7	Using Fig. , design a problem to help other students better understand general second-order circuits. 
Sol 7	(a) When the switch is at *A*, the circuit has reached steady state. Under this condition, the circuit is as shown below.  (a) When the switch is at *A*, $i(0^-) = 9[(40x50)/(40+50)]/50 = 4$ A and $v(0^-) = 50i(0^-) = 200$ V. Since the current flowing through the inductor cannot change in zero time, $i(0^+) = i(0^-) =$ **4 A**. Since the voltage across the capacitor cannot change in zero time, $v(0^-) = v(0^-) =$ **200 V**.

(b) For the inductor, $v_L = L(di/dt)$ or $di(0^+)/dt = v_L(0^+)/0.02$.

At $t = 0^+$, the right hand loop becomes,

$$-200 + 50x4 + v_L(0^+) = 0 \text{ or } v_L(0^+) = 0 \text{ and } (di(0^+)/dt) = \underline{0}.$$

For the capacitor, $i_C = C(dv/dt)$ or $dv(0^+)/dt = i_C(0^+)/0.01$.

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$$((200-0)/20) + i_C + 4 = 0 \text{ or } i_C = -14 \text{ A}.$$

Therefore,

$$dv(0^+)/dt = -14/0.01 = \mathbf{-1.4\ kV/s}.$$

(c) When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

Thus,

$$i(\infty) = \mathbf{0\ A} \text{ and } v(\infty) = \mathbf{0\ V}.$$