Electric Circuits II – Assignment 04

#	Student ID	Student Name	Grade (10)
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Delivery Date	

١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

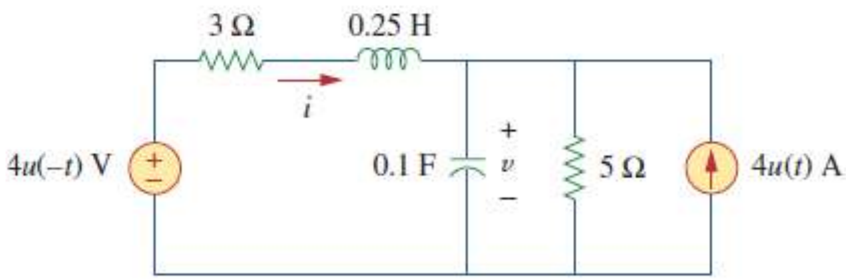
Q1

In the circuit of Fig. , find:

(a) $v(0^+)$ and $i(0^+)$,

(b) $dv(0^+)/dt$ and $di(0^+)/dt$,

(c) $v(\infty)$ and $i(\infty)$.

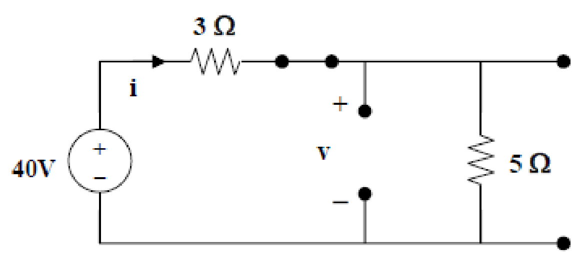


Sol 1

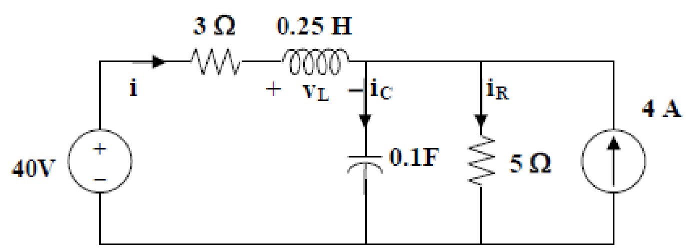
(a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

$$i(0^-) = 40/(3 + 5) = 5\text{A}, \text{ and } v(0^-) = 5i(0^-) = 25\text{V}.$$

Hence,
$$i(0^+) = i(0^-) = \mathbf{5A}$$
$$v(0^+) = v(0^-) = \mathbf{25V}$$



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A,\ i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

$$dv(0^+)/dt = 4/0.1 = \mathbf{40\ V/s}$$

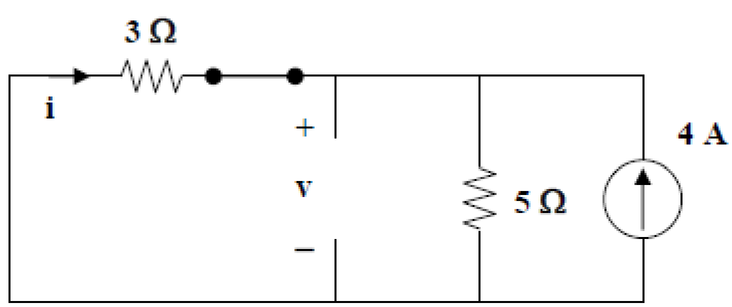
Similarly, $v_L = Ldi/dt$ which leads to $di(0^+)/dt = v_L(0^+)/L$

$$3i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$15 + v_L(0^+) + 25 = 0 \text{ or } v_L(0^+) = -40$$

$$di(0^+)/dt = -40/0.25 = \mathbf{-160\ A/s}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (c).



(c)

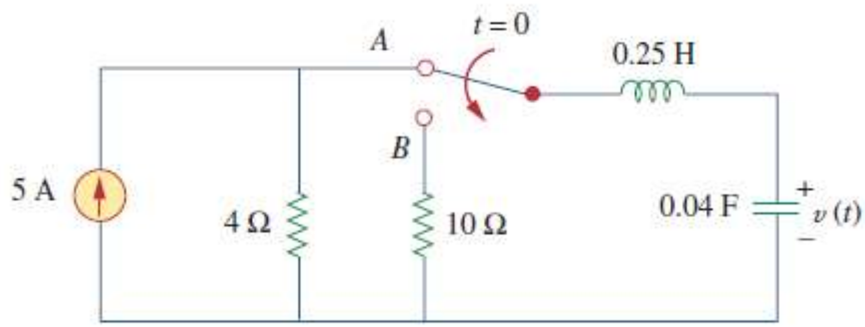
$$i(\infty) = -5(4)/(3 + 5) = \mathbf{-2.5\ A}$$

$$v(\infty) = 5(4 - 2.5) = \mathbf{7.5\ V}$$

Q2	The current in an *RLC* circuit is described by $$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0$$ If $i(0) = 10$ A and $di(0)/dt = 0$, find $i(t)$ for $t > 0$.
Sol 2	$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \frac{-10 \pm \sqrt{10-10}}{2} = -5$, repeated roots. $$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 10 = A$$ $$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$ $$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$ $$\text{Therefore, } i(t) = \mathbf{[(10 + 50t)e^{-5t}]\ A}$$

Q3

In the circuit of Fig. , the switch instantaneously moves from position A to B at $t = 0$. Find $v(t)$ for all $t \geq 0$.



Sol 3

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x5 = 20$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+20) = -80$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ which\,is > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

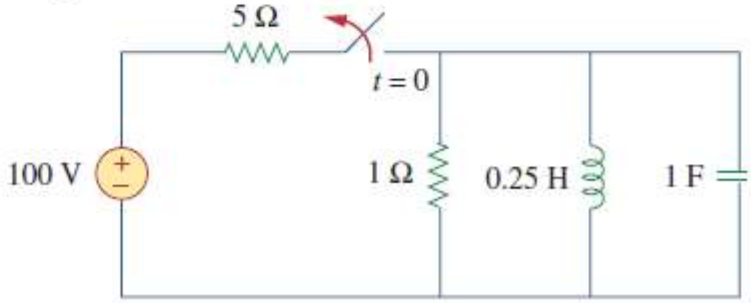
$$i(0) = 0 = A_1 + A_2,\ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -80$$

$$This\,leads\,to\ A_1 = -2.309 = -A_2$$

$$i(t) = 2.309\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$Since,\ v(t) = \frac{1}{C}\int_0^t i(t)dt + 20,\ we\ get$$

$$\mathbf{v(t) = [21.55e^{-2.679t} - 1.55e^{-37.32t}]\ V}$$

Q4	Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. . Assume steady-state conditions exist at $t = 0^-$. 
Sol 4	....

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When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \quad \alpha = \frac{1}{2RC} = 0.5$$

$$\alpha < \omega_o \quad \longrightarrow \quad \text{underdamped case} \quad \omega_d = \sqrt{\omega_o^{\,2} - \alpha^2} = \sqrt{4-0.25} = 1.936$$

$I_o(0) = i(0) =$ initial inductor current $= 100/5 = 20$ A

$V_o(0) = v(0) =$ initial capacitor voltage $= 0$ V

$$v(t) = e^{-\alpha t}(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)) = e^{-0.5\alpha t}(A_1\cos(1.936t) + A_2\sin(1.936t))$$

$v(0) = 0 = A_1$

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1\cos(1.936t) + A_2\sin(1.936t)) + e^{-0.5\alpha t}(-1.936A_1\sin(1.936t) + 1.936A_2\cos(1.936t))$$

$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+20)}{1} = -20 = -0.5A_1 + 1.936A_2 \quad \longrightarrow \quad A_2 = -10.333$$

Thus,

$$\underline{v(t) = [-10.333e^{-0.5t}\sin(1.936t)]volts}$$

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