Electric Circuits II – Assignment 03

Step Response of RC/RL Circuits

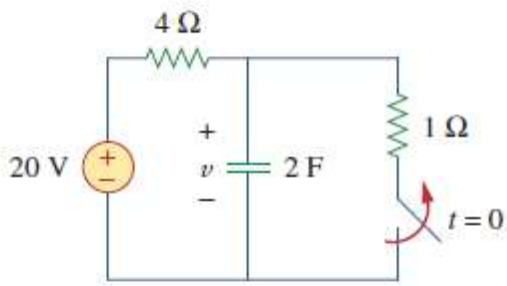
#	Student ID	Student Name	Grade (10)
-			

Delivery Date	

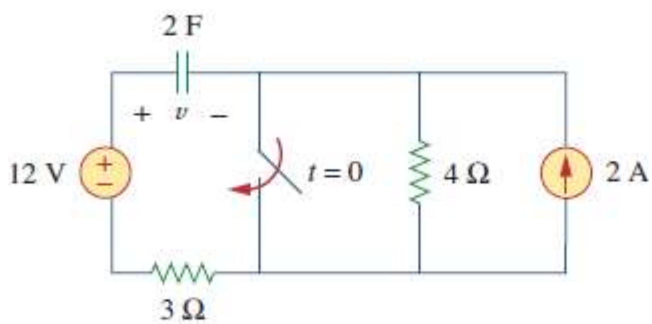
١. يتم تسليم التمرين محلولا في خلال أسبوع من تاريخ التمرين، و يتم حذف درجتين من التمرين عن كل أسبوع تأخير
٢. يتم التسليم لمعيد المقرر مباشرة
٣. تتم أجابه التمرين في نفس ورق الأسئلة

Q1

Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.



(a)



(b)

Sol 1

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \mathbf{4\ V}$$

After t = 0,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)\, e^{-t/8}$$

$$v(t) = 20 - 16\, e^{-t/8}\ \text{V}$$

....

....

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$v_1 = 12$ V

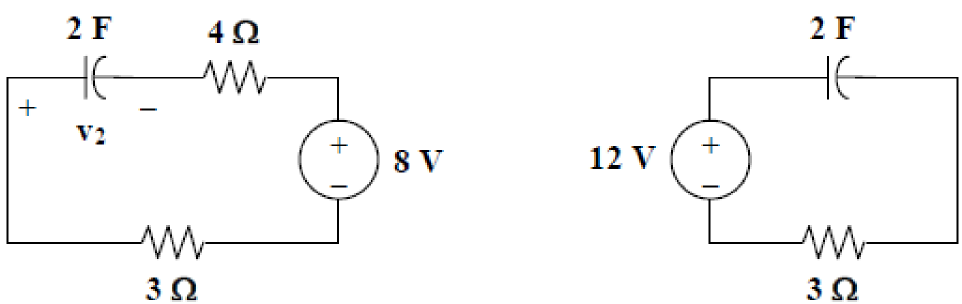
To get v_2, transform the current source as shown in Fig. (a).

$v_2 = -8$ V

Thus,

$v = 12 - 8 = \mathbf{4\ V}$

After t = 0, the circuit becomes that shown in Fig. (b).



(a) (b)

$v(t) = v(\infty) + [\,v(0) - v(\infty)\,]\, e^{-t/\tau}$

$v(\infty) = 12, \quad v(0) = 4, \quad \tau = RC = (2)(3) = 6$

$v(t) = 12 + (4 - 12)\, e^{-t/6}$

$v(t) = \mathbf{12 - 8e^{-t/6}\ V}$

....

....

....

....

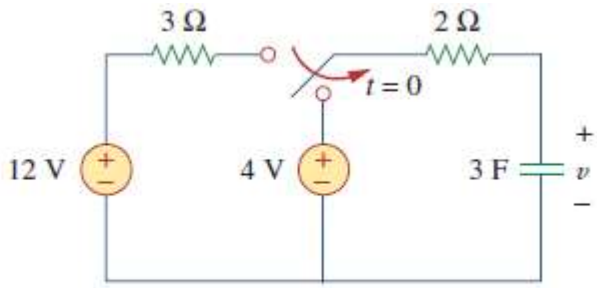
....

....

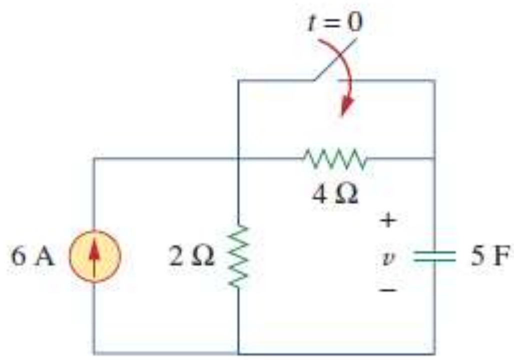
....

Q2

Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.



(a)



(b)

Sol 2

(a) Before $t = 0$, $v = \mathbf{12\ V}$.

After $t = 0$, $v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$

$$v(\infty) = 4, \quad v(0) = 12, \quad \tau = RC = (2)(3) = 6$$

$$v(t) = 4 + (12 - 4)\, e^{-t/6}$$

$$v(t) = \mathbf{4 + 8e^{-t/6}\ V}$$

....

(a) Before $t = 0$, $v = \mathbf{12\ V}$.

After $t = 0$, $v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$

$v(\infty) = 4$, $v(0) = 12$, $\tau = RC = (2)(3) = 6$

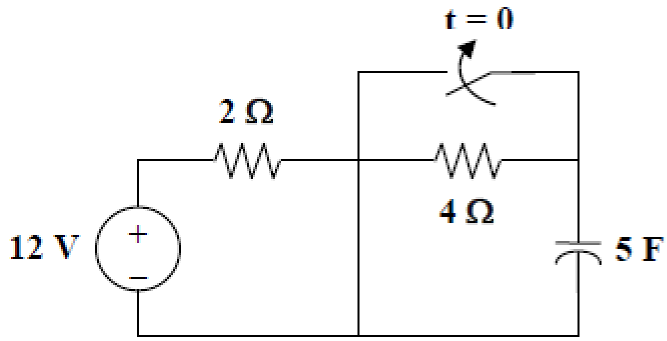
$v(t) = 4 + (12 - 4)\, e^{-t/6}$

$v(t) = \mathbf{4 + 8e^{-t/6}\ V}$

(b) Before $t = 0$, $v = \mathbf{12\ V}$.

After $t = 0$, $v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$

After transforming the current source, the circuit is shown below.



$v(0) = 12$, $v(\infty) = 12$, $\tau = RC = (2)(5) = 10$

$v = \mathbf{12\ V}$

....

....

....

....

....

....

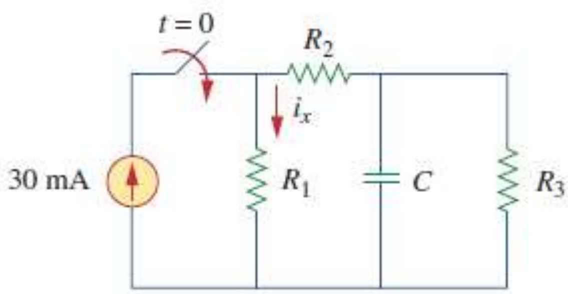
....

....

....

Q3

In the circuit of Fig. ', find i_x for $t > 0$. Let $R_1 = R_2 = 1\ \text{k}\Omega$, $R_3 = 2\ \text{k}\Omega$, and $C = 0.25$ mF.



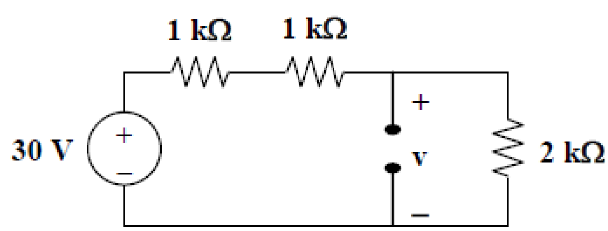
Sol 3

For the capacitor voltage,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = 0$$

For t > 0, we transform the current source to a voltage source as shown in Fig. (a).



(a)

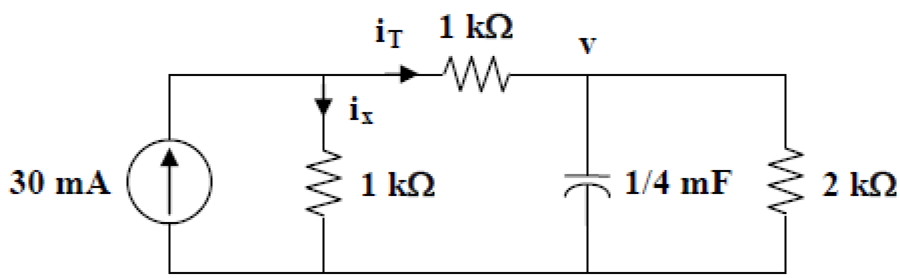
$$v(\infty) = \frac{2}{2+1+1}(30) = 15\ \text{V}$$

$$R_{th} = (1+1) \,||\, 2 = 1\ \text{k}\Omega$$

$$\tau = R_{th}C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4}$$

$$v(t) = 15\left(1 - e^{-4t}\right), \quad t > 0$$

We now obtain i_x from v(t). Consider Fig. (b).



(b)

$$i_x = 30 \text{ mA} - i_T$$

But $$i_T = \frac{v}{R_3} + C\frac{dv}{dt}$$

$$i_T(t) = 7.5\left(1 - e^{-4t}\right) \text{ mA} + \frac{1}{4} \times 10^{-3} (-15)(-4) e^{-4t} \text{ A}$$

$$i_T(t) = 7.5\left(1 + e^{-4t}\right) \text{ mA}$$

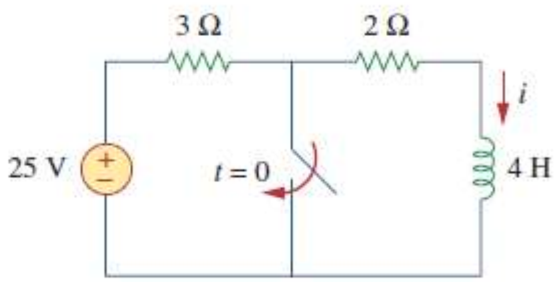
Thus,

$$i_x(t) = 30 - 7.5 - 7.5 e^{-4t} \text{ mA}$$

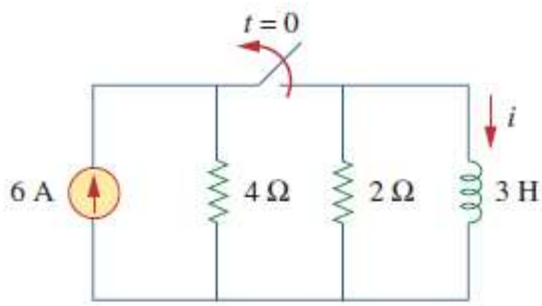
$$i_x(t) = \mathbf{7.5\left(3 - e^{-4t}\right) mA, \quad t > 0}$$

Q4

Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig.



(a)



(b)

Sol 4

(a) Before t = 0, $i = \frac{25}{3+2} = \mathbf{5\ A}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \mathbf{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$$i(t) = \mathbf{6\ A}$$

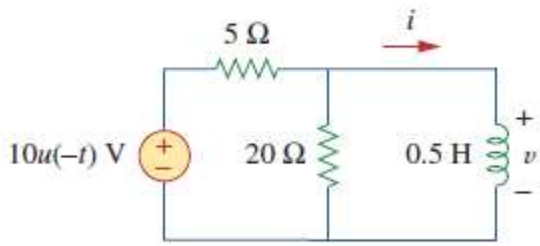
After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

$$i(t) = \mathbf{6e^{-2t/3}\ u(t)A}$$

Q5

Obtain $v(t)$ and $i(t)$ in the circuit of Fig.



Sol 5

For $t < 0$, $\quad u(-t) = 1, \quad i(0) = \dfrac{10}{5} = 2$

For $t > 0$, $\quad u(-t) = 0, \quad i(\infty) = 0$

$$R_{th} = 5 \,||\, 20 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \mathbf{2e^{-8t}u(t)\ A}$$

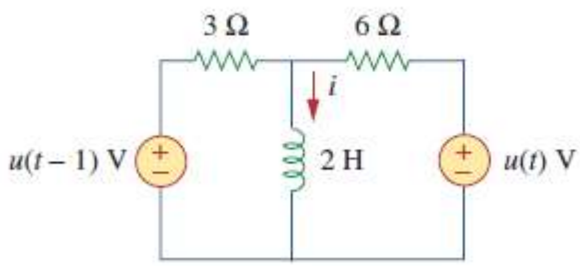
$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)\,e^{-8t}$$

$$v(t) = \mathbf{-8e^{-8t}u(t)\ V}$$

$$\mathbf{2e^{-8t}u(t)\ A,\ -8e^{-8t}u(t)\ V}$$

Q6

For the circuit in Fig. , calculate $i(t)$ if $i(0) = 0$.



Sol 6

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 \,||\, 6} = 1$$

For $0 < t < 1$, $u(t-1) = 0$ so that

$$i(0) = 0\,, \qquad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)$$

For $t > 1$, $\quad i(1) = \frac{1}{6}\left(1 - e^{-1}\right) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946e^{-(t-1)}$$

Thus,

$$i(t) = \begin{cases} \frac{1}{6}\left(1 - e^{-t}\right)\text{ A} & 0 < t < 1 \\ 0.5 - 0.3946e^{-(t-1)}\text{ A} & t > 1 \end{cases}$$