



Lecture (06)

Circuit Theorems (2)

By:
Dr. Ahmed ElShafee

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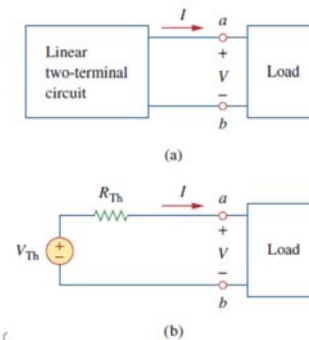
- Thevenin
- Norton
- Max power transfere

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Thevenin

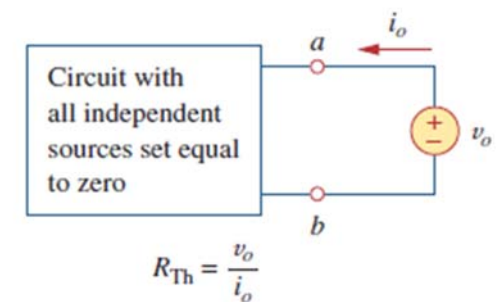
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} , where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



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- To find V_{th} , calculate the open circuit voltage between a, and b

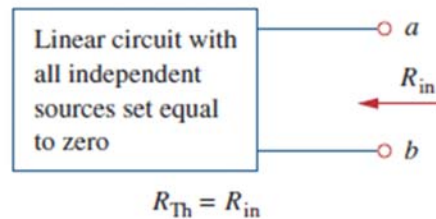


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To find R_{th} there are two cases:

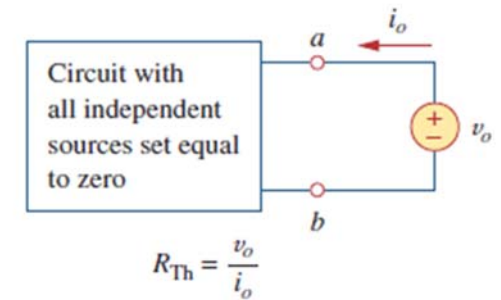
■ **CASE 1** If the network has no dependent sources, we turn off all independent sources. is the input resistance of the network looking between terminals a and b



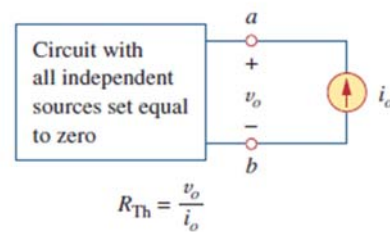
■ **CASE 2** If the network has dependent sources, we turn off all independent sources.

As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables.
1. We apply a voltage source at terminals a and b and determine the resulting current .

Then $R_{th} = V_o / i_o$



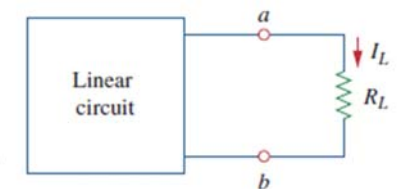
2. insert a current source at terminals a - b as shown in figure and find the terminal voltage $R_{th} = V_o / i_o$



- It often occurs that takes a negative value.
- In this case, the negative resistance ($V = -IR$) implies that the circuit is supplying power.
- This is possible in a circuit with dependent sources.
- Thevenin's theorem is very important in circuit analysis, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load.

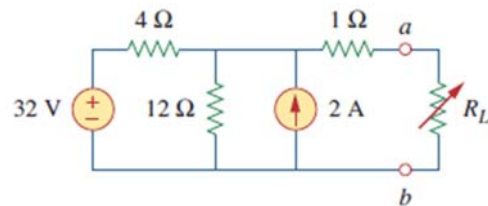
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$



Example 01

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals $a-b$. Then find the current through $R_L = 6, 16,$ and 36Ω .



1) Find V_{th}

KCL @ V_1

$$\frac{32-v_1}{4} - \frac{v_1}{12} + 2 = 0 \quad \times 12$$

$$9 - 3v_1 - v_1 + 24 = 0$$

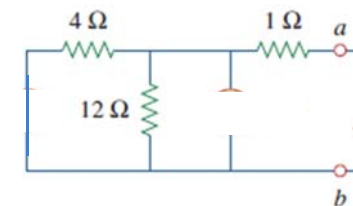
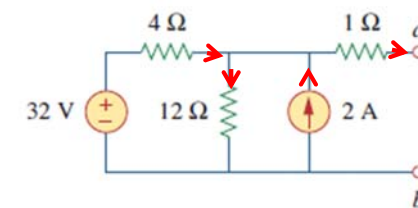
$$-4v_1 = -120$$

$$v_1 = 30 \text{ V} = V_{th}$$

2) Find R_{th}

$$4 \parallel 12 = 3$$

$$3 \equiv 1 = 4 \text{ ohm}$$

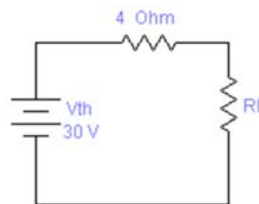


$$I_l = \frac{30}{4 + R_l}$$

$$I_{6ohm} = \frac{30}{4+6} = 3 \text{ Amp}$$

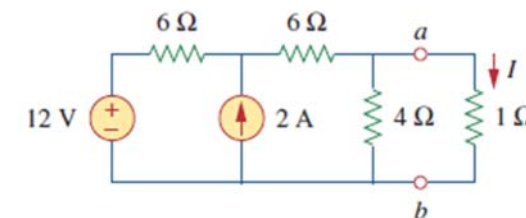
$$I_{16ohm} = \frac{30}{4 + 16} = 1.5 \text{ Amp}$$

$$I_{24ohm} = \frac{30}{4 + 24} = 1.07 \text{ Amp}$$



Example 02

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit of Fig. 4.30. Then find I .



$$V_{Th} = 6 \text{ V}, R_{Th} = 3 \Omega, I = 1.5 \text{ A.}$$

1) R_{th}

$$R_{th} = (6+6) \parallel 4 = 12 \parallel 4 = 3 \text{ Ohm}$$

2) V_{th}

KCL@V1

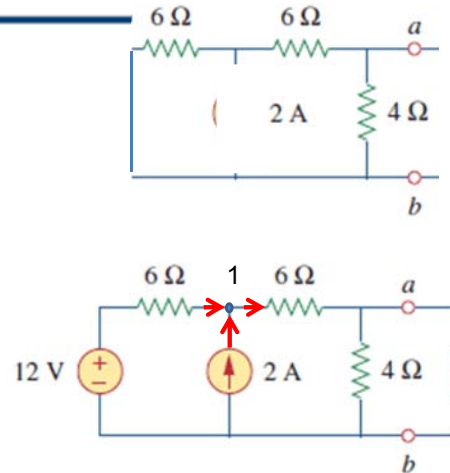
$$\frac{12-V_1}{6} + 2 - \frac{V_1}{10} = 0 \quad \times 60$$

$$120V_1 - 10V_1 + 120 - 6V_1 = 0$$

$$V_1 = 15V$$

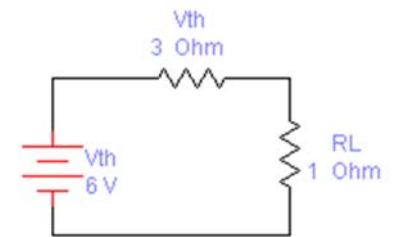
V.D.

$$V_{4ohm} = 15 \times \frac{4}{4+6} = 6 \text{ Volts}$$



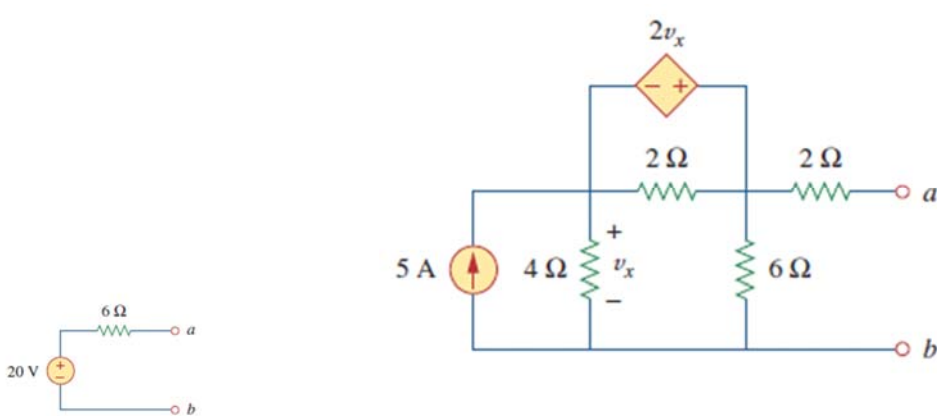
Ohm

$$I = \frac{V_{th}}{3+1} = \frac{6}{4} = 1.5 \text{ Amp}$$



Example 03

Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals $a-b$.



1) V_{th}

KCL@V1

$$5 + \frac{V_1 - V_2}{2} - \frac{V_1}{4} = 0 \quad \times 4$$

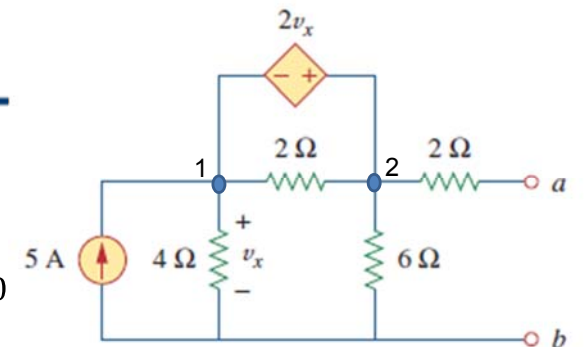
$$20 + 2V_1 - 2V_2 - V_1 = 0$$

$$3v_1 - 2v_2 = -20 \rightarrow 1$$

KCL@V2

$$V_2 - V_1 = 2V_x = 2V_1$$

$$V_2 = 3V_1 \rightarrow 2$$



Sub 2 in 1

$$V2 - 2V2 = -20$$

$$-V2 = -20$$

$$V2 = 20 = V_{th}$$

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Rth, neglecting sources add $V_0=1V$, finding corresponding I_0

KVL@1

$$4I1 + 2(I1 - I2) + 6(I1 - I3) = 0$$

$$4I1 + 2I1 - 2I2 + 6I1 - 6I3 = 0$$

$$12I1 - 2I2 - 6I3 = 0 \rightarrow 1$$

KVL@2

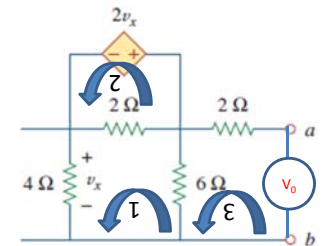
$$2Vx + 2(I2 - I1) = 0$$

$$8I1 + 2I2 - 2I1 = 0$$

$$6I1 + 2I2 = 0 / 3$$

$$3I1 + I2 = 0$$

$$I2 = -3I1 \rightarrow 2$$



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Sub 2,3 in 1

$$12I1 + 6I1 - \frac{6(1+6I1)}{8} = 0 \times 4$$

$$48I1 + 24I1 - 3 - 18I1 = 0$$

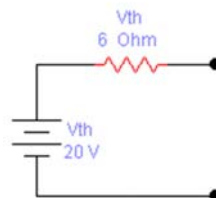
$$54I1 = 3$$

$$I1 = \frac{3}{54} = 0.0555 \text{ Amp}$$

Sub in 3

$$I3 = \frac{1 + 6(0.0555)}{8} = 0.167 \text{ Amp} = I_0$$

$$R_{th} = \frac{1}{I_0} = 6 \text{ Ohm}$$

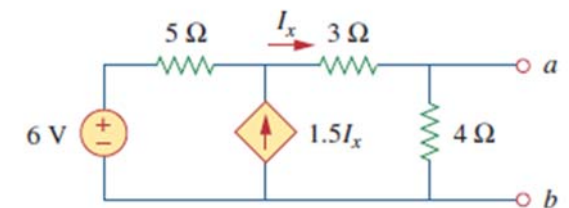


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Example 04

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.



$$: V_{Th} = 5.333 \text{ V}, R_{Th} = 444.4 \text{ m}\Omega.$$

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Find V_{th}

Ohm

$$I_x = V_1/7$$

KCL@V1

$$\frac{6-V_1}{5} + 1.5 I_x - \frac{V_1}{7} = 0 \quad \times 70$$

Sub 1 in 2

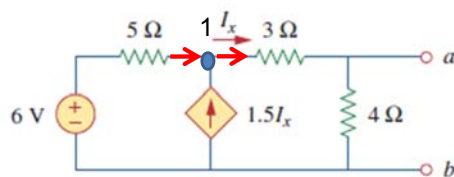
$$84 - 14V_1 + 15V_1 - 10 V_1 = 0$$

$$9V_1 = 84$$

$$V_1 = 9.333 \text{ Volt}$$

V.D.

$$V_{4ohm} = 9.333 \times \frac{4}{3+4} = 5.33 \text{ Volts} = V_{th}$$



Find R_{th}

Ignore all independent sources

connect terminals to current source I_0

and find corresponding V_0

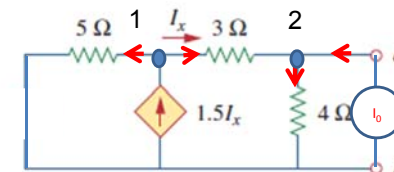
$$-\frac{v_1}{5} + 1.5 \frac{v_1 - v_2}{3} - \frac{v_1 - v_2}{3} = 0$$

$$-\frac{v_1}{5} + 0.5 \frac{v_1 - v_2}{3} = 0 \quad \times 30$$

$$-6v_1 + 5v_1 - 5v_2 = 0$$

$$-v_1 - 5v_2 = 0 \quad \times 4$$

$$-4v_1 - 20v_2 = 0 \rightarrow 1$$



KCL@2

$$\frac{V_1 - V_2}{3} - \frac{V_2}{4} + 1 = 0 \quad \times 12$$

$$4v_1 - 4v_2 - 3v_2 = -12$$

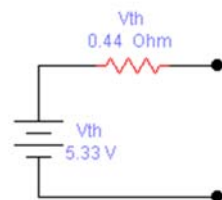
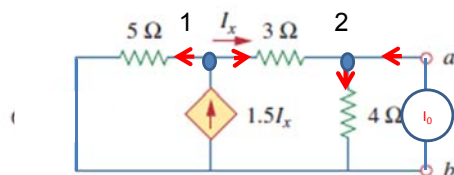
$$-4v_1 - 7v_2 = -12 \rightarrow 2$$

Add 1,2

$$-27V_2 = -12$$

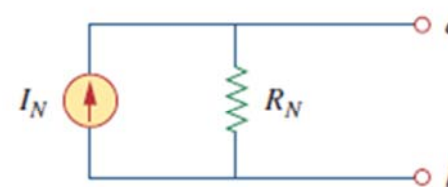
$$V_2 = 0.44 \text{ volt}$$

$$R_{th} = 0.44 \text{ ohm}$$

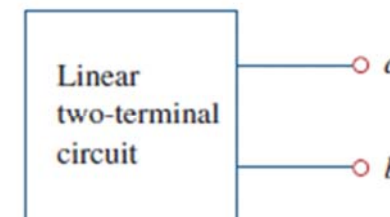


Norton Theorem

• **Norton's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

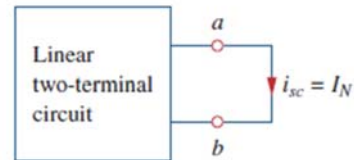


(b)



- To find the Norton current we determine the short-circuit current flowing from terminal a to b

$$I_N = i_{sc}$$



- To find R_N , it's the same as V_{Th} calculated in the previous section taking into consideration the two case (independent, and dependent sources)

$$R_N = R_{Th}$$

$$I_N = \frac{V_{Th}}{R_{Th}}$$

Summary:

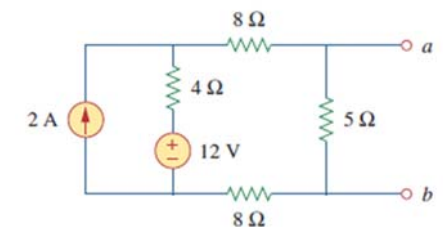
$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

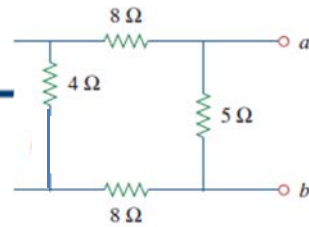
$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

Example 07

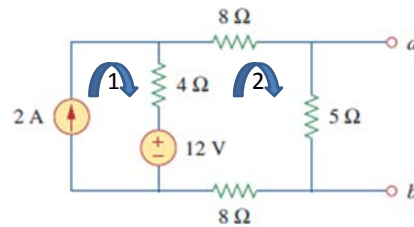
Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals a - b .



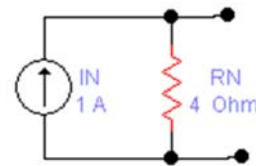
1) R_N
 $8 \equiv 4 \equiv 8 = 20$
 $5 \parallel 20 = 4 \text{ ohm}$



2) I_N
 KVL@1
 $I_1 = 2 \text{ Amp}$



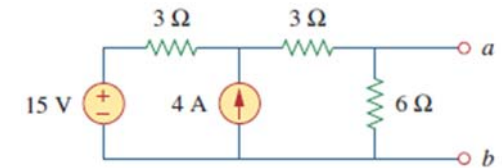
KVL@2
 $4(I_2 - 2) + 16I_2 - 12 = 0$
 $4I_2 - 8 + 16I_2 - 12 = 0$
 $20I_2 = 20$
 $I_2 = 1 \text{ Amp}$



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Example 8

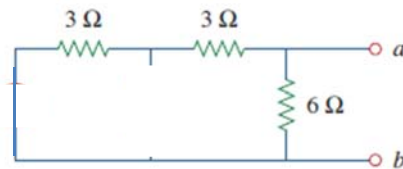
Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals $a-b$.



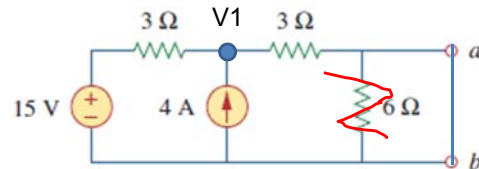
$R_N = 3 \Omega, I_N = 4.5 \text{ A.}$

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1) R_N
 $3 \equiv 3 \parallel 6 = 3 \text{ ohm}$



2) I_N
 KCL@v1
 $\frac{15-V_1}{3} + 4 - \frac{V_1}{3} = 0 \quad \times 3$
 $15 - V_1 + 12 - V_1 = 0$
 $15 - 2V_1 + 12 = 0$
 $27 - 2V_1 = 0$
 $V_1 = \frac{27}{2} = 13.5 \text{ Volt}$
 $I_N = \frac{13.5}{3} = 4.5 \text{ Amp}$

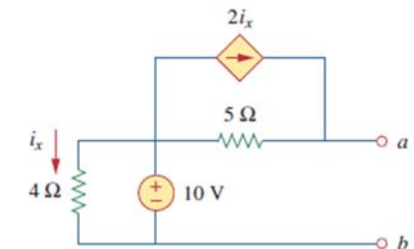


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Example 9

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals $a-b$.



$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5 \Omega$

$I_N = 7 \text{ A}$

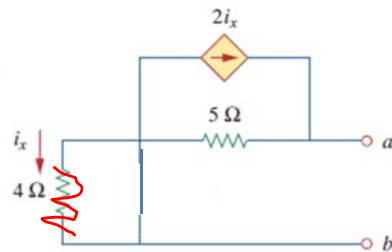
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To find R_N

Neglect all independent sources

$$R_N = 5 \text{ ohm}$$



To find I_N

Add short circuit at terminals, find

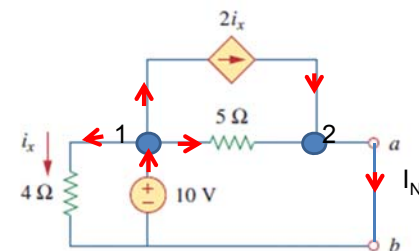
Isc

KCL@V1

$$V1 = 10 \text{ Volts}$$

Ohm

$$I_x = \frac{V1}{4} = 2.5 \text{ Amp}$$

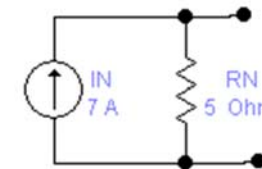
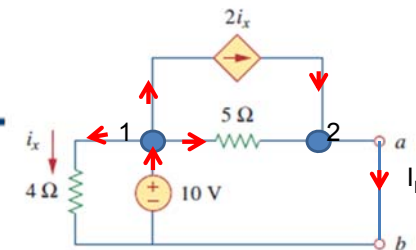


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KCL@V2

$$\frac{10}{5} + 2 \times 2.5 - I_N = 0 \times 5$$

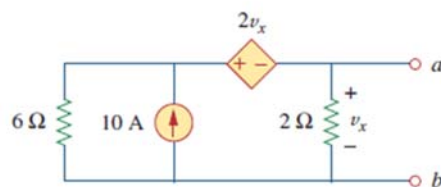
$$I_N = 7 \text{ Amp}$$



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Example 10

Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals $a-b$.



$$: R_N = 1 \Omega, I_N = 10 \text{ A.}$$

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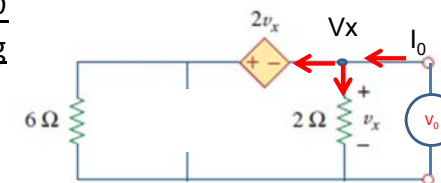
To find R_N , ignore all independent sources, add voltage source V_0 @ terminal, calculate corresponding I_0

KCL@V1

$$I_0 - \frac{V_x}{2} - \frac{V_x + 2V_x}{6} = 0$$

$$I_0 = V_x = V_0$$

$$R_N = \frac{V_0}{I_0} = 1 \text{ ohm}$$

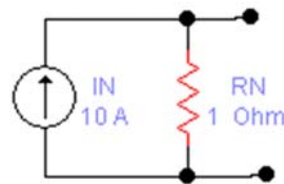
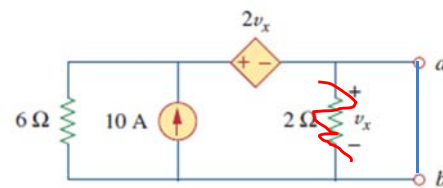


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To find I_N , connect short circuit between terminals, calculate I_{sc}

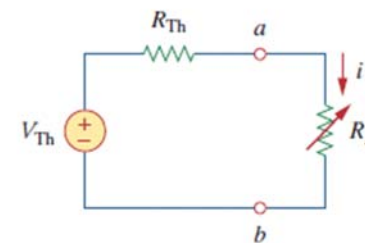
KVL@2

$I_N = 10 \text{ Amp}$



Maximum Power Transfer

- The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.
- We assume that we can adjust the load resistance. If the entire circuit is replaced by its Thevenin equivalent except for the load



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

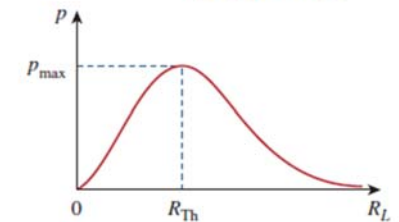


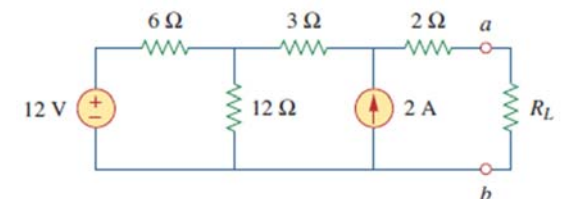
Figure 4.49 Power delivered to the load as a function of R_L .

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 11

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.



$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9 \Omega$$

$$V_{Th} = 22 \text{ V}$$

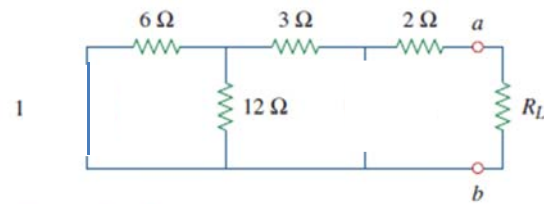
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

Find R_{th}

$$6 \parallel 2 = 4 \text{ ohm}$$

$$4 \equiv 3 \equiv 2 = 9 \text{ ohm}$$

$$R_L = R_{th} = 9 \text{ ohm}$$



Find V_{th}

KCL@1

$$\frac{12 - V_1}{6} - \frac{V_1}{12} - \frac{V_1 - V_2}{3} = 0 \quad \times 12$$

$$24 - 2V_1 - V_1 - 4V_1 + 4V_2 = 0 \quad 12 \text{ V}$$

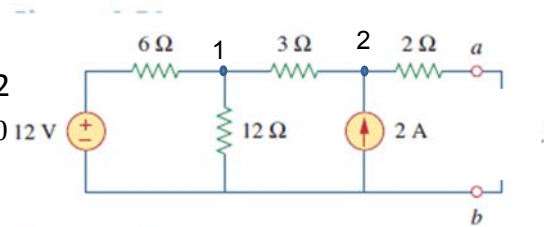
$$-7V_1 + 4V_2 = -24 \rightarrow 1$$

KCL@2

$$\frac{V_1 - V_2}{3} + 2 = 0 \quad \times 3$$

$$V_1 - V_2 + 6 = 0 \quad \times 7$$

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$$7V_1 - 7V_2 = -42 \rightarrow 2$$

Add 1,2

$$-3V_2 = -66$$

$$V_2 = 22 \text{ volts} = V_{th}$$

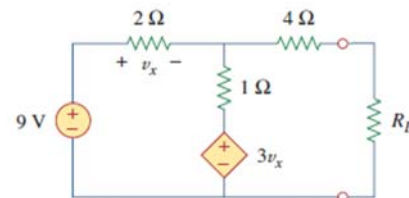
$$P_{max} = \frac{V_{th}^2}{4 R_{th}} = \frac{22^2}{4 \times 9} = 13.44 \text{ w}$$

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Example 12

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

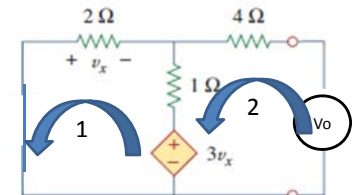


4.222 Ω , 2.901 W.

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To find R_{th} , ignore all independent sources, connect V_0 at terminals calculate I_0



KVL@1

$$2I_1 + (I_1 - I_2) - 3V_x = 0$$

$$2I_1 + I_1 - I_2 + 6I_1 = 0$$

$$9I_1 - I_2 = 0 \quad \times 7$$

$$63I_1 - 7I_2 = 0$$

KVL@2

$$4I_1 + (I_2 - I_1) - 1 + 3V_x = 0$$

$$4I_2 + I_2 - I_1 - 1 - 6I_1 = 0$$

$$7I_1 + 5I_2 = 1 \quad \times 9$$

$$-63I_1 + 45I_2 = 9$$

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Add 1,2

$$38 I_2 = 9$$

$$I_2 = 0.237 \text{ Amp} = I_0$$

$$R_{th} = \frac{1}{0.237} = 4.222 \text{ ohm} = R_l$$

Find V_{th}

Ohm:

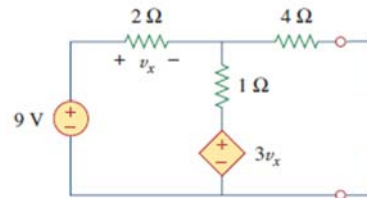
$$V_x = 2I_1$$

KVL

$$-9 + 3I_1 + 3V_x = 0$$

$$3I_1 + 6I_1 = 9$$

$$I_1 = 1 \text{ Amp}$$



$$V_{th} = (1 \times 1) + (3 \times 2) = 7 \text{ volts}$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{49}{4 \times 4.222} = 2.901 \text{ W}$$



Thanks,..

See you next week (ISA),...