



Lecture (04) Mesh Analysis

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Agenda

- Mesh analysis with voltage sources
- Mesh Analysis with Current Sources

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Mesh analysis with voltage sources

- Determine meshes in the circuit.
- Set the current and direction of each mesh.
- Use KVL to build system of equations (equation/mesh), use ohm's law to express voltage in terms of mesh current.
- Solve system of equations, to find the current of each mesh.
- Convert mesh current to then normal current form.

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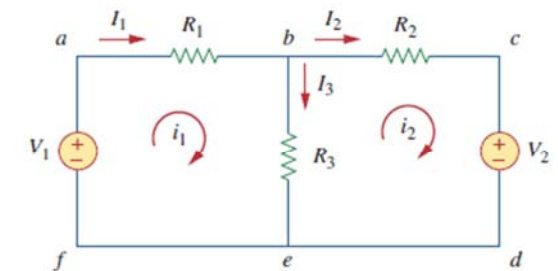
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$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

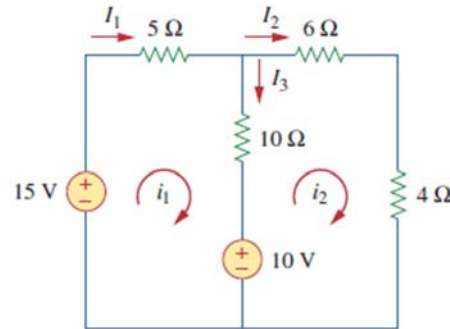


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Example 01

For the circuit in Fig. 3.18, find the branch currents I_1 , I_2 , and I_3 using mesh analysis.



$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \quad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

KVL@1

$$-15 + 5i_1 + (i_1 - i_2)10 + 10 = 0$$

$$-5 + 5i_1 + 10i_1 - 10i_2 = 0$$

$$15i_1 - 10i_2 = 5 \quad /5$$

$$3i_1 - 2i_2 = 1 \rightarrow 1$$

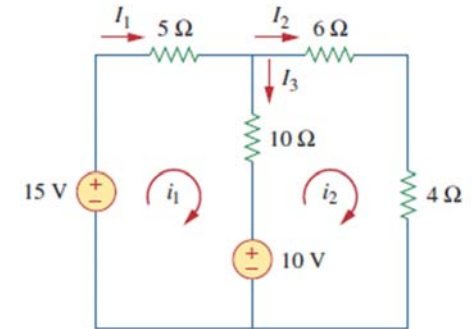
KVL@2

$$-10 + 10(i_2 - i_1) + 6i_2 + 4i_2 = 0$$

$$10i_2 - 10i_1 + 6i_2 + 4i_2 = 10$$

$$20i_2 - 10i_1 = 10 \quad /10$$

$$-i_1 + 2i_2 = 1 \rightarrow 2$$



Add 1,2

$$2i_1 = 2$$

$$i_1 = 1 \text{ Amp}$$

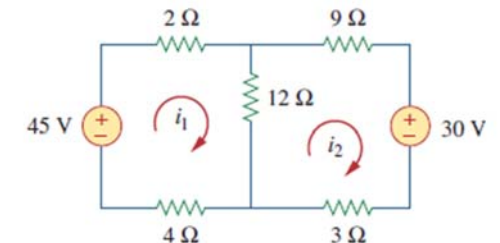
Sub in 2

$$2i_2 = 2$$

$$i_2 = 1 \text{ amp}$$

Example 02

Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 3.19.



$$i_1 = 2.5 \text{ A}, \quad i_2 = 0 \text{ A.}$$

KVL@1

$$-45 + 2i_1 + 12(i_1 - i_2) + 4i_1 = 0$$

$$2i_1 + 12i_1 - 12i_2 + 4i_1 = 45$$

$$18i_1 - 12i_2 = 45 \quad /3$$

$$6i_1 + 4i_2 = 15 \rightarrow 1$$

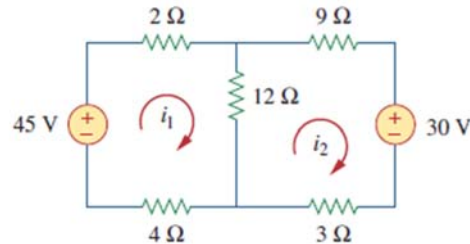
KVL@2

$$12(i_2 - i_1) + 9i_2 + 3i_2 + 30 = 0$$

$$12i_2 - 12i_1 + 9i_2 + 3i_2 = -30$$

$$-12i_1 + 24i_2 = -30 \quad /6$$

$$-2i_1 + 4i_2 = -5 \rightarrow 2$$



Add 1,2

$$4i_1 = 10$$

$$i_1 = 2.5 \text{ Amp}$$

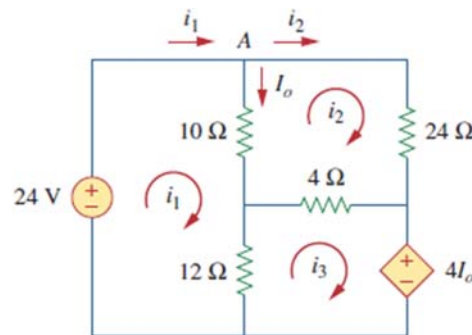
Sub in 2

$$4i_2 = -5 + (2 \times 2.5) = 0$$

$$i_2 = 0 \text{ Amp}$$

Example 03

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.



$$\text{Thus, } I_o = i_1 - i_2 = 1.5 \text{ A.}$$

KVL@1

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 24$$

$$10i_1 - 10i_2 + 12i_1 - 12i_3 = 24$$

$$= 24$$

$$22i_1 - 10i_2 - 12i_3 = 24 \quad /2$$

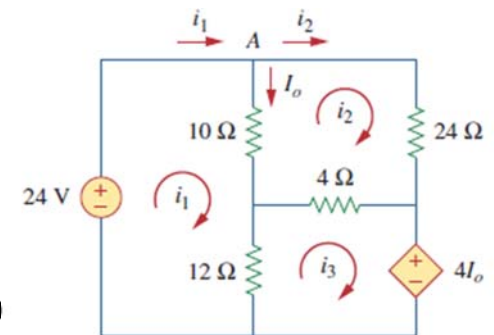
$$11i_1 - 5i_2 - 6i_3 = 12 \rightarrow 1$$

KVL@2

$$10(i_2 - i_1) + 24i_2 + 4(i_2 - i_3) = 0$$

$$10i_2 - 10i_1 + 24i_2 + 4i_2 - 4i_3 = 0$$

$$-10i_1 + 38i_2 - 4i_3 = 0 \quad /2$$



$$-5i_1 + 19i_2 - 2i_3 = 0 \rightarrow 2$$

KVL@3

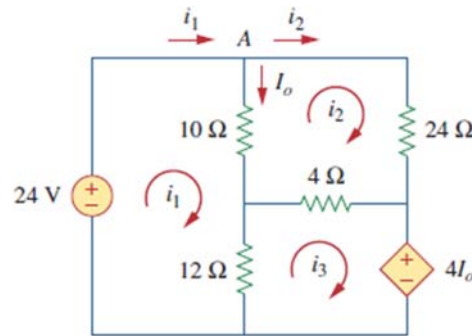
$$12(i_3 - i_1) + 4(i_3 - i_2) + 4i_0 = 0$$

$$12i_3 - 12i_1 + 4i_3 - 4i_2 + 4(i_1 - i_2) = 0$$

$$12i_3 - 12i_1 + 4i_3 - 4i_2 + 4i_1 - 4i_2 = 0$$

$$-8i_1 - 8i_2 + 16i_3 = 0 \quad /8$$

$$-i_1 - i_2 + 2i_3 = 0 \rightarrow 3$$



$$\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$$

Cramer

$$\begin{pmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta = 11(38 - 2) + 5(-10 - 6) - 1(10 + 114) = 396 - 80 - 124 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{vmatrix} = 12(38 - 2) = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{vmatrix} = -12(-10 - 2) = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 10 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 12(5 + 19) = 288$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ Amp}$$

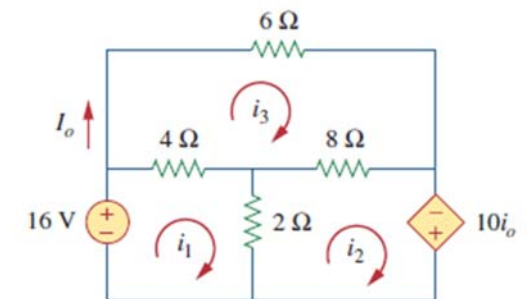
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ Amp}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ Amp}$$

$$i_0 = i_1 - i_2 = 2.25 - 0.75 = 1.5 \text{ Amp}$$

Example 04

Using mesh analysis, find I_o in the circuit of Fig. 3.21.



Answer: -4 A.

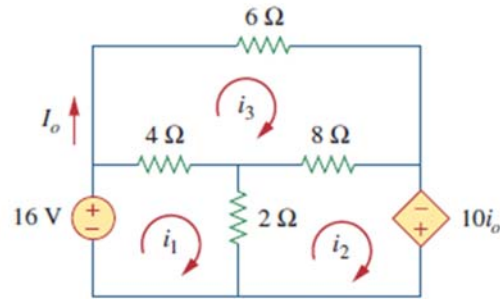
KVL@1

$$\begin{aligned}
 -16 + 4(i_1 - i_3) \\
 + 2(i_1 - i_2) &= 0 \\
 4i_1 - 4i_3 + 2i_1 - 2i_2 &= 16 \\
 6i_1 - 2i_2 + 4i_3 &= 16 \quad /2 \\
 3i_1 - i_2 - 2i_3 &= 4 \rightarrow 1
 \end{aligned}$$

KVL@2

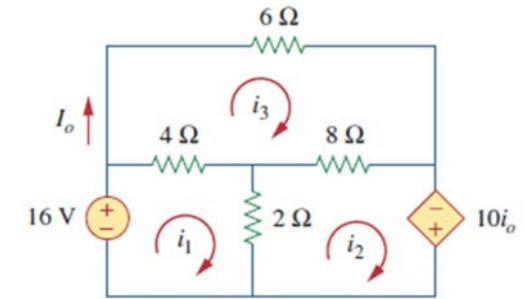
$$\begin{aligned}
 2(i_2 - i_1) + 8(i_2 - i_3) \\
 - 10i_o &= 0 \\
 2i_2 - 2i_1 + 8i_2 - 8i_3 - 10i_o \\
 &= 0 \\
 -2i_1 + 10i_2 - 18i_3 &= 0 \quad /2
 \end{aligned}$$

$$-i_1 + 5i_2 - 9i_3 = 0 \rightarrow 2$$



KVL@3

$$\begin{aligned}
 6i_3 + 4(i_3 - i_1) + 8(i_3 - i_2) \\
 &= 0 \\
 6i_3 + 4i_3 - 4i_1 + 8i_3 - 8i_2 \\
 &= 0 \\
 -4i_1 - 8i_2 + 18i_3 &= 0 \quad /2 \\
 -2i_1 - 4i_2 + 9i_3 &= 0 \rightarrow 3
 \end{aligned}$$



Cramer

$$\begin{pmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta = 3(45 - 36) + (-9 - 8) - 2(9 + 10) = 27 - 17 - 38 = -27$$

$$\Delta_1 = \begin{vmatrix} 4 & -1 & -2 \\ 0 & 5 & -9 \\ 0 & -4 & 9 \end{vmatrix} = 4(45 - 36) = 36$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{36}{-27} = -1.33 \text{ Amp}$$

$$\Delta_2 = \begin{vmatrix} 3 & 4 & -2 \\ -1 & 0 & -9 \\ -2 & 0 & 9 \end{vmatrix} = -4(-9 - 18) = 108$$

$$i_2 = \frac{108}{-27} = -4 \text{ Amp} = i_o$$

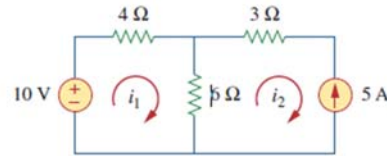
$$\Delta_3 = \begin{vmatrix} 3 & -1 & 4 \\ -1 & 5 & 0 \\ -2 & -4 & 0 \end{vmatrix} = 4(4 + 10) = 56$$

$$i_3 = \frac{56}{-27} = -2.07 \text{ Amp}$$

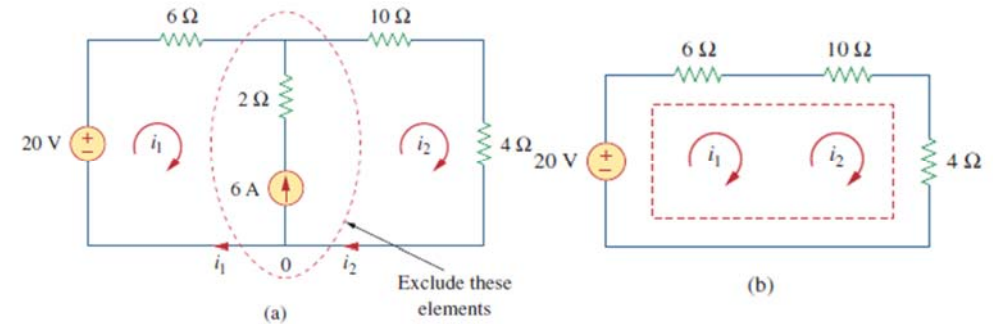
Mesh Analysis with Current Sources

- Case 1: When a current source exists only in one mesh, mesh current = source current

$$i_2 = -5 \text{ A}$$



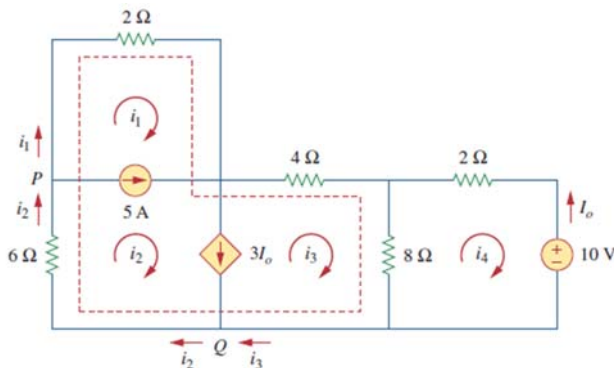
- Case 2: When a current source exists between two meshes; We create a *supermesh* by excluding the current source and any elements connected in series with it,



- Apply KCL : $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

Example 05

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.



$$i_1 = -7.5 \text{ A}, \quad i_2 = -2.5 \text{ A}, \quad i_3 = 3.93 \text{ A}, \quad i_4 = 2.143 \text{ A}$$

KVL@1,2,3 super mesh

$$2i_1 + 6i_2 + 4i_3 + 8(i_3 - i_4) = 0$$

$$2i_1 + 6i_2 + 4i_3 + 8i_3 - 8i_4 = 0$$

$$2i_1 + 6i_2 + 12i_3 - 8i_4 = 0 \quad /2$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \rightarrow 1$$

KCL@ super 1,2

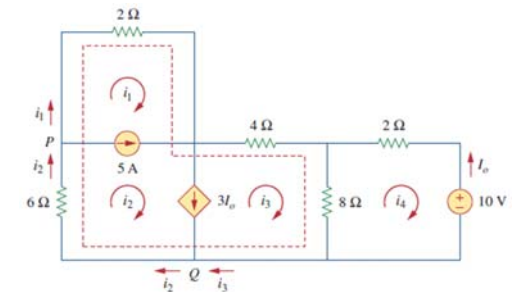
$$i_2 = 5 + i_1 \rightarrow 2$$

KCL@ super 2,3

$$i_2 - i_3 = 3i_0$$

$$\text{But } i_0 = -i_4$$

$$i_2 - i_3 + 3i_4 = 0 \rightarrow 3$$



KVL@4

$$8(i_4 - i_3) + 2i_4 + 10 = 0$$

$$8i_4 - 8i_3 + 2i_4 = -10$$

$$-8i_3 + 10i_4 = -10 \quad /2$$

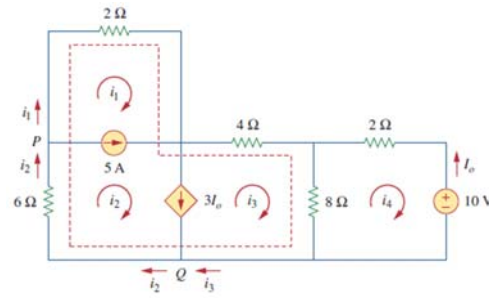
$$-4i_3 + 5i_4 = -5 \rightarrow 4$$

Sub 2 in 1,3

$$i_1 + 3(i_1 + 5) + 6i_3 - 4i_4 = 0$$

$$4i_1 + 6i_3 - 4i_4 = -15 \rightarrow 5$$

$$i_1 - i_3 + 3i_4 = -5 \rightarrow 6$$



Cramer

$$\begin{pmatrix} 4 & 6 & -4 \\ 1 & -1 & 3 \\ 0 & -4 & 5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -15 \\ -5 \\ -5 \end{pmatrix}$$

$$\Delta = 4(-5 + 12) - (30 - 16) = 28 - 14 = 14$$

$$\Delta_1 = \begin{vmatrix} -15 & 6 & -4 \\ -5 & -1 & 3 \\ -5 & -4 & 5 \end{vmatrix}$$

$$= -15(-5 + 12) + 5(30 - 16) - 5(18 - 4)$$

$$= -105 + 70 - 70 = -105$$

$$i_1 = \frac{\Delta_1}{\Delta} = -\frac{105}{14} = -7.5 \text{ Amp}$$

From 2

$$i_2 = 5 - 7.5 = -2.5 \text{ Amp}$$

$$\Delta_3 = \begin{vmatrix} 4 & -15 & -4 \\ 1 & -5 & 3 \\ 0 & -5 & 5 \end{vmatrix} = 4(-25 + 15) - (-75 - 20)$$

$$= -40 + 95 = 55$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{55}{14} = 3.93 \text{ Amp}$$

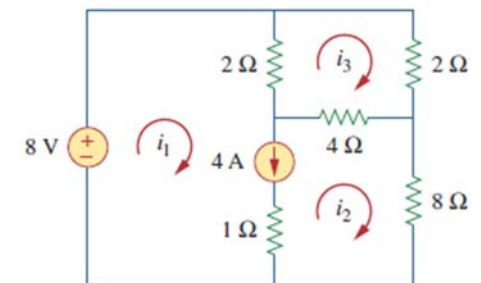
$$\Delta_4 = \begin{vmatrix} 4 & 6 & -15 \\ 1 & -1 & -5 \\ 0 & -4 & -5 \end{vmatrix} = 4(5 - 20) - (-30 + 60) = 60 - 30$$

$$= 30$$

$$i_4 = \frac{\Delta_4}{\Delta} = \frac{30}{14} = 2.14 \text{ Amp}$$

Example 06

Use mesh analysis to determine i_1 , i_2 , and i_3 in Fig. 3.25.



$$i_1 = 4.632 \text{ A}, i_2 = 631.6 \text{ mA}, i_3 = 1.4736 \text{ A.}$$

KVL@1,2 super mesh

$$-8 + 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2 = 0$$

$$2i_1 - 3i_2 + 4i_2 - 4i_3 + 8i_2 = 8$$

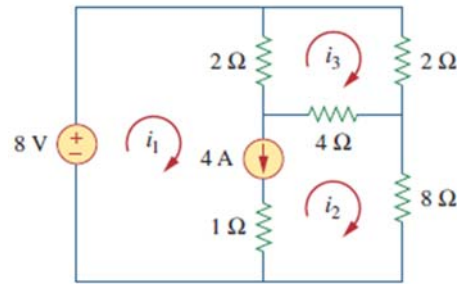
$$2i_1 + 12i_2 - 6i_3 = 8 \quad /2$$

$$i_1 + 6i_2 - 3i_3 = 4 \quad \rightarrow 1$$

KCL @ super mesh 1,2

$$i_1 - i_2 = 4$$

$$i_1 = 4 + i_2 \quad \rightarrow 2$$



KVL@3

$$2(i_3 - i_1) + 2i_3 + 4(i_3 - i_2) = 0$$

$$2i_3 - 2i_1 + 2i_3 + 4i_3 + 4i_3 - 4i_2 = 0$$

$$-2i_1 - 4i_2 + 8i_3 = 0 \quad /2$$

$$-i_1 - 2i_2 + 4i_3 = 0 \quad \rightarrow 2$$

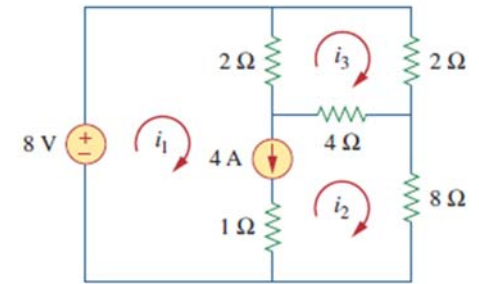
Sub 2 in 1,2

$$4 + i_2 + 6i_2 - 3i_3 = 4$$

$$7i_2 - 3i_3 = 0 \quad \rightarrow 4$$

$$-4 - i_2 - 2i_2 + 4i_3 = 0$$

$$-3i_2 + 4i_3 = 4 \quad \rightarrow 3$$



Mult 4 x 3 and 3 x 4

$$28i_2 - 12i_3 = 0$$

$$-9i_2 + 12i_3 = 12$$

$$19i_2 = 12$$

$$i_2 = \frac{12}{19} = 0.632 \text{ Amp}$$

Sub in 2

$$i_1 = 4 + 0.632 = 4.632 \text{ Amp}$$

Sub in 1

$$i_3 = \frac{i_1 + 6i_2 - 4}{3} = \frac{4.632 + 6(0.632) - 4}{3} = 1.47 \text{ Amp}$$

