Lecture (01)
Basic laws – P1

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Agenda

- Ohm's Law
- Nodes, Branches, and Loops
- Kirchhoff's Laws

Ohm's Law

- Materials in general have a characteristic behavior of resisting the flow of electric charge, ability to resist current, is known as *resistance* and is represented by the symbol *R*.
- in mathematical form

$$R = \rho \frac{\ell}{A}$$

- ρ *resistivity*
- A *area*
- L *length*
- Good conductors, such as copper and aluminum, have low resistivity, while insulators, such as mica and paper, have high

- Resistivity of common materials

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

- <u>Ohm's</u> law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor. "Georg Simon Ohm (1787–1854), a German physicist"

$$v \propto i$$

$$v = iR$$

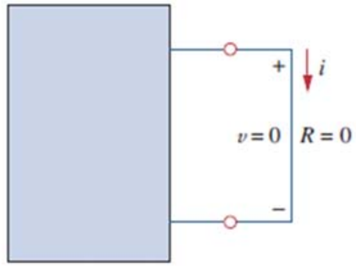
- *R* in Eq. is measured in the unit of ohms, designated .
- <u>The *resistance R* </u>of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).
- The direction of current *i* and the polarity of voltage *v* must conform with the passive sign convention

- This implies that current flows from a higher potential to a lower potential in order for

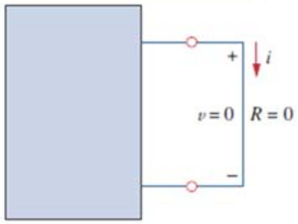
$$v = iR.$$

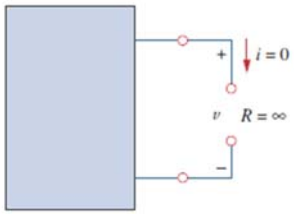
- If current flows from a lower potential to a higher potential

$$v = -iR$$

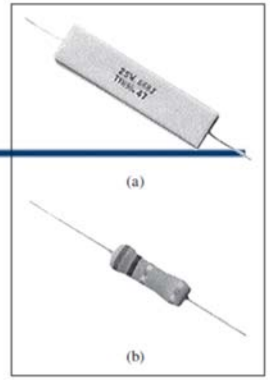


- the value of *R* can range from zero to infinity
- An element with *R=0* is called a *short circuit, V=0, I =?*
- <u>A short circuit is a circuit element with resistance approaching zero</u>
- an element with R = ∞ is known as an *open circuit*
- indicating that the current is zero though the voltage could be anything
- <u>An open circuit is a circuit element with</u>

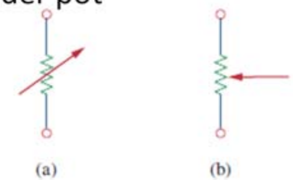




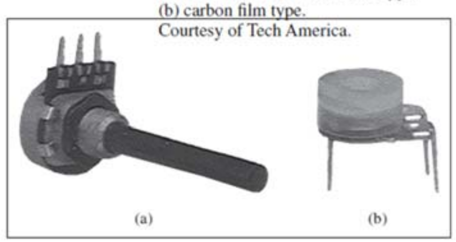
- Resistor types
  - Fixed
    - wirewound
    - composition
- Variable (*potentiometer* or *pot*)
  - Composition
  - slider pot



Fixed resistors: (a) wirewound type, (b) carbon film type.
Courtesy of Tech America.

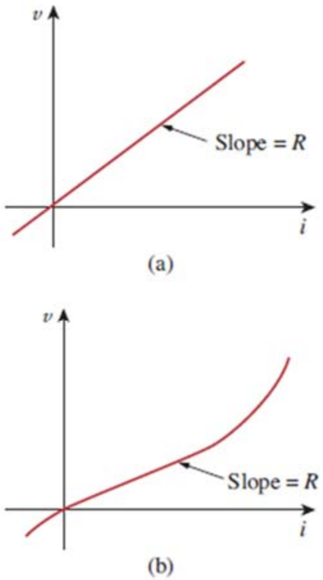


Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.



Variable resistors: (a) composition type, (b) slider pot.
Courtesy of Tech America.

- Device mounting classification
  - surface mounted or
  - Integrated
- Linear resistor has a constant resistance and thus its current-voltage is straight has a R slope
- A *nonlinear* resistor does not obey Ohm's law.



The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

- The conductance is a measure of how well an element will conduct electric current, unit of conductance is the *mho* ℧ or siemens (S),

$$G = \frac{1}{R} = \frac{i}{v}$$

$$1\ \text{S} = 1\ ℧ = 1\ \text{A/V}$$

$$i = Gv$$

$$p = vi = i^2R = \frac{v^2}{R}$$

$$p = vi = v^2G = \frac{i^2}{G}$$

- Dissipated power properties
  - The power dissipated in a resistor is a nonlinear function of either current or voltage.
  - Since *R* and *G* are positive quantities, the power dissipated in a resistor is always positive.
  - Thus, a resistor always absorbs power from the circuit.
  - This confirms the idea that a resistor is a passive element, incapable of generating energy.

Example 1

- An electric iron draws 2 A at 120 V. Find its resistance.

An electric iron draws 2 A at 120 V. Find its resistance.

Solution

$$R = \frac{v}{i} = \frac{120}{2} = 60\,\Omega$$

Example 02

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

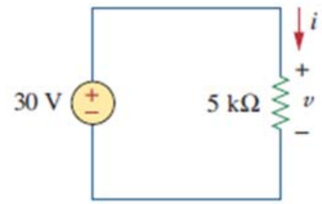
The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

Answer

- I = V/R = 110/15 = 7.33 ohm

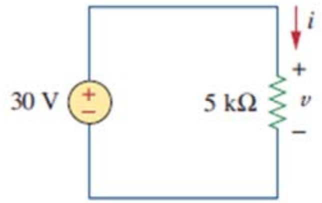
Example 03

In the circuit shown in Fig. , calculate the current i, the conductance G, and the power p.



In the circuit shown in Fig. , calculate the current i, the conductance G, and the power p.

Answer



$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

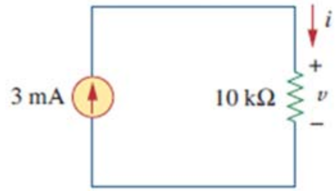
$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Example 04

For the circuit shown in Fig. calculate the voltage v, the conductance G, and the power p.

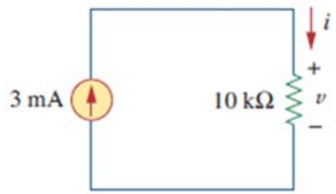


For the circuit shown in Fig. calculate the voltage v, the conductance G, and the power p.

Answer

- V= I.R = 3 x 10^{-3} x 10 x 10^3 = 30 volt
- G = 1/R = 1 / 10 x 10^3 =

  100 u siemens
- P = i^2 x R = (3 x 10^{-3})^2 x 10 x 10^3 = 0.09 w



Example 05

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Answer

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

example 06

A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R.

A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R.

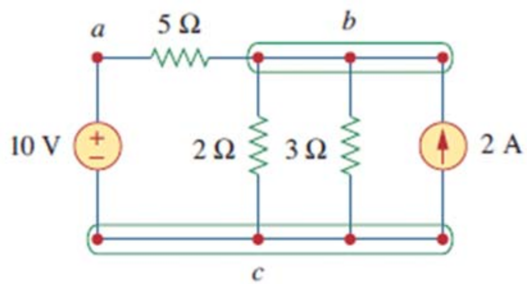
Answer

I = P/V = 30 $\cos^2$(t) / 15 cos (t) = 2 cos (t) Amp

R = V^2/P = 225 $\cos^2$(t)/30 $\cos^2$(t) = 7.5 ohm

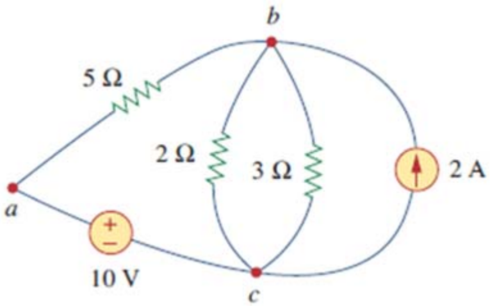
Nodes, Branches, and Loops

- A ***branch*** represents a single element such as a voltage source or a resistor.
- In other words, a branch represents any two-terminal element



- five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

- A <u>node</u> is the point of connection between two or more branches (indicated by a dot in a circuit)
- If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.
- Below circuit has three nodes a, b , and *c*.

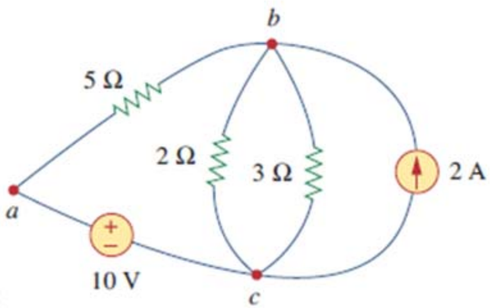


- A <u>loop</u> is any closed path in a circuit.
- A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- *<u>Independent loop</u>;* contains at least one branch which is not a part of any other independent loop
- Independent loops or paths result in independent sets of equations

- <u>fundamental theorem of network topology</u>
- A network with *b* branches, *n* nodes, and *l* independent loops will satisfy that

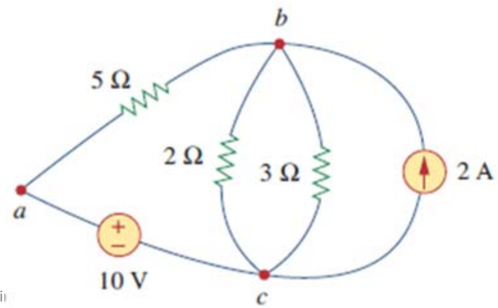
L = b-n+1

b=5
n=3
l = 3
Independent loops:
Abca == 5 Ω - 2 Ω – 10v
Abca == 5 Ω - 3 Ω – 10v
Abca == 5 Ω – 2A – 10V



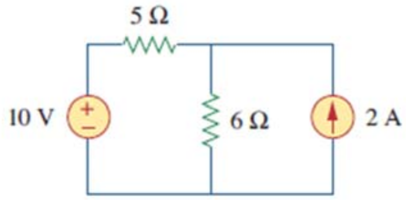
- Two or more elements are in <u>series</u> if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in <u>parallel</u> if they are connected to the same two nodes and consequently have the same voltage across them

Connection type	
In series	5Ω – 10V
In parallel	2Ω - 3Ω – 2A
In parallel	(5Ω – 10V) - 2Ω - 3Ω – 2A



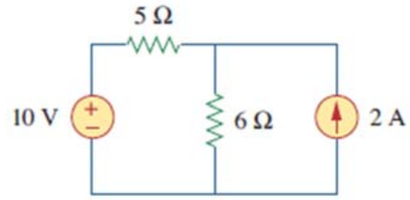
Example 07

Determine the number of branches and nodes in the circuit shown in Fig. Identify which elements are in series and which are in parallel.



Determine the number of branches and nodes in the circuit shown in Fig. Identify which elements are in series and which are in parallel.

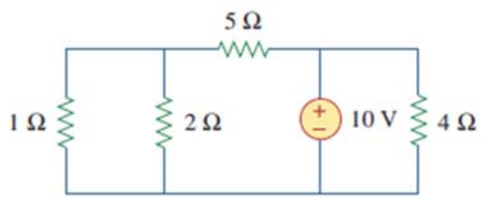
- Solution
- b = 4
- n =2
- l =b-n+1=3
- Independent loops : 10V-5Ω-6Ω , 10V-5Ω-2A, 6Ω-2A



Connection type	
In series	10V - 5Ω
In parallel	6Ω – 2A
In parallel	(10V-5Ω) - 6Ω – 2A

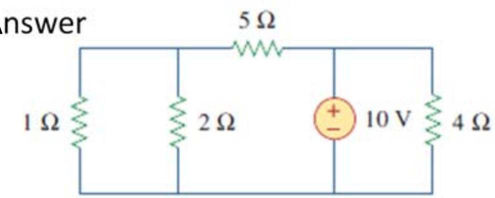
Example 8

- How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.



How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.

Answer



- b = 5
- n =3
- l =5-3+1=3
- Independent loops : 1Ω-5Ω-10V , 1Ω-5Ω-4Ω, 4Ω-5Ω-2Ω

Connection type	
In parallel	1Ω- 2Ω
In parallel	4Ω – 10V

Kirchhoff's Laws

- Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\sum_{n=1}^{N} i_n = 0$$

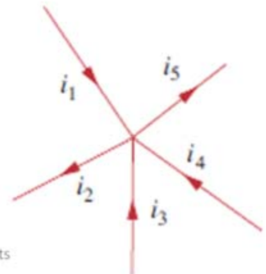
- where N is the number of branches connected to the node and i_n is the n^{th} current entering (or leaving) the node.

- By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

- The sum of the currents entering a node is equal to the sum of the currents leaving the node.

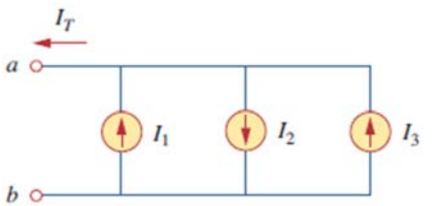
$$i_1 + i_3 + i_4 = i_2 + i_5$$



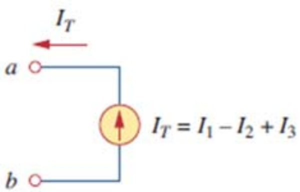
- The combined or equivalent current source can be found by applying KCL to node *a*.

$$I_T + I_2 = I_1 + I_3$$

$$I_T = I_1 - I_2 + I_3$$



(a)



- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

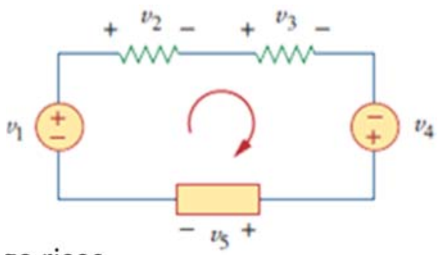
$$\sum_{m=1}^{M} v_m = 0$$

- M is the number of voltages in the loop (or the number of branches in the loop) and V_m is the *m*th voltage.

- The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- We can start with any branch and go around the loop either clockwise or counterclockwise.
- 1st clockwise:

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

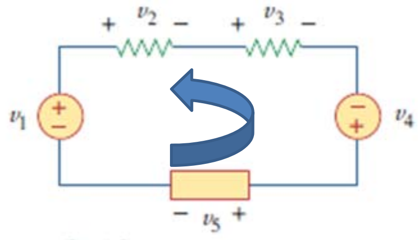
$$v_2 + v_3 + v_5 = v_1 + v_4$$



- Sum of voltage drops Sum of voltage rises

- 2nd counter clockwise:

$$v_1 - v_2 - v_3 + v_4 = 0$$

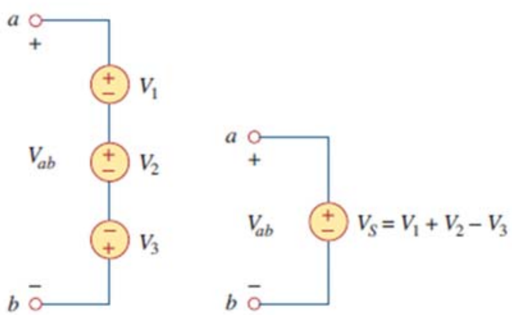


- When voltage sources are connected in series

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

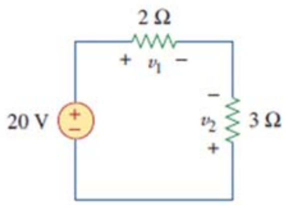
$$V_{ab} = V_1 + V_2 - V_3$$

-

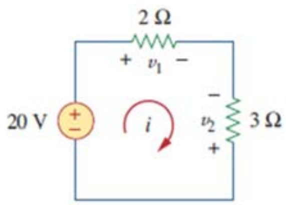


Example 9

For the circuit in Fig.   , find voltages v_1 and v_2.

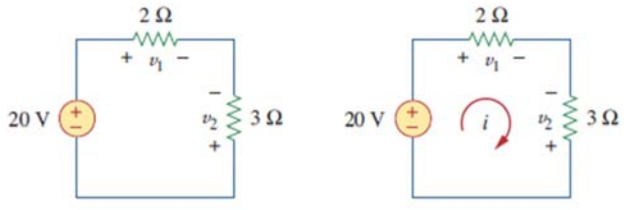


(a)



(b)

For the circuit in Fig. , find voltages v_1 and v_2.



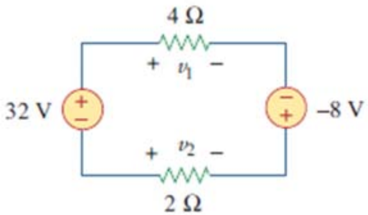
Answer

- Ohm's law $v_1 = 2i, \quad v_2 = -3i$
- KVL $-20 + v_1 - v_2 = 0$
- substitute $-20 + 2i + 3i = 0$

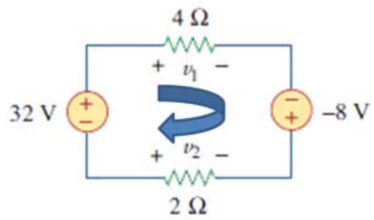
- $5i = 20 \quad \Rightarrow \quad i = 4$ A
- $v_1 = 8$ V, $\quad v_2 = -12$ V

Example 10

Find v_1 and v_2 in the circuit of Fig.



Find v_1 and v_2 in the circuit of Fig.



Answer

- Ohm's law : $V_1=4i$, $V_2 = -2i$
- KVL : $-32+ V_1 +8 - V_2 = 0$

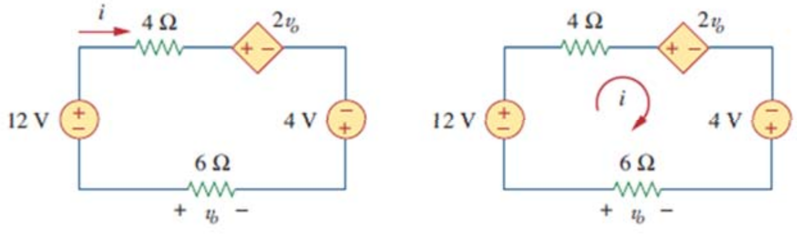
  $-32+4i +8 + 2i = 0$

  $6i = 24$

  $i = 4$ Amp
- $V_1 = 16$volt
- $V_2 = -8$ Volt

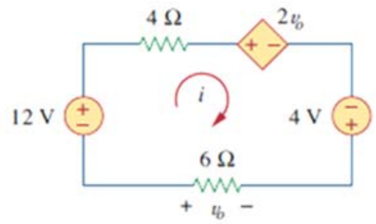
Example 11

- Determine v_o and i in the circuit shown in Fig



(a) (b)

- Determine v_o and i in the circuit shown in Fig



(a)

- Ohm's low : $V0 = -6i$
- KVL : $-12 + V_{4\Omega} + 2\ V0 - 4 - V_{6\Omega} = 0$

$-12 + 4i + 2\ V0 - 4 + 6i = 0$

$-12 + 4i - 12i - 4 + 6i = 0$

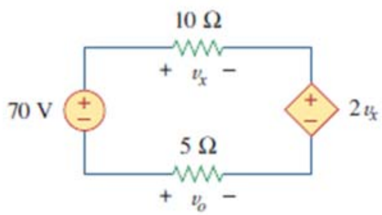
$-16 = 2i$

$i = -8$ Amp

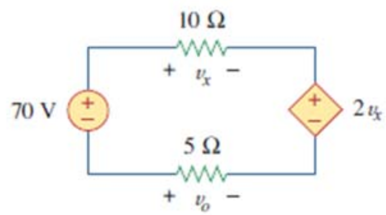
$V0 = 48$ volt

Example 12

Find v_x and v_o in the circuit of Fig.



Find v_x and v_o in the circuit of Fig.



Answer

- Ohms law: $V_x = 10i$

$V_0 = -5i$

- KVL: $-70 + V_x + 2\ Vx - V_0 = 0$

$-70 + 10i + 2\ Vx + 5i = 0$

$-70 + 10i + 20i + 5i = 0$

$70 = 35i$

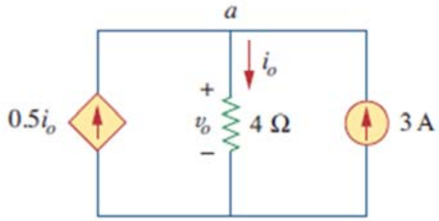
$i = 2$ Amp

$V_x = 20$ V

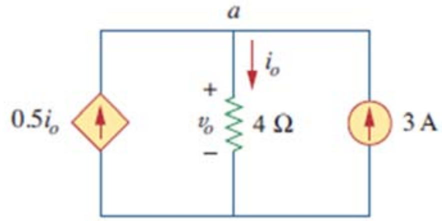
$V_0 = -10$ V

Example 13

Find current i_o and voltage v_o in the circuit shown in Fig.



Find current i_o and voltage v_o in the circuit shown in Fig.

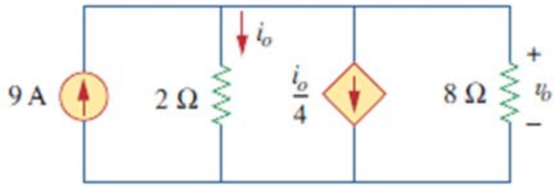


Answer

- KCL @ a : $I_0 = 0.5I_0 + 3$
  $0.5\ I_0 = 3$
  $I_0 = 6$ Amp
- Ohm: $V_0 = 4 \times I_0 = 24$ Volt

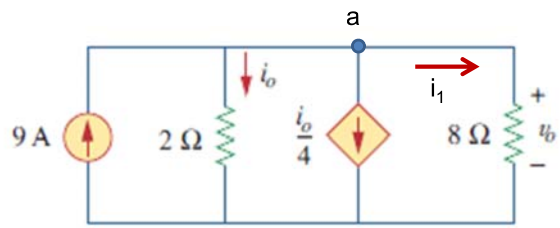
Example 14

Find v_o and i_o in the circuit of Fig.



Find v_o and i_o in the circuit of Fig.

Solution:



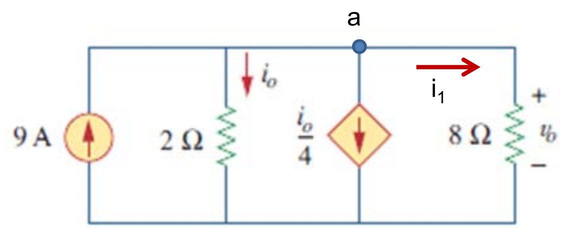
KCL @ a : $9 - i_0 - (i_0 / 4) - i_1 = 0$

KVL @(2 Ω - 8 Ω) : $V_{2\Omega} = V_{8\Omega}$

$2\ i_0 = 8\ i_0$

$i_1 = i_0/4$

Find v_o and i_o in the circuit of Fig.



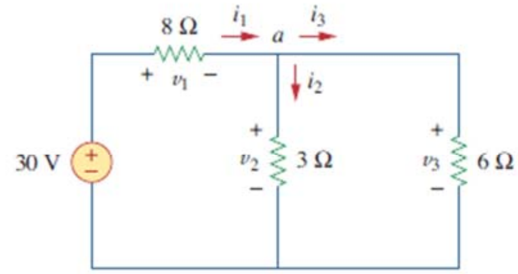
$9 - i_0 - i_0/4 - i_0/4 = 0$

$9 = 1.5\ i0$ → $i_0 = 6$ Amp

$i_1 = 1.5$ Amp → $V_0 = 1.5 \times 8 = 12$ Volt

Example 15

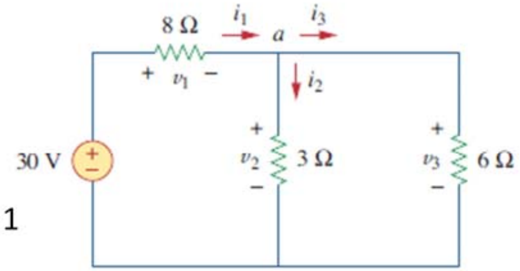
Find currents and voltages in the circuit shown in Fig



(a)

Find currents and voltages in the circuit shown in Fig

<u>Answer</u>



(a)

Ohm: V1 = 8 i1

V2 = 3 i2

V3 = 3 i3

KCL @a: i1 – i2 – i3=0 →1

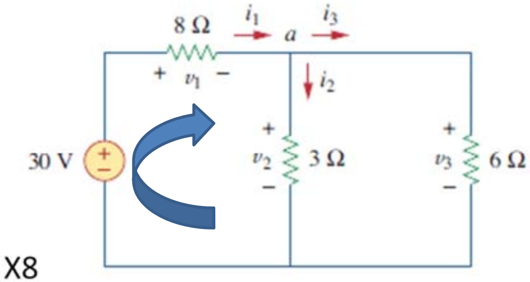
KVL @ left: -30 + V1 + V2 =

30 – 8 i1 – 3 i2 = 0

i2 = (30 – 3 i2)/8 →2

Find currents and voltages in the circuit shown in Fig

<u>Answer</u>



(a)

KVL@ right : V2 = V3

3 i2 = 6 i3

i3 = 0.5 i2 →3

Substitute 2,3 in 1

(30 – 3 i2)/8 – i2 – i2/2 = 0 X8

30 – 3 i2 – 8i2 – 4i2 = 0

i2 = 2 Amp

i3 = 1 Amp

i1 = 3 Amp

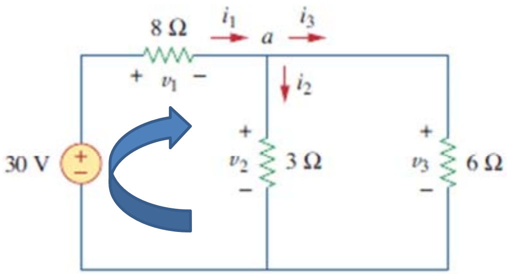
Find currents and voltages in the circuit shown in Fig

<u>Answer</u>

$V1 = 24$ Volt

$V2 = 6$ Volt

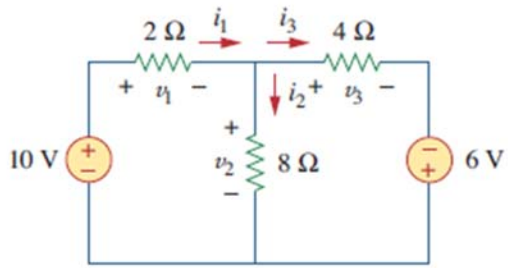
$V3 = 6$ Volt



(a)

Example 16

Find the currents and voltages in the circuit shown in Fig



Find the currents and voltages in the circuit shown in Fig

<u>Answer</u>

Ohm: $V1 = 2\, i1$

$V2 = 8\, i2$

$V3 = 4\, i3$

KCL @ node: $i1 - i2 - i3 = 0 \rightarrow 1$

KVL @ Left: $-10 + V1 + V2 = 0$

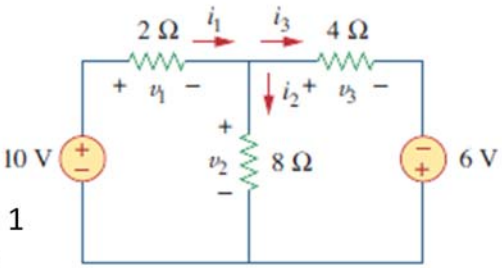
$-10 + 2\, i1 + 8\, i2 = 0$

$i1 = 5 - 4\, i2 \rightarrow 2$

KVL @ right: $-6 - V2 + V3 = 0$

$-6 - 8\, i2 + 4\, i3 = 0$

$i3 = (6 + 8\, i2)/4 \rightarrow 3$



Find the currents and voltages in the circuit shown in Fig

<u>Answer</u>

Substitute 2,3 in 1

$5 - 4\, i2 - i2 - (6 - 8\, i2)/4 = 0$

$20 - 16\, i2 - 4\, i2 - 6 - 8\, i2 = 0$

$14 = 28\, i2$

$i2 = 0.5$ Amp

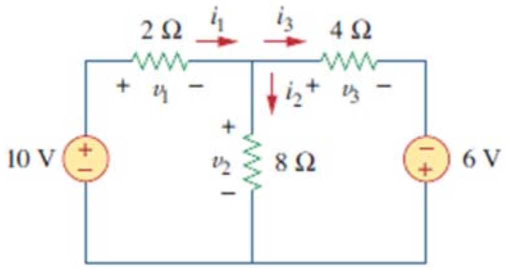
$i1 = 3$ Amp

$i3 = 2.5$ Amp

$V1 = 6$ V

$V2 = 4$ V

$V3 = 10$ V



Thanks,..

See you next week (ISA),…