Lecture (5)
Digital Coding techniques (I)
Analog to digital conversion &
Digital Data transmission

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Agenda

- The need for digital coding
- Analog to digital conversion
- Digital Data transmission

The need for digital coding

- Have already noted in that both analog and digital information can be encoded as digital signals.

Why?

- Advantages of digital signals
  - cheaper than analog signaling
  - less susceptible to noise interference
- disadvantages of digital signals
  - suffer more from attenuation than do analog signals
  - Takes higher band width than analog signal

The need for digital coding (cont,..)

- so we will start learning how to convert the analog data to digital data (analog to digital conversion).
- If your original data is digital origin, so there is no need for the 1^{st} step.
- Then we will discover different codes that presents digital data, advantages and disadvantages of each code.

Analog to digital conversion

- A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified.
- For this reason, the tendency today is to change an analog signal to digital data.
- In this section we describe two techniques, pulse code modulation and delta modulation.

Analog to digital conversion (cont,..)

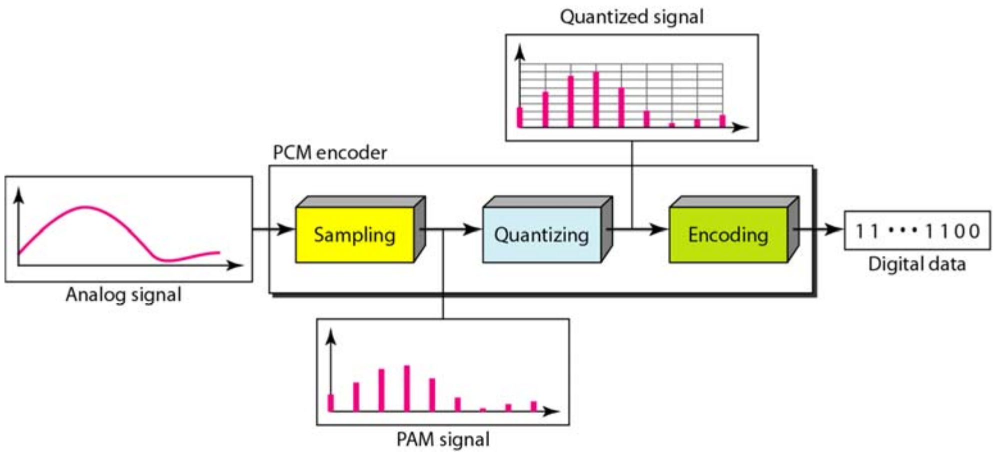
Analog 2 Digital conversion (plan A)

PCM

- PCM consists of three steps to digitize an analog signal:
  - Sampling
  - Quantization
  - Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.
- Filtering should ensure that we do not distort the signal, ie remove high frequency components that affect the signal shape.

Analog to digital conversion (cont,..)

Components of PCM encoder



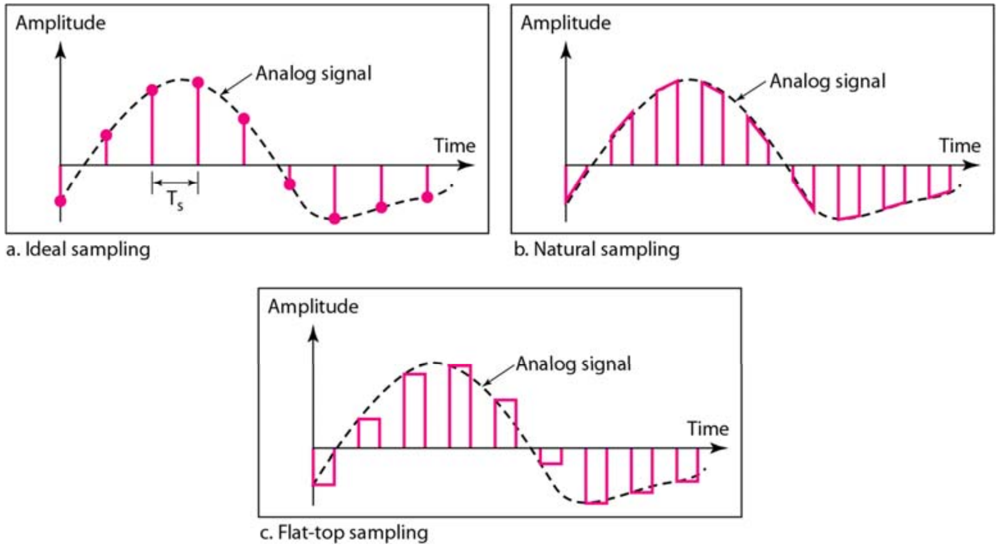
Analog to digital conversion (cont,..)

Step 1: Sampling

- Analog signal is sampled every T_S secs.
- T_s is referred to as the sampling interval.
- fs = 1/Ts is called the sampling rate or sampling frequency.
- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, like natural but with single amplitude value
- The process is referred to as pulse amplitude modulation PAM and the outcome is a signal with analog (non integer) values

Analog to digital conversion (cont,..)

Three different sampling methods for PCM

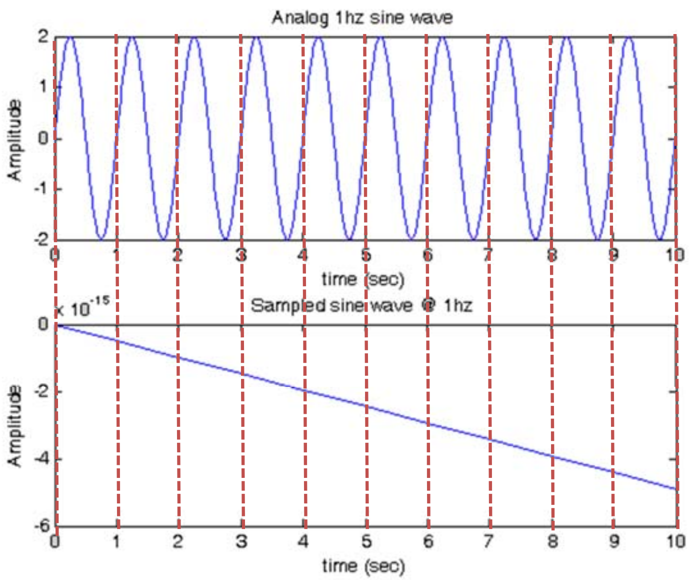


Analog to digital conversion (cont,..)

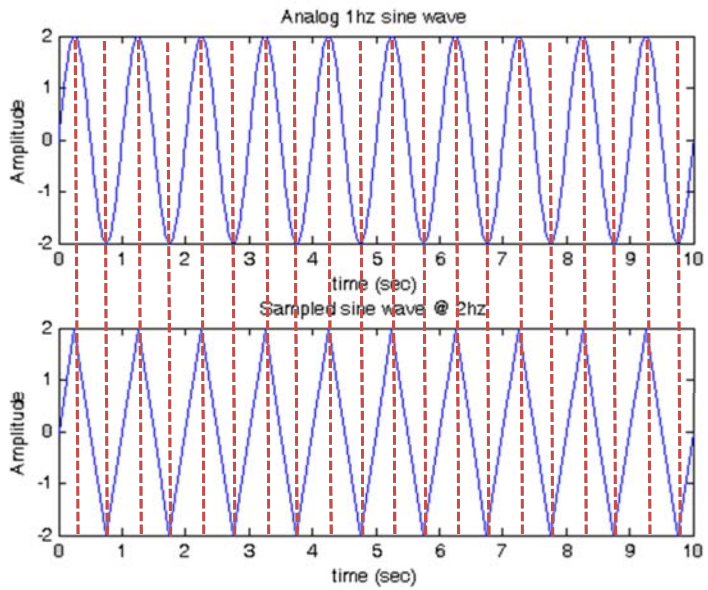
Nyquist–Shannon sampling theorem

If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart.

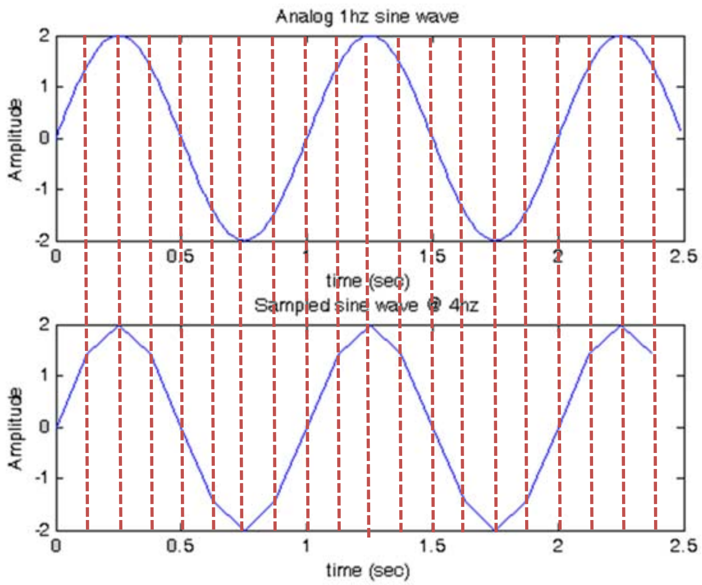
Analog to digital conversion (cont,..)



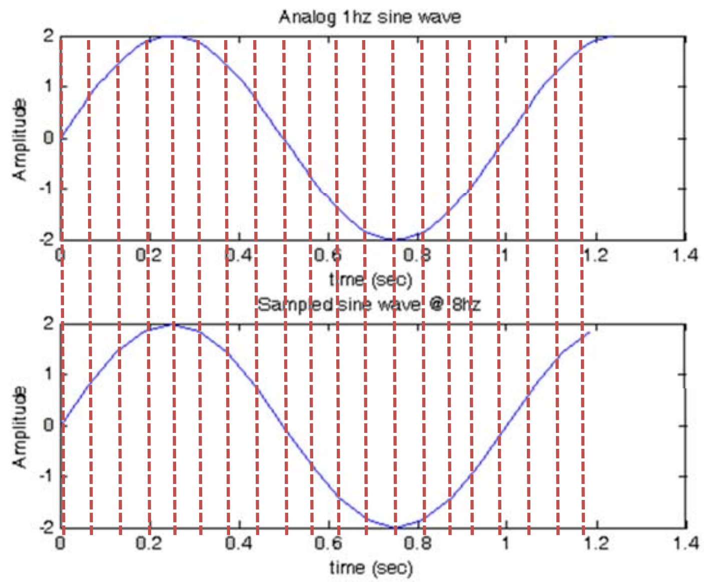
Analog to digital conversion (cont,..)



Analog to digital conversion (cont,..)



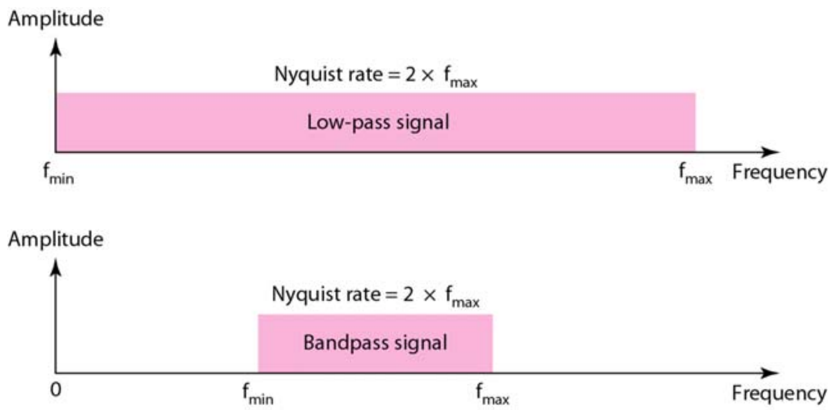
Analog to digital conversion (cont,..)



Analog to digital conversion (cont,..)

So According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

Nyquist sampling rate for low-pass and band-pass signals



Analog to digital conversion (cont,..)

Sine wave example again,..

For an intuitive example of the Nyquist theorem,

let us sample a simple sine wave at three sampling rates:

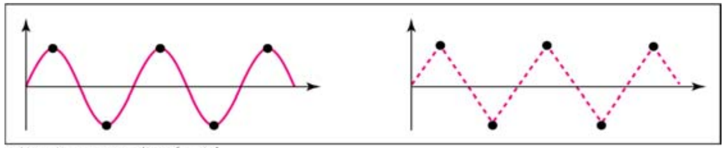
fs = 4f (2 times the Nyquist rate),

fs = 2f (Nyquist rate), and

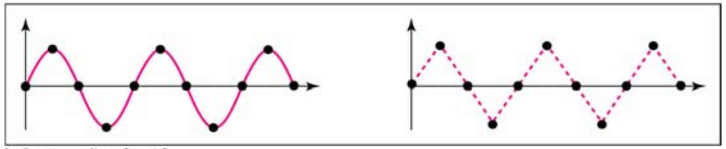
fs = f (one-half the Nyquist rate).

Figure shows the sampling and the subsequent recovery of the signal.

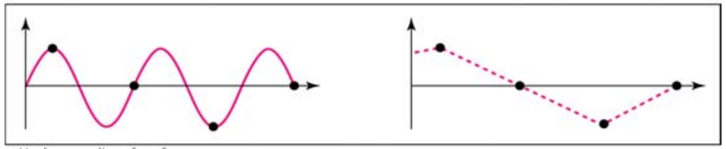
Analog to digital conversion (cont,..)



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$



c. Undersampling: $f_s = f$

Analog to digital conversion (cont,..)

- It can be seen that sampling at the Nyquist rate can create a good approximation of the original sine wave (part a).
- Oversampling in part b can also create the same approximation, but it is redundant and unnecessary.
- Sampling below the Nyquist rate (part c) does not produce a signal that looks like the original sine wave.

Analog to digital conversion (cont,..)

Clock Example:

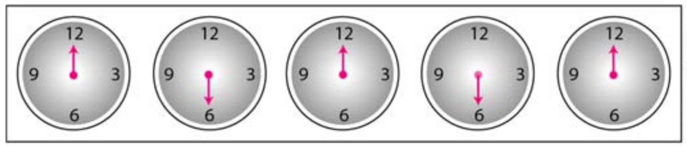
Consider a hand clock, has a period of 60 seconds, and you will are allowed to take a look at time every short period of time (take a sample)

In part a, according to the Nyquist theorem, to know the time we need to take a sample the hand every 30 s (Ts = .5 T or fs = 2f).

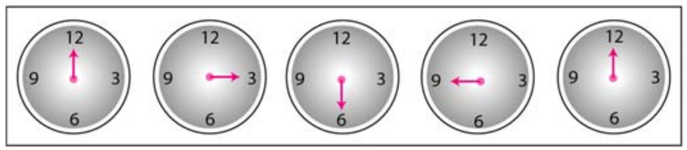
The resulted sample points (readings), in order, are 12, 6, 12, 6, 12, and 6.

The receiver of the samples cannot tell if the clock is moving forward or backward.

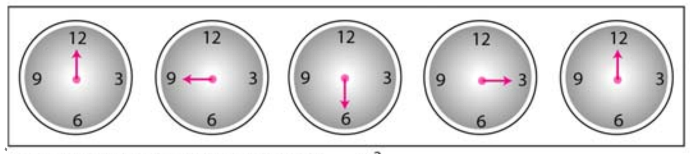
Analog to digital conversion (cont,..)



a. Sampling at Nyquist rate: $T_s = T\frac{1}{2}$



b. Oversampling (above Nyquist rate): $T_s = T\frac{1}{4}$



c. Undersampling (below Nyquist rate): $T_s = T\frac{3}{4}$

Analog to digital conversion (cont,..)

In part b, we sample at double the Nyquist rate (every 15 s) (Ts = .25 T or fs = 4f).

The sample points are 12, 3, 6, 9, and 12.

The clock is moving forward.

In part c, we sample below the Nyquist rate (Ts = T or fs = f).

The sample points are 12, 9, 6, 3, and 12.

Although the clock is moving forward, the receiver thinks that the clock is moving backward.

Analog to digital conversion (cont,..)

wagon-wheel effect

An example related is the seemingly backward rotation of the wheels of a forward-moving car in a movie.

This can be explained by under-sampling.

A movie is filmed at 24 frames per second.

If a wheel is rotating more than 12 times per second, the under-sampling creates the impression of a backward rotation.

Analog to digital conversion (cont,..)

Telephone example

Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz.

The sampling rate therefore is 8000 samples per second.

Analog to digital conversion (cont,..)

Example

A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal.

Therefore, we can sample this signal at 2 times the highest frequency (200 kHz).

The sampling rate is therefore 400,000 samples per second.

Analog to digital conversion (cont,..)

Example

A complex band pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends.

We do not know the maximum frequency in the signal.

Analog to digital conversion (cont,..)

2nd step: Quantization

Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.

The amplitude values are infinite between the two limits.

We need to map the infinite amplitude values onto a finite set of known values.

This is achieved by dividing the distance between min and max into L zones, each of height Δ

$$\Delta = (\max - \min)/L$$

Analog to digital conversion (cont,..)

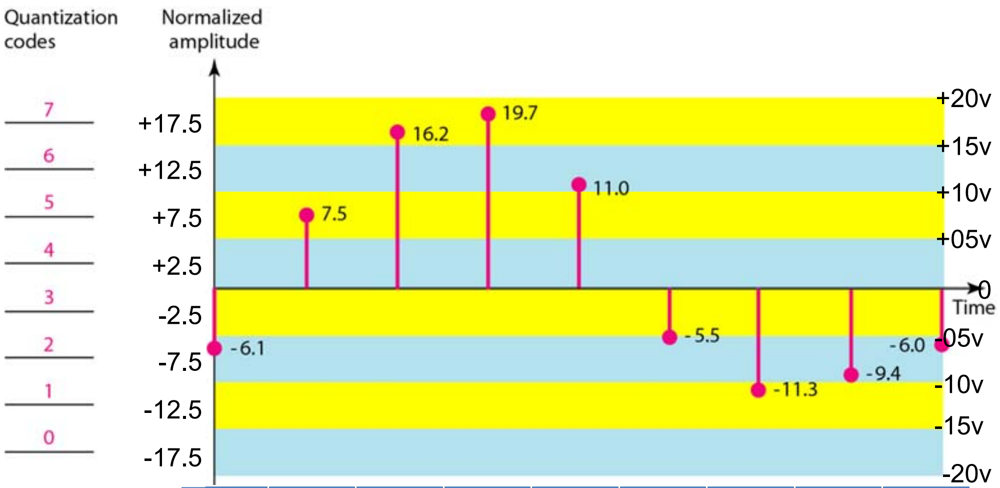
Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Analog to digital conversion (cont,..)

Quantization Zones

- Assume we have a voltage signal with amplitutes V_{min}=-20V and V_{max}=+20V.
- We want to use L=8 quantization levels.
- Zone width Δ = (20 - -20)/8 = 5
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5



Normalized PAM values	**-6.1**	**7.5**	**16.2**	**19.7**	**11**	**-5.5**	**-11.3**	**-9.4**	**-6**
Normalized quantized values	-7.5	7.5	17.5	17.5	12.5	-7.5	-12.5	-7.5	-7.5
Normalized error	-1.4	0	1.3	-2.2	1.5	-2.0	-1.2	2.1	-1.5

Analog to digital conversion (cont,..)

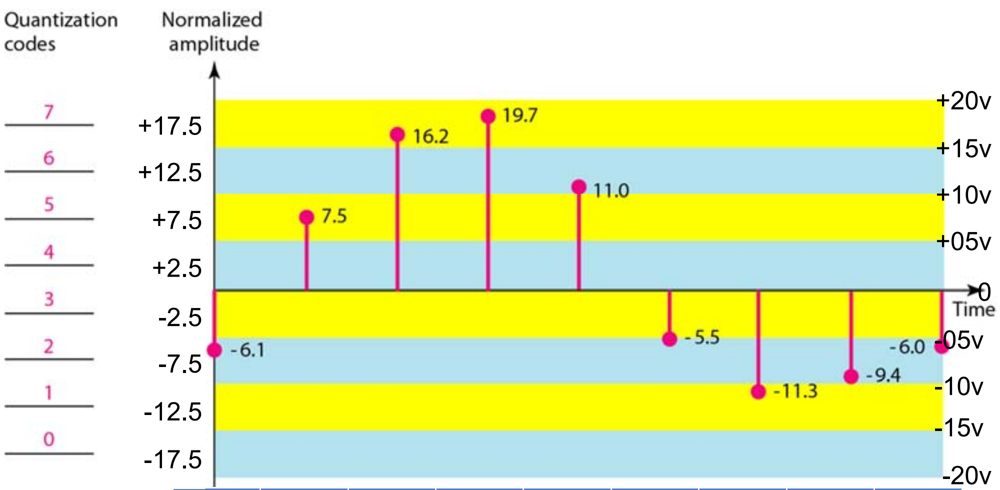
3rd step: encoding

Assigning Codes to Zones

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$n_b = \log_2 L$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.



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Normalized quantized values	-7.5	7.5	17.5	17.5	12.5	-7.5	-12.5	-7.5	-7.5
Normalized error	-1.4	0	1.3	-2.2	1.5	-2.0	-1.2	2.1	-1.5
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Analog to digital conversion (cont,..)

Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples -> higher bit rate

Analog to digital conversion (cont,..)

Example

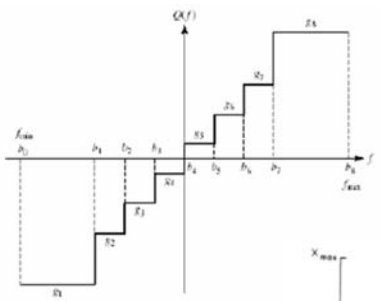
So	0.1	5.1	10.1	15.1
Sq	2.5	7.5	12.5	17.5
Noise	2.4	2.4	2.4	2.4
S/N	2.5/2.4=	7.5/2.4=	12.5/2.4	17.5/2.4

Analog to digital conversion (cont,..)

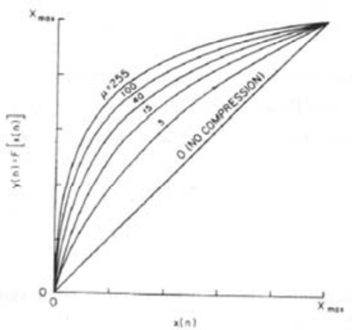
Quantization Error and SN_QR

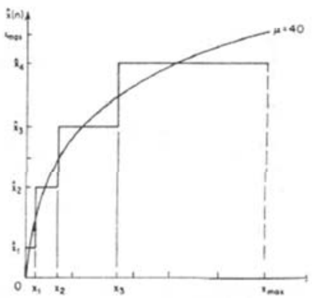
- Signals with lower amplitude values will suffer more from quantization error as the error range: $\Delta/2$, is fixed for all signal levels.
- Non linear quantization is used to alleviate this problem.
- Goal is to keep SN_QR fixed for all sample values.
- Two approaches:
  - The quantization levels follow a logarithmic curve. Smaller Δ's at lower amplitudes and larger Δ's at higher amplitudes.
  - Companding: The sample values are compressed at the sender into logarithmic zones, and then expanded at the receiver. The zones are fixed in height.

Analog to digital conversion (cont,..)



$Q(f) = g_l, \text{ if } f \in B_l$





Analog to digital conversion (cont,..)

Bit rate and bandwidth requirements of PCM

- The bit rate of a PCM signal can be calculated form the number of bits per sample x the sampling rate

$$\text{Bit rate} = n_b \times f_s$$

- A digitized signal will always need more bandwidth than the original analog signal.
- Price we pay for robustness and other features of digital transmission.

Analog to digital conversion (cont,..)

Example

We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?

Solution

The human voice normally contains frequencies from 0 to 4000 Hz. So the sampling rate and bit rate are calculated as follows:

$$\text{Sampling rate} = 4000 \times 2 = 8000 \text{ samples/s}$$
$$\text{Bit rate} = 8000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

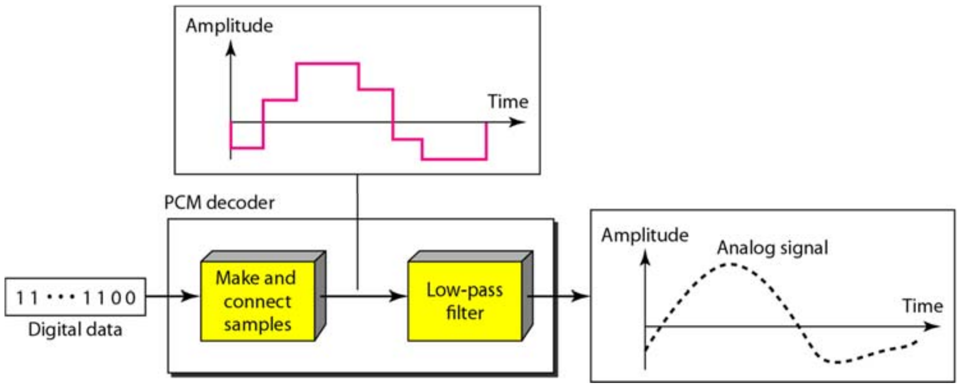
Analog to digital conversion (cont,..)

PCM Decoder

- To recover an analog signal from a digitized signal we follow the following steps:
  – We use a hold circuit that holds the amplitude value of a pulse till the next pulse arrives.
  – We pass this signal through a low pass filter with a cutoff frequency that is equal to the highest frequency in the pre-sampled signal.
- The higher the value of L, the less distorted a signal is recovered.

Analog to digital conversion (cont,..)

Components of a PCM decoder



Analog to digital conversion (cont,..)

Example

We have a low-pass analog signal of 4 kHz.

If we send the analog signal, we need a channel with a minimum bandwidth of 4 kHz.

If we digitize the signal and send 8 bits per sample, we need a channel with a minimum bandwidth of 8 × 4 kHz = 32 kHz.

Analog to digital conversion (cont,..)

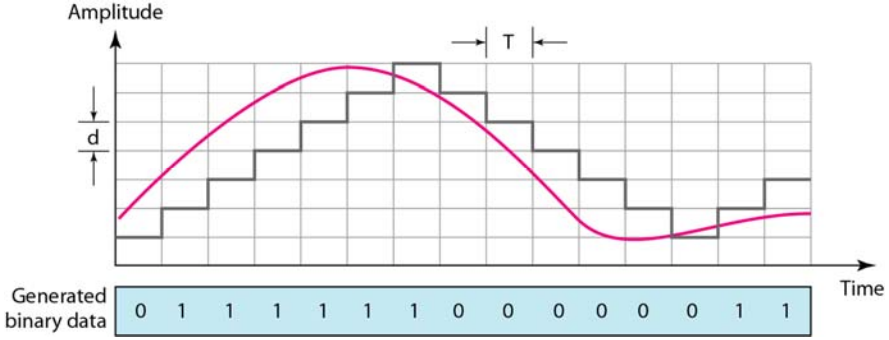
Analog 2 Digital conversion (plan B)

Delta Modulation

- This scheme sends only the difference between pulses, if the pulse at time t_{n+1} is higher in amplitude value than the pulse at time t_n, then a single bit, say a “1”, is used to indicate the positive value.
- If the pulse is lower in value, resulting in a negative value, a “0” is used.
- This scheme works well for small changes in signal values between samples.
- If changes in amplitude are large (or faster), this will result in large errors.

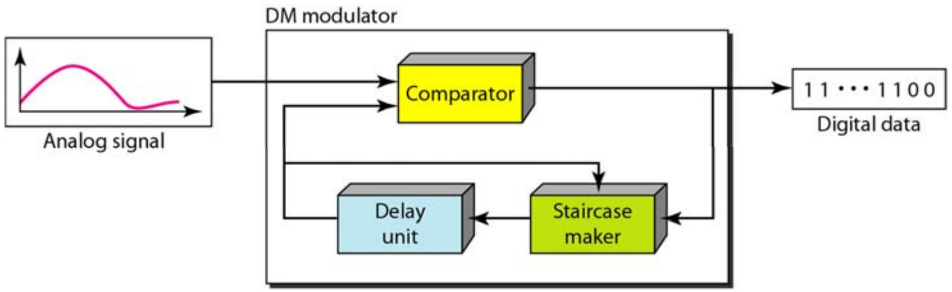
Analog to digital conversion (cont,..)

The process of delta modulation



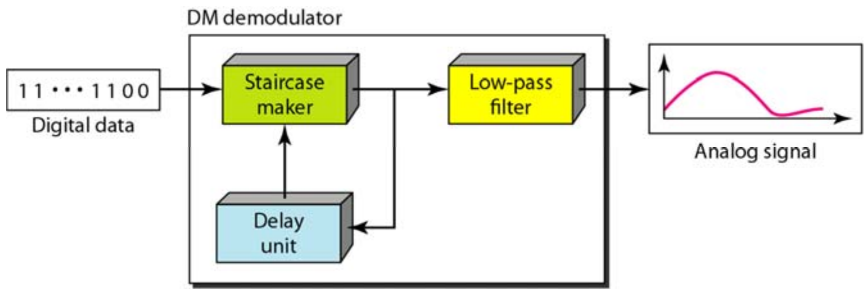
Analog to digital conversion (cont,..)

Delta modulation components



Analog to digital conversion (cont,..)

Delta demodulation components



Analog to digital conversion (cont,..)

Analog 2 Digital conversion (plan C)

Delta PCM (DPCM)

- Instead of using one bit to indicate positive and negative differences, we can use more bits -> quantization of the difference.
- Each bit code is used to represent the value of the difference.
- The more bits the more levels -> the higher the accuracy.

Digital Data transmission

The transmission of binary data across a link can be accomplished in either parallel or serial mode.

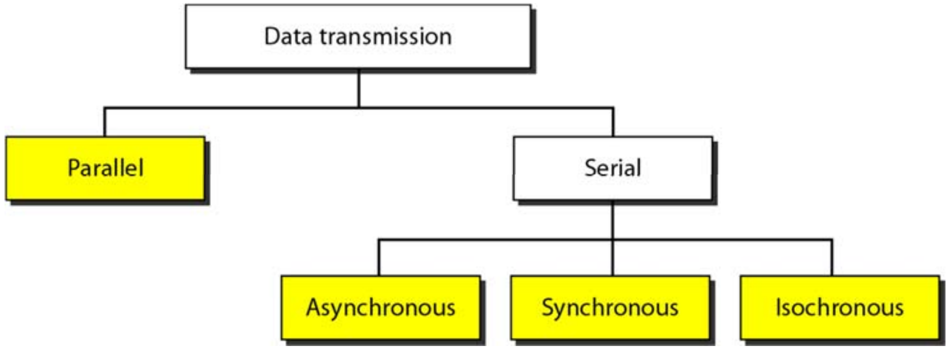
In parallel mode, multiple bits are sent with each clock tick.

In serial mode, 1 bit is sent with each clock tick.

While there is only one way to send parallel data, there are three subclasses of serial transmission: asynchronous, synchronous, and isochronous.

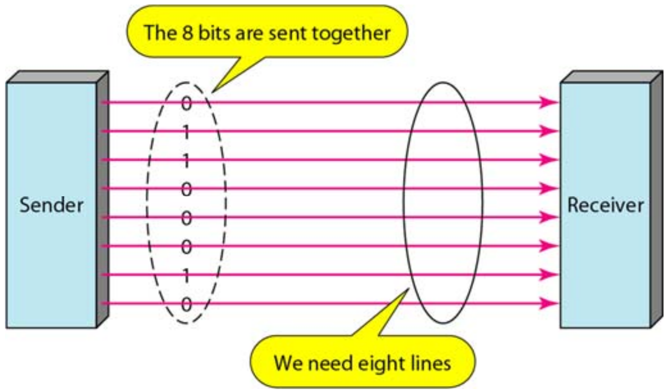
Digital Data transmission (cont,..)

Data transmission and modes



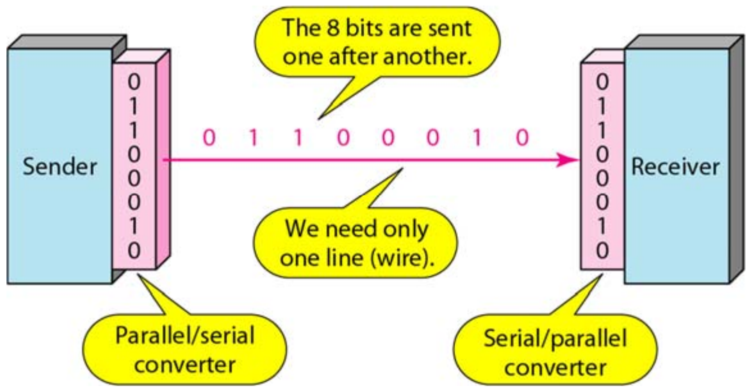
Digital Data transmission (cont,..)

Parallel transmission



Digital Data transmission (cont,..)

Serial transmission

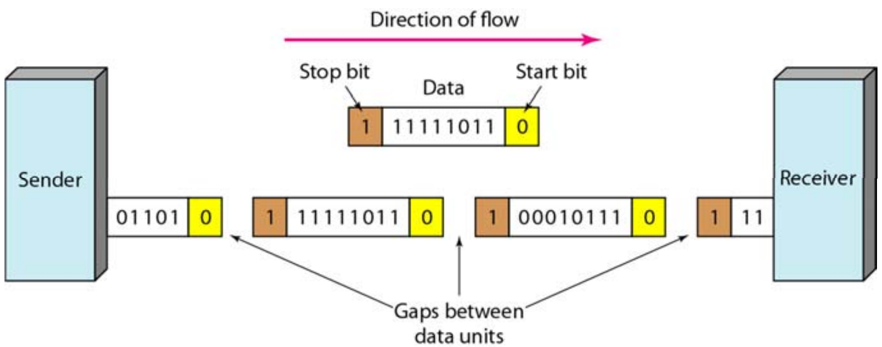


Digital Data transmission (cont,..)

Asynchronous transmission

- In asynchronous transmission, we send 1 start bit (0) at the beginning and 1 or more stop bits (1s) at the end of each byte.
- There may be a gap between each byte.
- Asynchronous here means "asynchronous at the byte level," but the bits are still synchronized; their durations are the same.

Digital Data transmission (cont,..)

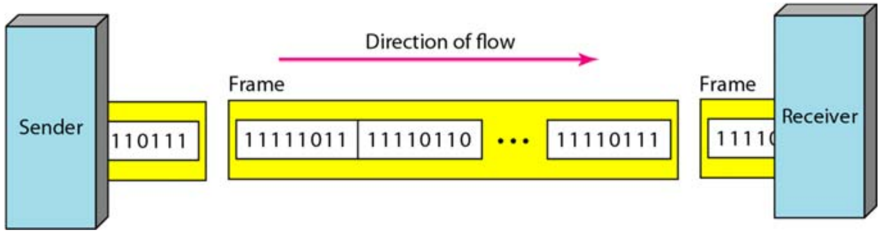


Digital Data transmission (cont,..)

synchronous transmission

- In synchronous transmission, we send bits one after another without start or stop bits or gaps.
- It is the responsibility of the receiver to group the bits.
- The bits are usually sent as bytes and many bytes are grouped in a frame.
- A frame is identified with a start and an end byte(s).

Digital Data transmission (cont,..)



Thanks,…