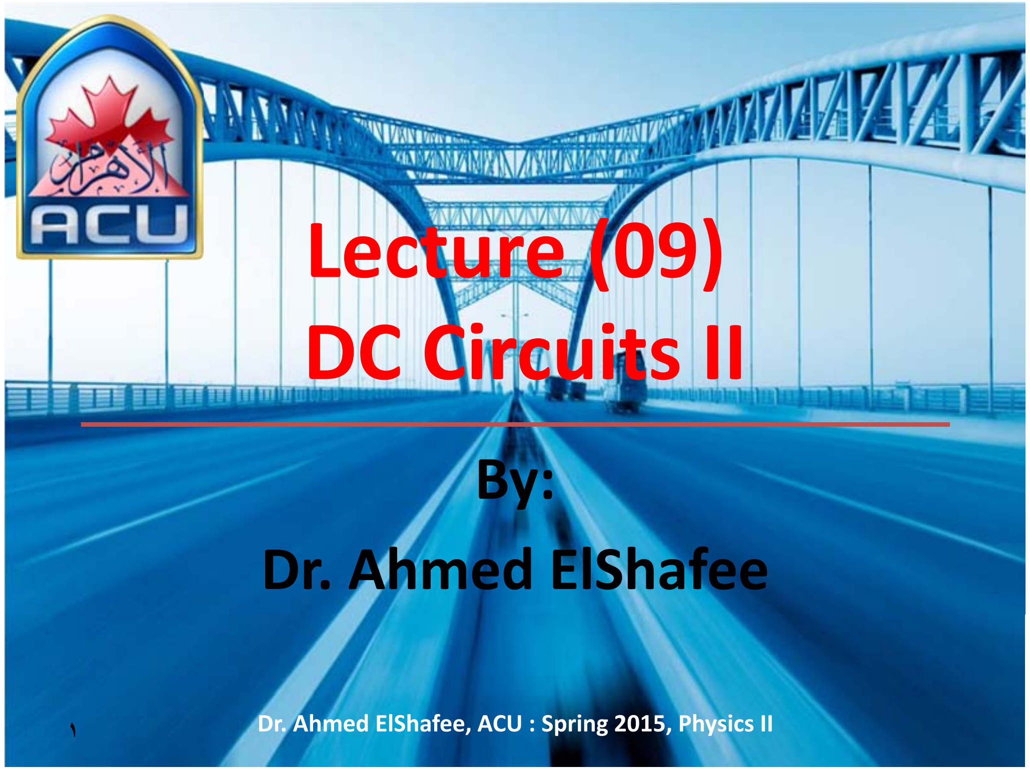


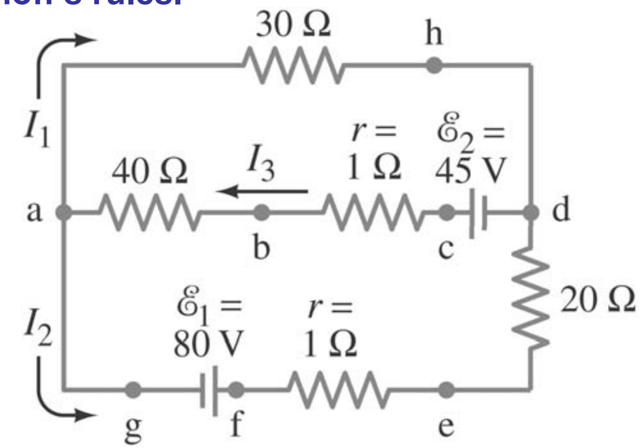
Lecture (09)
DC Circuits II

By:

Dr. Ahmed ElShafee

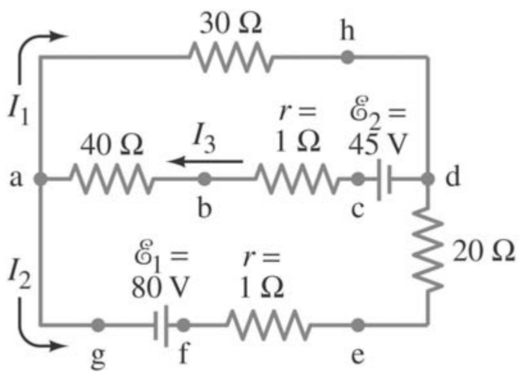
Kirchhoff's rules and DC currents

- **Some circuits cannot be broken down into series and parallel connections. For these circuits we use Kirchhoff's rules.**



- **Junction rule: The sum of currents entering a junction equals the sum of the currents leaving it (i.e., charge does not pile up).**

- $I_{in} = Iou_t$
- $I_3 = I_1 + I_2$
- $I_3 - I_2 - I_1 = 0$



Top loop

$45 - (40 + 1)I_3 - (30)I_1 = 0$

$45 - 41\,I_3 - 30\,I_1 = 0$

$I_1 = 1.5\ - 1.37\,I_3$ → (1)

Buttom loop

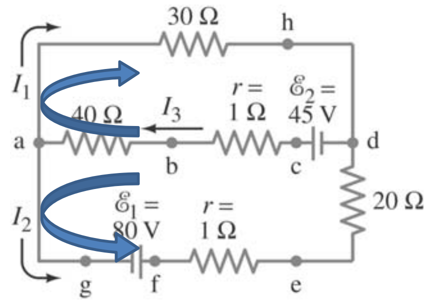
$45 + 80 - (1 + 20)I2 - (41)I3 = 0$

$125 - 21\,I2 - 41\,I3 = 0$

$I2 = 5.95\ - 1.95\,I3$ → (2)

Node

$I3 = I2 + I1$ →(3)



Substitute 1, and 2 in 3

$I3 = (1.5 + 5.95) - (1.37 + 1.95)I3$

$I3 = 7.45 - 3.32\ I3$

$4.32\ I3 = 7.45$

$I3 = 1.72\ Amp\ \rightarrow$

$from\ 1$

$I1 = 1.5\ - (1.37\ \times 1.72) = -0.86\ Amp$

$from\ 2$

$I2 = 5.95\ - (1.95\ \times 1.72) = 2.6\ Amp$

example

- **Using Kirchhoff's rules.**

a) **Calculate the currents (call them I_1, I_2, and I_3) through the three batteries of the circuit in the figure.**

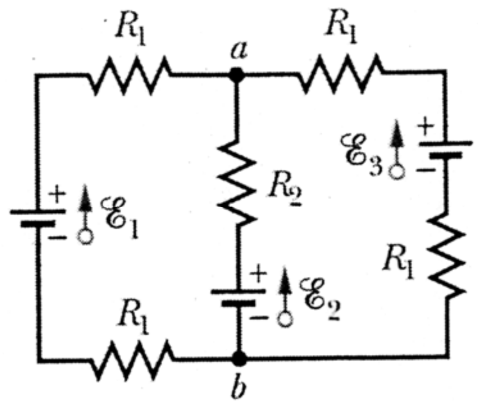
b) **What is V_a-V_b?**

$$\xi_1 = 2.0\,V$$

$$\xi_2 = \xi_3 = 4.0\,V$$

$$R_1 = 1.0\,\Omega$$

$$R_2 = 2.0\,\Omega$$



$\underline{left}$

$-2 - 2I_1 + 4 - 2I_2 = 0$

$2 - 2I_1 - 2I_2 = 0$

$I_1 = 1 - I_2$

$\underline{right}$

$-4 - 2\ I_3 + 4 - 2\ I_2 = 0$

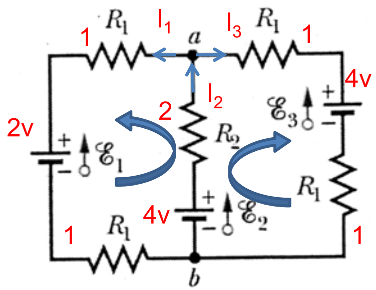
$I_3 = -I_2$

$\underline{node}$

$I_2 = I_1 + I_3$

$I_2 = 1 - I_2\ - I_2$

$I_2 = {}^1/_3\ amp$



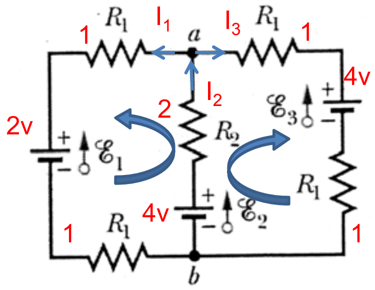
$from\ 2$:

$I3 = -{}^1/_3\ amp$

$from\ 1$:

$I1 = 1 - {}^1/_3 = {}^2/_3\ amp$

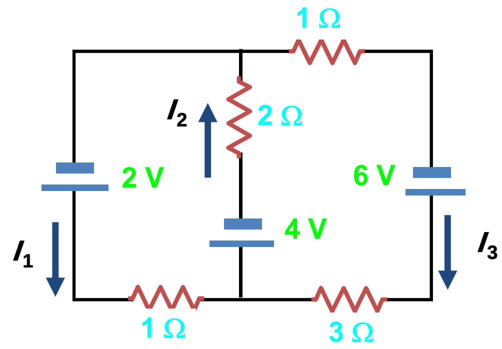
- $V_{R2} = I_2 \times 2 = {}^1/_3 \times 2 = {}^2/_3 \ volt$
- $Vab = 4 - {}^2/_3 = 3\,{}^1/_3 \ volt$



MCQ

- Which of the equations is valid for the circuit below?

1) $2 + I_1 + 2I_2 = 0$
2) $2 + 2I_1 + 2I_2 + 4I_3 = 0$
3) $2 + I_1 - 4 + 2I_2 = 0$
4) $I_3 - 4 + 2I_2 + 6 = 0$
5) $2 + I_1 + 3I_3 - 6 = 0$



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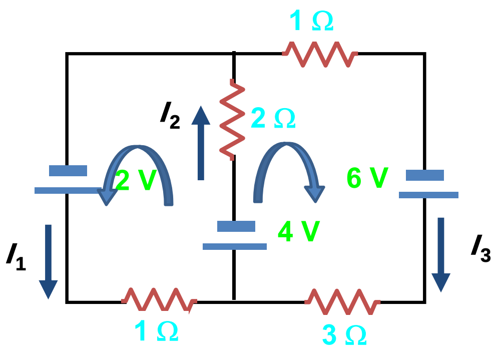
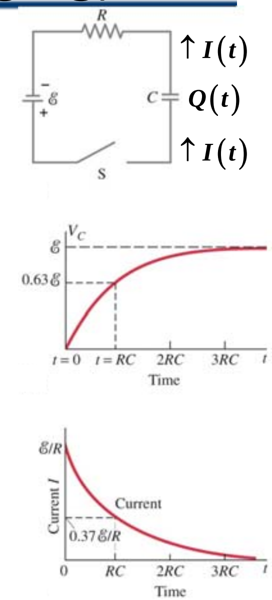


Circuits Containing Resistor and Capacitor (*RC* Circuits charging)

- **When the switch is closed, the capacitor will begin to charge.**
- **As it does, the voltage across it increases, and the current through the resistor decreases.**
- After the switch S closes in the *RC* circuit shown in (a),
- the voltage across the capacitor increases with time as shown in (b),
- and the current through the resistor decreases with time as shown in (c).



- **The charges across the capacitor is**

$Q_c = \varepsilon C\,(1 - e^{-t/RC})$

- **The voltage across the capacitor is $V_C = Q/C$:**

$V_c = \varepsilon\,(1 - e^{-t/RC})$

- **The current flow through capacitor**

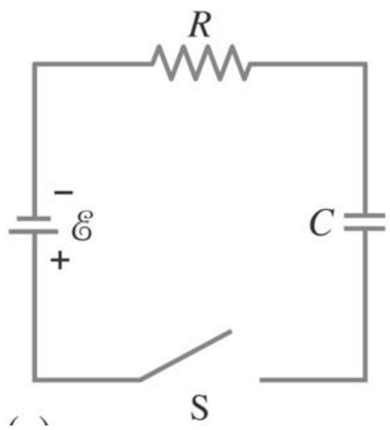
$I_c = \frac{\varepsilon}{R}\, e^{-t/RC}$

- **Time constant**

$\tau = RC$

Example

- **The capacitance in the circuit shown is $C = 0.30\ \mu F$, the total resistance is 20 kΩ, and the battery emf is 12 V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when the current I is 0.20 its maximum value.**



$C = 0.3\ uF$
$R = 20\ kOhm$
$\varepsilon = 12V$

$Q_c = \varepsilon C\,(1 - e^{-t/RC})$
$V_c = \varepsilon\,(1 - e^{-t/RC})$
$I_c = \frac{\varepsilon}{R}\, e^{-t/RC}$

a. $\tau = RC = 20 \times 10^3 \times 0.3 \times 10^{-6} = 6 \times 10^{-3}\ sec$

b. $Q_{max} = \varepsilon\, C = 12 \times 0.3 \times 10^{-6} = 3.6 \times 10^{-6}\ column$

c. Tim of 99% charging

$Q_{99\%} = 0.99\ Q_{max} = 0.99\ \varepsilon\ C = \varepsilon\ C(1 - e^{-t/RC})$

$0.99 = 1 - e^{-t/RC}$

$e^{-t/RC} = 0.01$

$\frac{t}{RC} = -\ln 0.01 = 4.06$

$t = 4.6 \times RC = 4.6 \times 6 \times 10^{-3} = 0.0276\ sec$

$C = 0.3\ uF$
$R = 20\ kOhm$
$\varepsilon = 12V$

$Q_c = \varepsilon C\,(1 - e^{-t/RC})$
$V_c = \varepsilon\,(1 - e^{-t/RC})$
$I_c = \frac{\varepsilon}{R}\, e^{-t/RC}$

d. Current when Q = 0.5 Q_{max}

$Q = 0.5\ Qmax = 0.5\ \varepsilon\ C$

$Q = \varepsilon\ C(1 - e^{-t/RC})$

$0.5 = (1 - e^{-t/RC})$

$e^{-t/RC} = 0.5$

$I = \frac{\varepsilon}{R}\, e^{-t/RC} = \frac{\varepsilon}{R} \times 0.5 = \frac{12}{20 \times 10^3} \times 0.5 = 3 \times 10^{-4}\ amp$

e. Max current

$Imax = \frac{\varepsilon}{R} = \frac{12}{20 \times 103} = 6 \times 10^{-4}\ amp$

$C = 0.3\ uF$
$R = 20\ kOhm$
$\varepsilon = 12V$

$Q_c = \varepsilon C\ (1 - e^{-t/RC})$
$V_c = \varepsilon\ (1 - e^{-t/RC})$
$I_c = \frac{\varepsilon}{R}\ e^{-t/RC}$

f. Q when I=0.2 I_{max}

$$I = \frac{\varepsilon}{R}\ e^{-t/RC} = I_{max} e^{-t/RC} = Im_{ax}\ 0.2$$

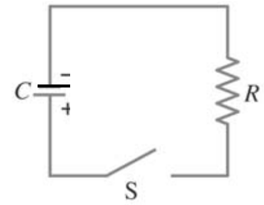
$$e^{-t/RC} = 0.2$$

$$Q_c = \varepsilon C\ (1 - e^{-t/RC}) = \varepsilon C\ (1 - 0.2) = \varepsilon C\ \times 0.8$$

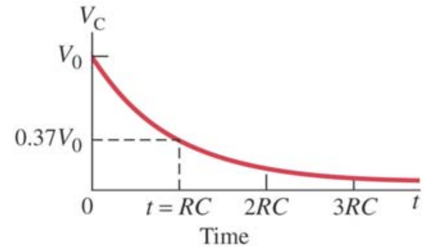
$$Q_c = 12\ \times 0.3\ \times\ 10^{-6}\ \times 0.8 = 2.88\ \times\ 10^{-6}\ f$$

Circuits Containing Resistor and Capacitor (*RC* Circuits discharging)

- **If an isolated charged capacitor is connected across a resistor, it discharges:**
- V_0 is the initial voltage drop on the capacitor
- $V_c(t) = V0\ e^{-t/RC}$
- $I_c(t) = \frac{V0}{R} e^{-t/RC}$
- $Q_c(t) = Q_0\ e^{-t/RC}$

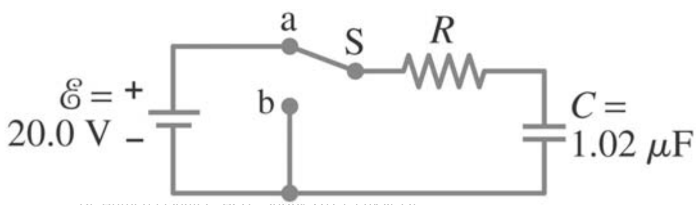


(a)



Example

- **In the *RC* circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at t = 0 the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance C = 1.02 μF. The current I is observed to decrease to 0.50 of its initial value in 40 μs. (a) What is the value of Q, the charge on the capacitor, at t = 0? (b) What is the value of R? (c) What is Q at t = 60 μs?**



$\varepsilon = V_0 = 20V$
$C = 1.02\ uF$
$I = 0.5\ I_0\ @\ 40\ uSec$

$V_c(t) = V0\ e^{-t/RC}$
$I_c(t) = \frac{V0}{R} e^{-t/RC}$
$Q_c(t) = Q_0\ e^{-t/RC}$

a. Q @ t=0

$$Q_0 = C\ V_0 = 1.02\ \times\ 10^{-6}\ \times 20 = 2.04\ \times\ 10^{-5}\ C$$

b. R ?

$$t = 40\ usec\ \rightarrow I = 0.5\ I_0$$

$$I_c(t) = \frac{V0}{R} e^{-t/RC} = 0.5\ \mathrm{I_0}$$

$$e^{-t/RC} = 0.5$$

$$\frac{t}{RC} = -\ln 0.5 = 0.693$$

$$R = \frac{t}{0.693\ \times C} = \frac{4\ \times\ 10^{-6}}{0.693\ \times 1.02\ \times\ 10^{-6}} = 57\ ohm$$

$\varepsilon = V_0 = 20V$
$C = 1.02\,uF$
$I = 0.5\,I_0 \,@\, 40\,uSec$

$V_c(t) = V0\, e^{-t/RC}$
$I_c(t) = \frac{V0}{R} e^{-t/RC}$
$Q_c(t) = Q_0\, e^{-t/RC}$

C. Q=? @ t=60usec

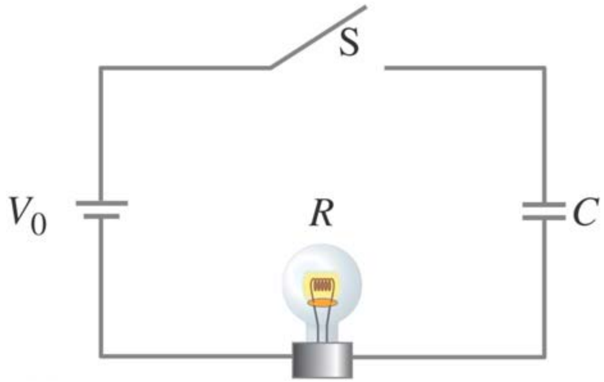
$$Q_c(t) = Q_0\, e^{-t/RC}$$

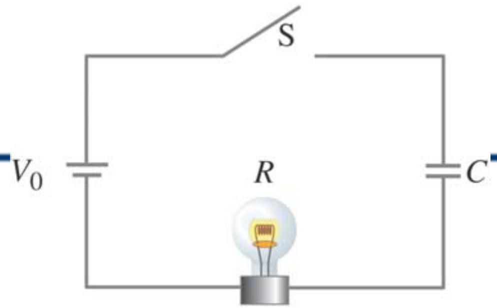
$$Q = 2.04 \times 10^{-5} \times e^{\frac{-60\times10^{-6}}{57\times1.02\times10^{-6}}} = 7.3\,uc$$

Example

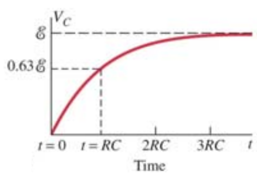
Bulb in *RC* circuit.

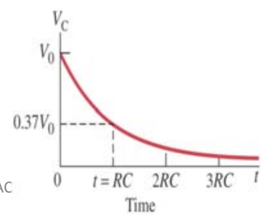
- **In the circuit shown, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.**

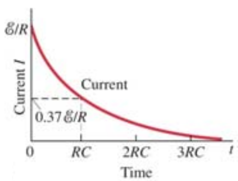




- When the switch is closed, the current is large and the bulb is bright.
- As the capacitor charges, the bulb dims; once the capacitor is fully charged the bulb is dark.



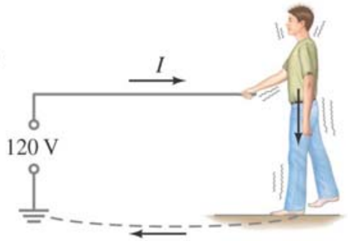




Electric Hazards

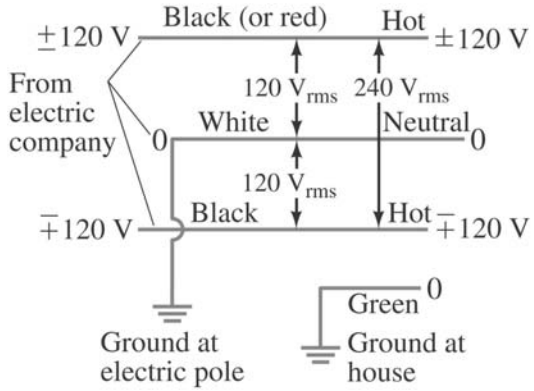
- **Most people can "feel" a current of 1 mA; a few mA of current begins to be painful.**
- **Currents above 10 mA may cause uncontrollable muscle contractions**
- **Currents around 100 mA passing through the body cause death by ventricular fibrillation.**
- **Higher currents may not cause fibrillation, but can cause severe burns.**
- **Household voltage can be lethal if you are wet and in good contact with the ground. Be careful!**

- A person receiving a shock has become part of a complete circuit.



(a)

- The safest plugs are those with three prongs; they have a separate ground line.
- Here is an example of household wiring – colors can vary, though! Be sure you know which is the hot wire before you do anything.



Am-, Volt-, and Ohm-meters

- An ammeter must be in series with the current it is to measure;
- a voltmeter must be in parallel with the voltage it is to measure.
- An ohmmeter measures resistance; it requires a battery to provide a current.