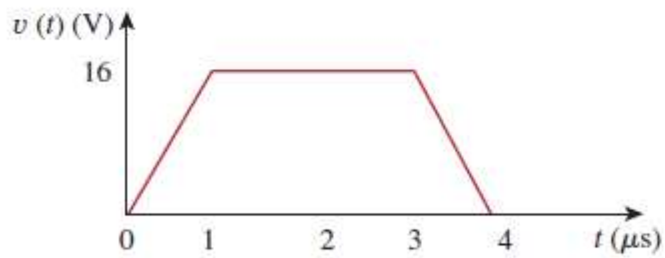
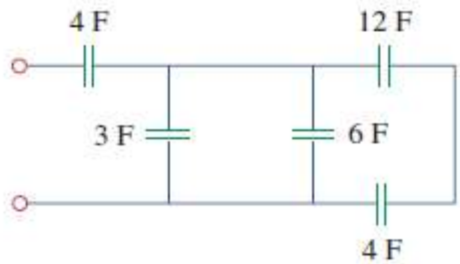
Circuits I – Tutorial 08

Capacitors and Inductors

Q1	**6.1** If the voltage across a 7.5-F capacitor is $2te^{-3t}$ V, find the current and the power.
Sol 1	$i = C\dfrac{dv}{dt} = 7.5\left(2e^{-3t} - 6te^{-3t}\right) = \mathbf{15(1-3t)e^{-3t}\ A}$ $p = vi = 15(1-3t)e^{-3t} \cdot 2t\, e^{-3t} = \mathbf{30t(1-3t)e^{-6t}\ W}$. $\mathbf{15(1-3t)e^{-3t}\ A,\ 30t(1-3t)e^{-6t}\ W}$

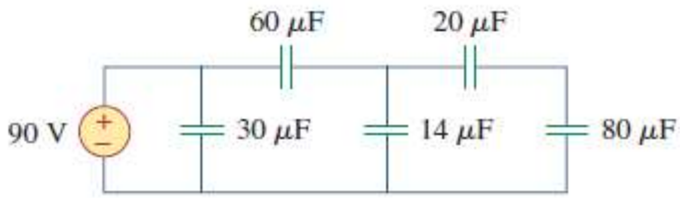
Q2	**6.10** The voltage across a 5-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor. 

Sol 2	$i = C\frac{dv}{dt} = 5x10^{-3}\frac{dv}{dt}$ $v = \begin{cases} 16t, & 0 < t < 1\mu s \\ 16, & 1 < t < 3\ \mu s \\ 64 - 16t, & 3 < t < 4\mu s \end{cases}$ $\frac{dv}{dt} = \begin{cases} 16x10^6, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -16x10^6, & 3 < t < 4\mu s \end{cases}$ $\underline{i(t) = \begin{cases} 80\ kA, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -80\ kA, & 3 < t < 4\mu s \end{cases}}$

Q3	**6.17** Determine the equivalent capacitance for each of the circuits of Fig. 6.51.  (a)
Sol 3	(a) 4F in series with 12F = 4 x 12/(16) = 3F 3F in parallel with 6F and 3F = 3+6+3 = 12F 4F in series with 12F = 3F i.e. C_{eq} = **3F**

Q4

6.24 For the circuit in Figure 6.58, determine (a) the voltage across each capacitor and (b) the energy stored in each capacitor.



Sol 4

20μF is series with 80μF = 20x80/(100) = 16μF

14μF is parallel with 16μF = 30μF

(a) $v_{30\mu F} = \mathbf{90V}$

$v_{60\mu F} = \mathbf{30V}$

$v_{14\mu F} = \mathbf{60V}$

$$v_{20\mu F} = \frac{80}{20+80} x60 = \mathbf{48V}$$

$v_{80\mu F} = 60 - 48 = \mathbf{12V}$

(b) Since $w = \frac{1}{2}Cv^2$

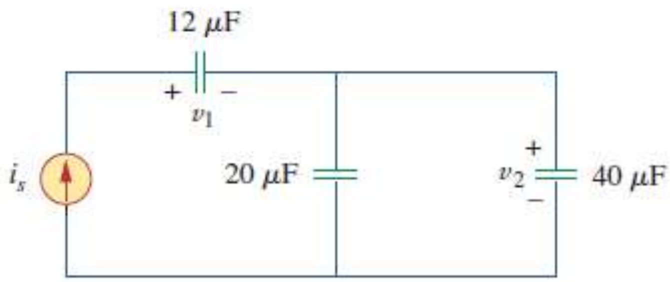
$w_{30\mu F} = 1/2 \text{ x } 30 \text{ x } 10^{-6} \text{ x } 8100 = \mathbf{121.5mJ}$

$w_{60\mu F} = 1/2 \text{ x } 60 \text{ x } 10^{-6} \text{ x } 900 = \mathbf{27mJ}$

$w_{14\mu F} = 1/2 \text{ x } 14 \text{ x } 10^{-6} \text{ x } 3600 = \mathbf{25.2mJ}$

$w_{20\mu F} = 1/2 \text{ x } 20 \text{ x } 10^{-6} \text{ x } (48)^2 = \mathbf{23.04mJ}$

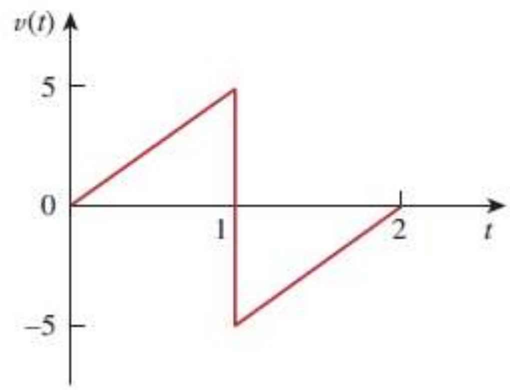
$w_{80\mu F} = 1/2 \text{ x } 80 \text{ x } 10^{-6} \text{ x } 144 = \mathbf{5.76mJ}$

Q5	**6.32** In the circuit of Fig. 6.64, let $i_s = 50e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s. 
Sol 5	(a) $C_{eq} = (12x60)/72 = 10\ \mu F$ $$v_1 = \frac{10^{-3}}{12x10^{-6}}\int_0^t 50e^{-2t}dt + v_1(0) = \underline{-2083e^{-2t}\ \Big\|_0^t} + 50 = \underline{-2083e^{-2t} + 2133V}$$ $$v_2 = \frac{10^{-3}}{60x10^{-6}}\int_0^t 50e^{-2t}dt + v_2(0) = \underline{-416.7e^{-2t}\ \Big\|_0^t} + 20 = \underline{-416.7e^{-2t} + 436.7V}$$ (b) At t=0.5s, $v_1 = -2083e^{-1} + 2133 = 1366.7, \quad v_2 = -416.7e^{-1} + 436.7 = 283.4$ $$w_{12\mu F} = \frac{1}{2}x12x10^{-6}x(1366.7)^2 = \underline{11.207\text{ J}}$$ $$w_{20\mu F} = \frac{1}{2}x20x10^{-6}x(283.4)^2 = \underline{803.2\text{ mJ}}$$ $$w_{40\mu F} = \frac{1}{2}x40x10^{-6}x(283.4)^2 = \underline{1.6063\text{ J}}$$

Q6	**6.34** The current through a 10-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.
Sol 6	$i = 10e^{-t/2}$ $v = L\frac{di}{dt} = 10x10^{-3}(10)\left(\frac{1}{2}\right)e^{-t/2}$ $= -50e^{-t/2}$ mV $v(3) = -50e^{-3/2}$ mV = **–11.157 mV** $p = vi = -500e^{-t}$ mW $p(3) = -500e^{-3}$ mW = **–24.89 mW**.

Q7

6.45 If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current $i(t)$. Assume $i(0) = 0$.



Sol 7	$i(t) = \frac{1}{L}\int_0^t v(t) + i(0)$ For $0 < t < 1$, $v = 5t$ $i = \frac{1}{10x10^{-3}}\int_0^t 5t\,dt + 0$ $= 250t^2 \ A$ For $1 < t < 2$, $v = -10 + 5t$ $i = \frac{1}{10x10^{-3}}\int_1^t (-10+5t)dt + i(1)$ $= \int_1^t (0.5t - 1)dt + 0.25kA$ $= [1 - t + 0.25t^2]\ kA$ $i(t) = \begin{bmatrix} 250t^2\ A, & 0 < t < 1s \\ [1 - t + 0.25t^2]\ kA, & 1 < t < 2s \end{bmatrix}$

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<td>Q8</td>
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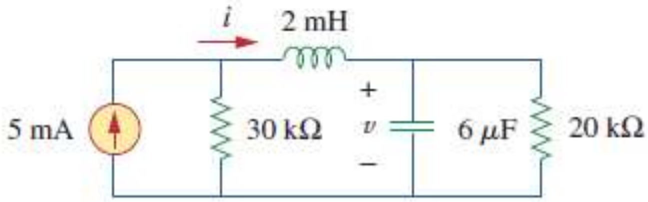
6.48 Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.



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<td>Sol 8</td>
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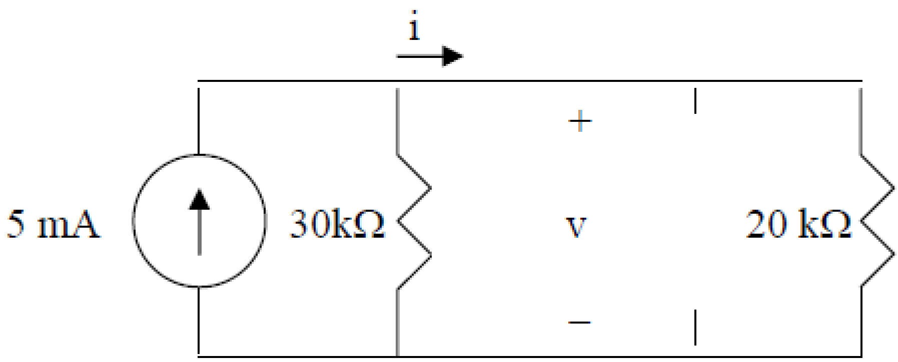

Using current division,

$$i = (30k/(30k+20k))(5mA) = \mathbf{3\ mA}$$

$$v = 20ki = \mathbf{60\ V}$$

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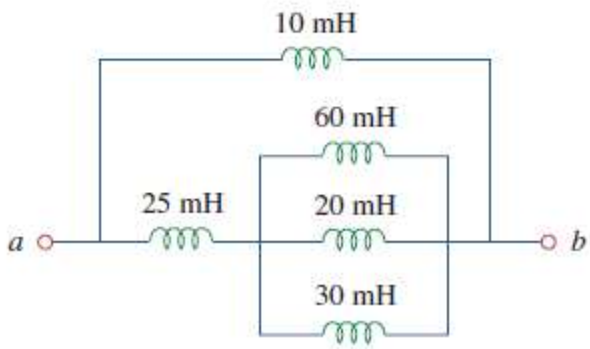
<tr><td>Q9</td><td>6.51 Determine L_{eq} at terminals a-b of the circuit in Fig. 6.73.

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<tr><td>Sol 9</td><td>$$\frac{1}{L}=\frac{1}{60}+\frac{1}{20}+\frac{1}{30}=\frac{1}{10} \qquad L = 10\text{ mH}$$
$$L_{eq}=10\|(25+10)=\frac{10\text{x}35}{45}$$
$$= \mathbf{7.778\ mH}$$</td></tr>
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