Circuits I – Tutorial 06

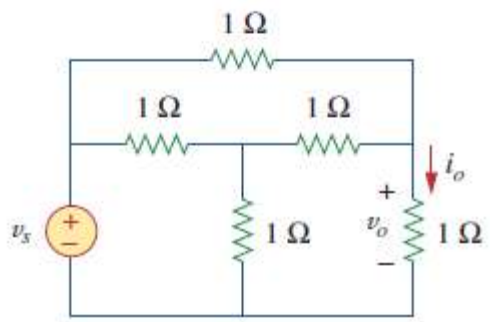
Circuit Theorems (1)

Q1

4.3 (a) In the circuit of Fig. 4.71, calculate v_o and i_o when $v_s = 1$ V.

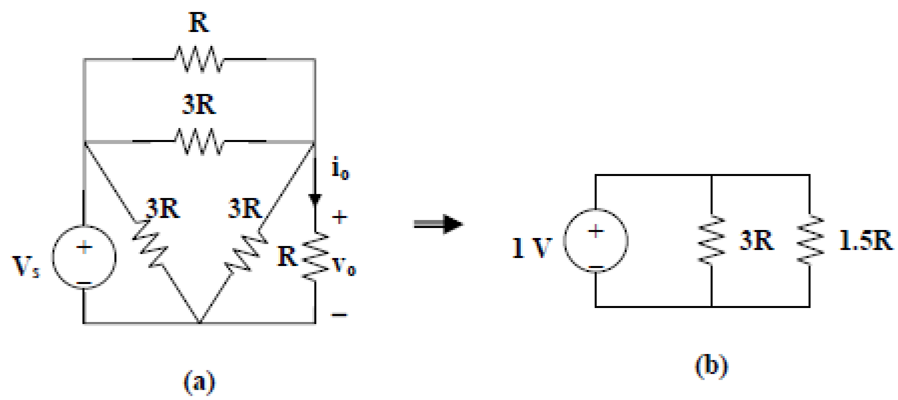
(b) Find v_o and i_o when $v_s = 10$ V.

(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?



linearity

Sol 1



(a) We transform the Y sub-circuit to the equivalent Δ.

$$R \| 3R = \frac{3R^2}{4R} = \frac{3}{4}R, \quad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$$v_o = \frac{v_s}{2} \text{ independent of R}$$

$i_o = v_o/(R)$

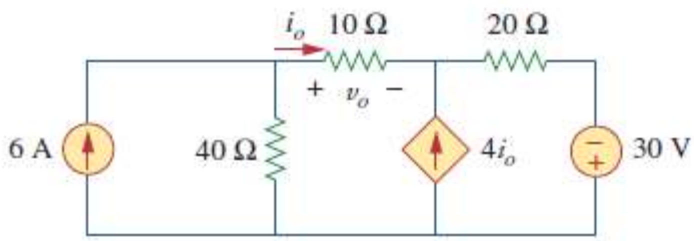
When $v_s = 1V$, $v_o = \mathbf{0.5V}$, $i_o = \mathbf{0.5A}$

(b) When $v_s = 10V$, $v_o = \mathbf{5V}$, $i_o = \mathbf{5A}$

(c) When $v_s = 10V$ and $R = 10\Omega$,
$v_o =$ **5V**, $i_o = 10/(10) =$ **500mA**

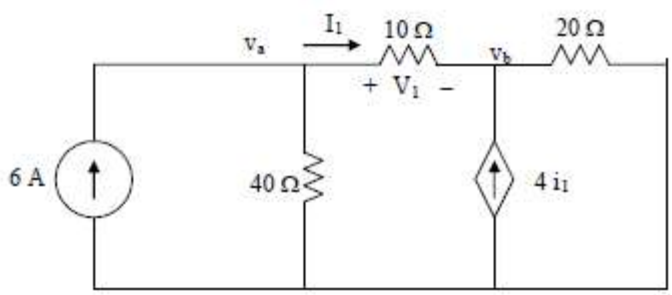
Q2

4.11 Use the superposition principle to find i_o and v_o in the circuit of Fig. 4.79.



Superposition

Sol 2



At node a,

$$6 = \frac{v_a}{40} + \frac{v_a - v_b}{10} \longrightarrow 240 = 5v_a - 4v_b \qquad (1)$$

At node b,

$$-I_1 - 4I_1 + (v_b - 0)/20 = 0 \text{ or } v_b = 100I_1$$

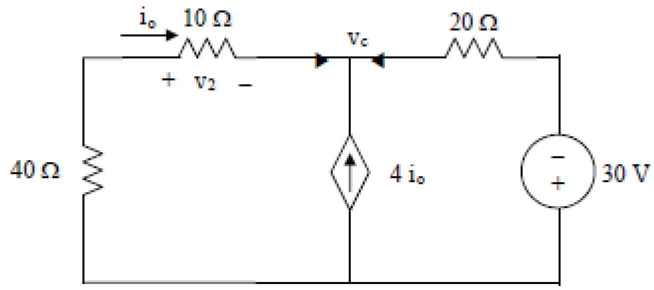
But $i_1 = \frac{v_a - v_b}{10}$ which leads to $100(v_a - v_b)10 = v_b$ or $v_b = 0.9091v_a$ (2)

Substituting (2) into (1),

$$5v_a - 3.636v_a = 240 \text{ or } v_a = 175.95 \text{ and } v_b = 159.96$$

However, $v_1 = v_a - v_b = 15.99$ V.

To find v_2, consider the circuit below.



$$\frac{0-v_c}{50}+4i_o+\frac{(-30-v_c)}{20}=0$$

But $i_o=\dfrac{(0-v_c)}{50}$

$$-\frac{5v_c}{50}-\frac{(30+v_c)}{20}=0 \longrightarrow v_c=-10\text{ V}$$

$$i_2=\frac{0-v_c}{50}=\frac{0+10}{50}=\frac{1}{5}$$

$$v_2=10i_2=2\text{ V}$$

$v_o = v_1 + v_2$ =15.99 + 2 = **17.99 V** and $i_o = v_o/10$= **1.799 A**.

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<td>Q3</td>
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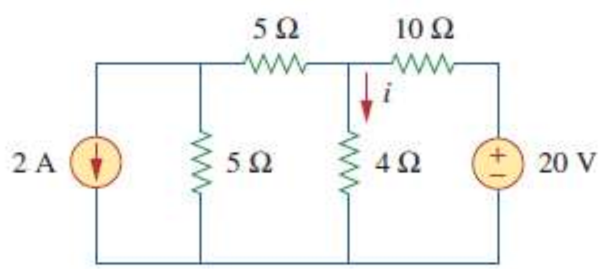
4.22 For the circuit in Fig. 4.90, use source transformation to find i.



Source transformation

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<td>Sol 3</td>
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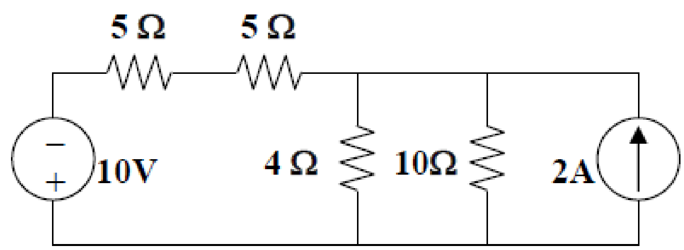
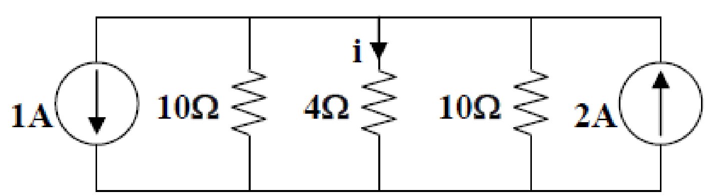
We transform the two sources to get the circuit shown in Fig. (a).



(a)



(b)

We now transform only the voltage source to obtain the circuit in Fig. (b).

$10||10 = 5$ ohms, $i = [5/(5 + 4)](2 – 1) = 5/9 =$ **555.5 mA**

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