# Circuits I – Tutorial 04

# Nodal Analysis

| # | Student ID | Student Name | Grade (10) |
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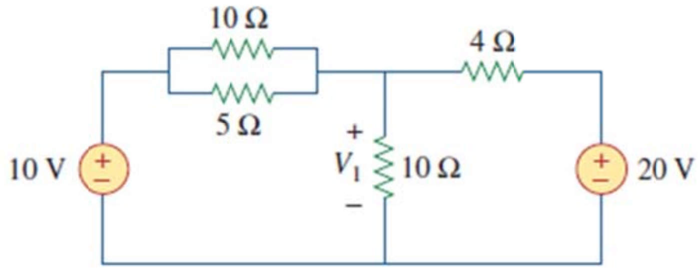
| Q1 | **3.6** Solve for $V_1$ in the circuit of Fig. 3.55 using nodal analysis.<br><br> |
| Sol 1 | $\frac{(V_1-(-10))}{5}+\frac{(V_1-(-10))}{10}+\frac{(V_1-0)}{10}+\frac{(V_1-20)}{10}=0$<br>Simplify and solve.<br>$\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)V_1=-\frac{10}{5}-\frac{10}{10}+\frac{20}{10}$<br>$(0.2+0.1+0.1+0.1)V_1=0.5V_1=-2-1+2=-1$<br>$V_1 = -2$ V. |

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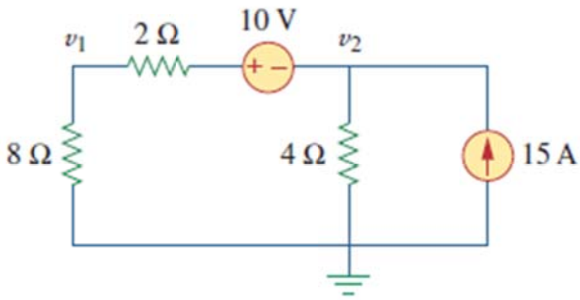
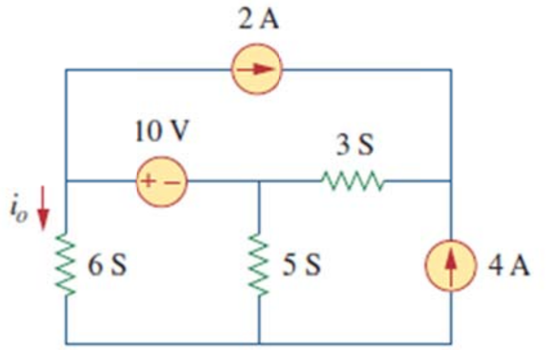
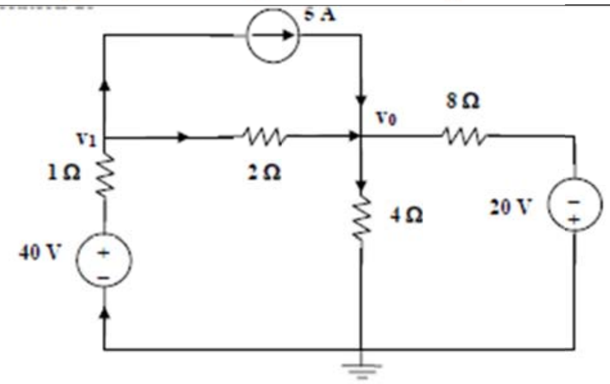
| Q2 | **3.13** Calculate $v_1$ and $v_2$ in the circuit of Fig. 3.62 using nodal analysis.   |
| Sol 2 | At node number 2, $[((v_2 + 10) – 0)/10] + [(v_2–0)/4] – 15 = 0$ or $(0.1+0.25)v_2 = 0.35v_2 = –1+15 = 14$ or <br> $v_2 =$ **40 volts**. <br> Next, $I = [(v_2 + 10) – 0]/10 = (40 + 10)/10 = 5$ amps and <br> $v_1 = 8x5 =$ **40 volts**. |

Q3

**3.15** Apply nodal analysis to find $i_o$ and the power dissipated in each resistor in the circuit of Fig. 3.64.




Sol 3




Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \longrightarrow 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \longrightarrow v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

$$i_0 = 6v_1 = \mathbf{29.45\ A}$$

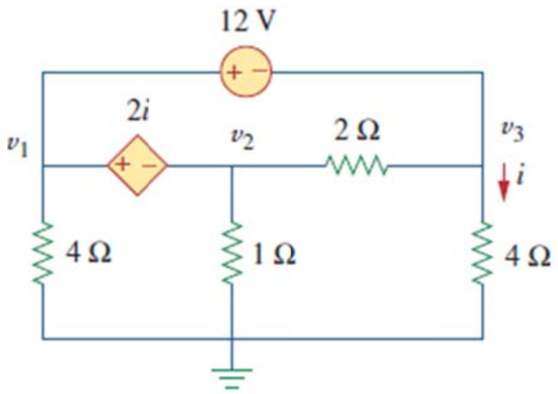
$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \mathbf{144.6\ W}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \mathbf{129.6\ W}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \mathbf{12\ W}$$

Q4

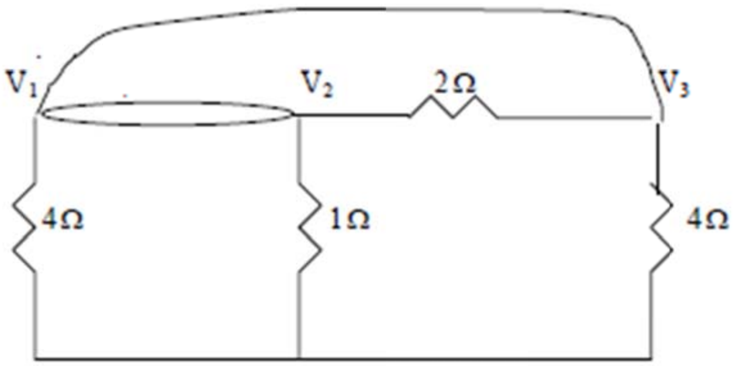
**3.20** For the circuit in Fig. 3.69, find $v_1$, $v_2$, and $v_3$ using nodal analysis.




Sol 4

Nodes 1 and 2 form a supernode; so do nodes 1 and 3. Hence

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \quad \longrightarrow \quad V_1 + 4V_2 + V_3 = 0 \qquad (1)$$




Between nodes 1 and 3,

$$-V_1 + 12 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_1 - 12 \qquad (2)$$

Similarly, between nodes 1 and 2,

$$V_1 = V_2 + 2i \qquad (3)$$

But $i = V_3/4$. Combining this with (2) and (3) gives

$$V_2 = 6 + V_1/2 \qquad (4)$$

Solving (1), (2), and (4) leads to

$$\underline{V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, V_3 = -15\text{V}}$$

Q5

**3.26** Calculate the node voltages $v_1$, $v_2$, and $v_3$ in the circuit of Fig. 3.75.

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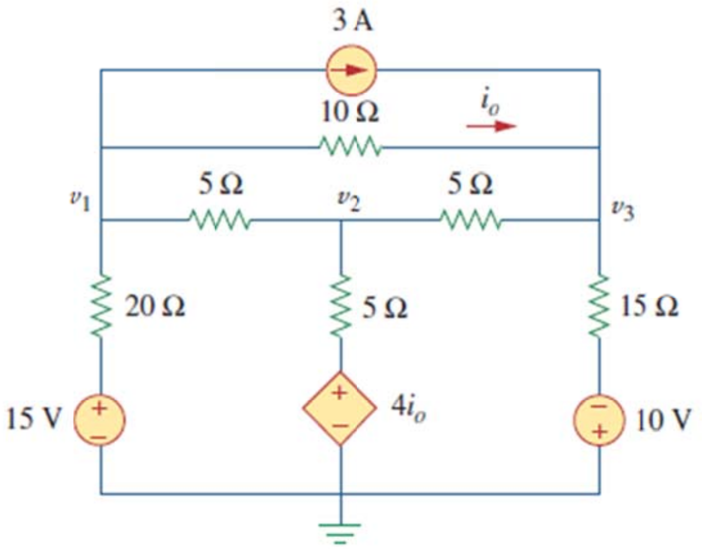


Sol 5

At node 1,

$$\frac{15-V_1}{20} = 3 + \frac{V_1-V_3}{10} + \frac{V_1-V_2}{5} \longrightarrow -45 = 7V_1 - 4V_2 - 2V_3 \quad (1)$$

At node 2,

$$\frac{V_1-V_2}{5} + \frac{4I_o-V_2}{5} = \frac{V_2-V_3}{5} \quad (2)$$

But $I_o = \frac{V_1-V_3}{10}$. Hence, (2) becomes

$$0 = 7V_1 - 15V_2 + 3V_3 \quad (3)$$

At node 3,

$$3 + \frac{V_1-V_3}{10} + \frac{-10-V_3}{15} + \frac{V_2-V_3}{5} = 0 \longrightarrow 70 = -3V_1 - 6V_2 + 11V_3 \quad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \longrightarrow AV = B$$

Using MATLAB leads to

$$V = A^{-1}B = \begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

$V_1 = -7.19V$; $V_2 = -2.78V$; $V_3 = 2.89V$.