Lecture (05)
Mesh Analysis

By:

Dr. Ahmed ElShafee

FACULTY OF ENGINEERING
AHRAM CANADIAN UNIVERSITY

Agenda

- Mesh analysis with voltage sources
- Mesh Analysis with Current Sources

Mesh analysis with voltage sources

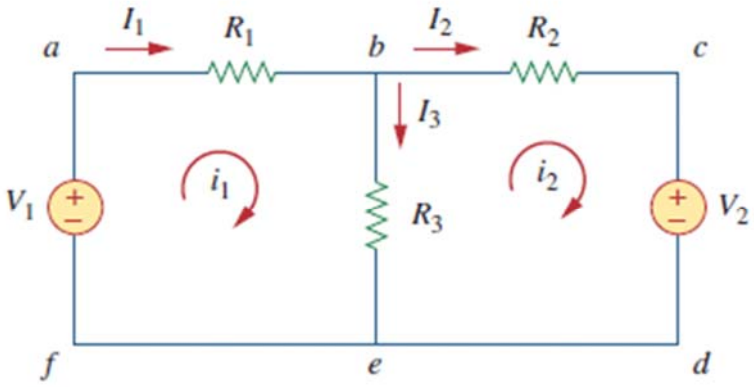
- Determine meshes in the circuit.
- Set the current and direction of each mesh.
- Use KVL to build system of equations (equation/mesh), use ohm's low to express voltage in terms of mesh current.
- Solve system of equations, to find the current of each mesh.
- Convert mesh current to then normal current form.

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

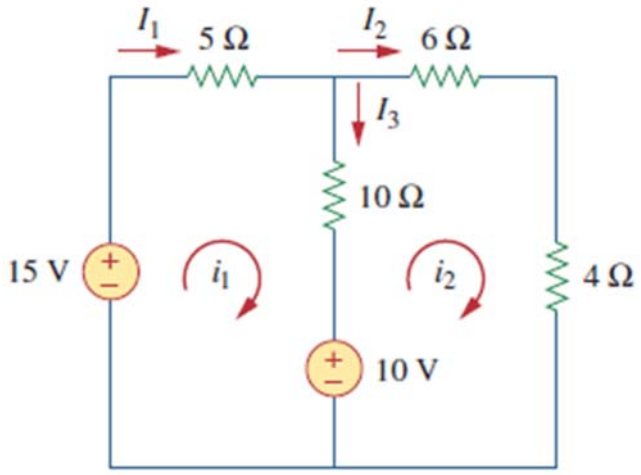
$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$



Example 01

For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.



$$i_1 = \frac{\Delta_1}{\Delta} = 1\ \text{A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1\ \text{A}$$

KVL@1

$$-15 + 5i1 + (i1 - i2)10 + 10 = 0$$

$$-5 + 5i1 + 10i1 - 10i2 = 0$$

$$15i1 - 10i2 = 5 \qquad /5$$

$$3i1 - 2i2 = 1 \rightarrow 1$$

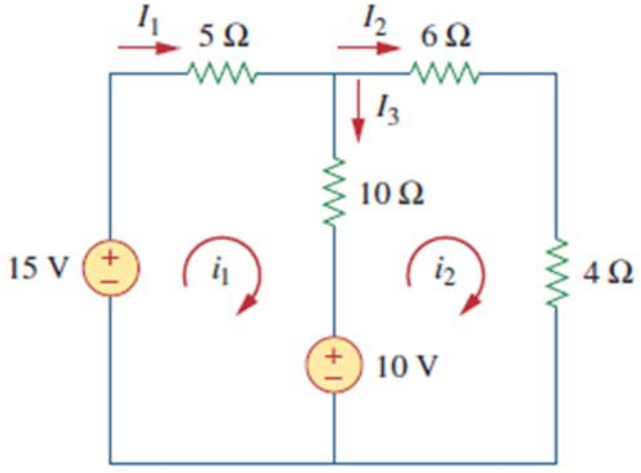
KVL@2

$$-10 + 10(i2 - i1) + 6i2 + 4i2 = 0$$

$$10i2 - 10i1 + 6i2 + 4i2 = 10$$

$$20i2 - 10i1 = 10 \quad /10$$

$$-i1 + 2i2 = 1 \rightarrow 2$$



Add 1,2

$2i1 = 2$

$i1 = 1\ Amp$

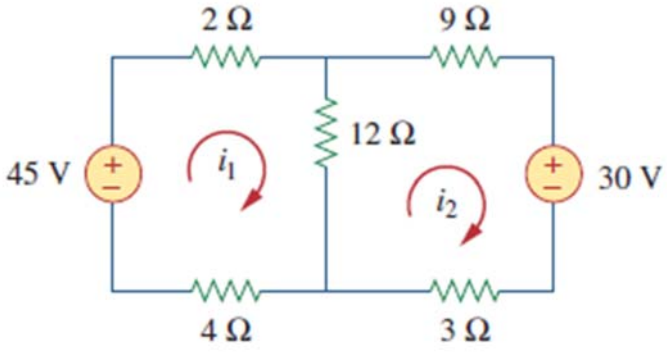
Sub in 2

$2i2 = 2$

$i2 = 1\ amp$

Example 02

Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 3.19.



$i_1 = 2.5$ A, $i_2 = 0$ A.

KVL@1

$$-45 + 2i1 + 12(i1 - i2) + 4i1 = 0$$

$$2i1 + 12i1 - 12i2 + 4i1 = 45$$

$$18i1 - 12i2 = 45 \quad /3$$

$$6i1 + 4i2 = 15 \rightarrow 1$$

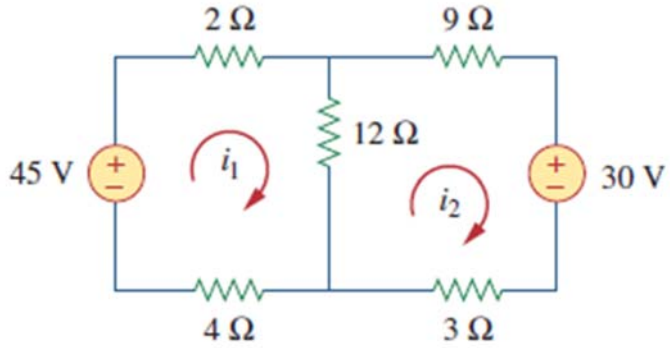
KVL@2

$$12(i2 - i1) + 9i2 + 3i2 + 30 = 0$$

$$12i2 - 12i1 + 9i2 + 3i2 = -30$$

$$-12i1 + 24i2 = -30 \quad /6$$

$$-2i1 + 4i2 = -5 \rightarrow 2$$



Add 1,2

$$4i1 = 10$$

$$i1 = 2.5\ Amp$$

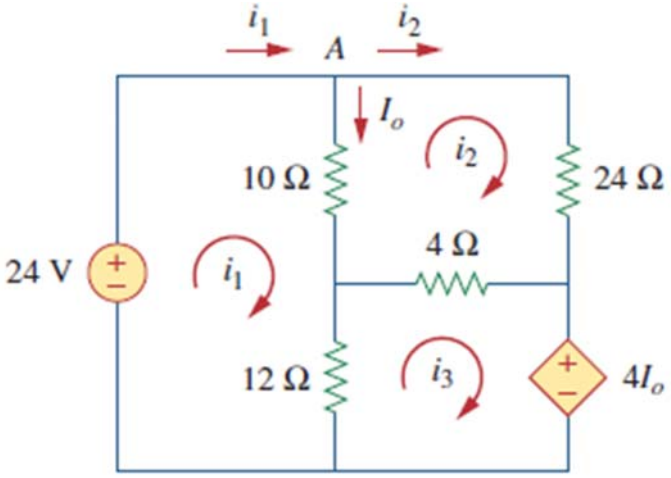
Sub in 2

$$4i2 = -5 + (2 \times 2.5) = 0$$

$$i2 = 0\ Amp$$

Example 03

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.



Thus, $I_o = i_1 - i_2 = 1.5$ A.

KVL@1

$$-24 + 10(i1 - i2) + 12(i1 - i3) = 0$$

$$10i1 - 10i2 + 12i1 - 12i3 = 24$$

$$22i1 - 10i2 - 12i3 = 24 \quad /2$$

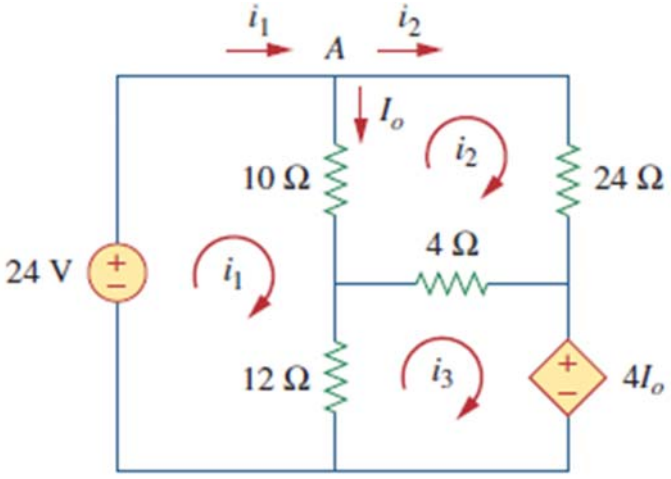
$$11i1 - 5i2 - 6i3 = 12 \quad \rightarrow 1$$

KVL@2

$$10(i2 - i1) + 24i2 + 4(i2 - i3) = 0$$

$$10i2 - 10i1 + 24i2 + 4i2 - 4i3 = 0$$

$$-10i1 + 38i2 - 4i3 = 0 \quad /2$$



$-5i1 + 19i2 - 2i3 = 0 \rightarrow 2$

KVL@3

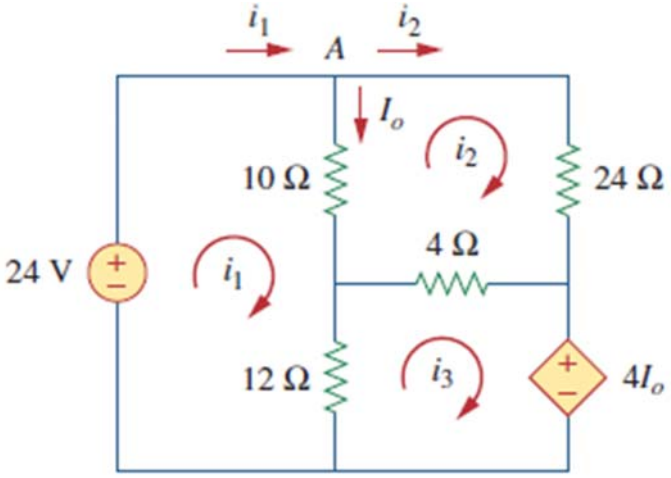
$12(i3 - i1) + 4(i3 - i2) + 4i0 = 0$

$12i3 - 12i1 + 4i3 - 4i2 + 4(i1 - i2) = 0$

$12i3 - 12i1 + 4i3 - 4i2 + 4i1 - 4i2 = 0$

$-8i1 - 8i2 + 16i3 = 0 \quad /8$

$-i1 - i2 + 2i3 = 0 \rightarrow 3$



$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} \\ \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

Crammer

$$\begin{pmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} i1 \\ i2 \\ i3 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix}$$

$\Delta = 11(38 - 2) + 5(-10 - 6) - 1(10 + 114)$

$= 396 - 80 - 124 = 192$

$$\Delta 1 = \begin{pmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{pmatrix} = 12(38 - 2) = 432$$

$$\Delta 2 = \begin{pmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{pmatrix} = -12(-10 - 2) = 144$$

$$\Delta 3 = \begin{pmatrix} 11 & -5 & 10 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{pmatrix} = 12(5 + 19) = 288$$

$$i1 = \frac{\Delta 1}{\Delta} = \frac{432}{192} = 2.25\ Amp$$

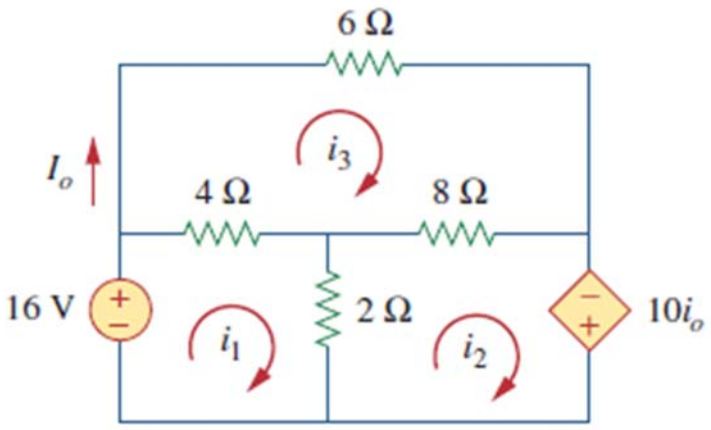
$$i2 = \frac{\Delta 2}{\Delta} = \frac{144}{192} = 0.75\ Amp$$

$$i3 = \frac{\Delta 3}{\Delta} = \frac{288}{192} = 1.5\ Amp$$

$$i0 = i1 - i2 = 2.25 - 0.75 = 1.5\ Amp$$

Example 04

Using mesh analysis, find I_o in the circuit of Fig. 3.21.



Answer: -4 A.

KVL@1

$$-16 + 4(i1 - i3) + 2(i1 - i2) = 0$$

$$4i1 - 4i3 + 2i1 - 2i2 = 16$$

$$6i1 - 2i2 + 4i3 = 16 \quad /2$$

$$3i1 - i2 - 2i3 = 4 \rightarrow 1$$

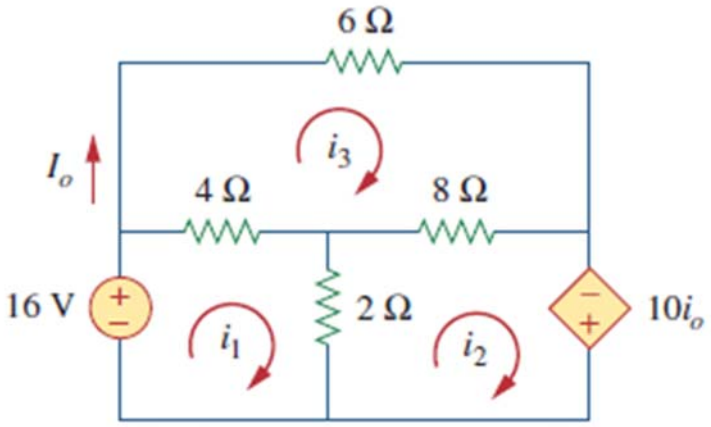
KVL@2

$$2(i2 - i1) + 8(i2 - i3) - 10i0 = 0$$

$$2i2 - 2i1 + 8i2 - 8i3 - 10i3 = 0$$

$$-2i1 + 10i2 - 18i3 = 0 \quad /2$$

$$-i1 + 5i2 - 9i3 = 0 \rightarrow 2$$



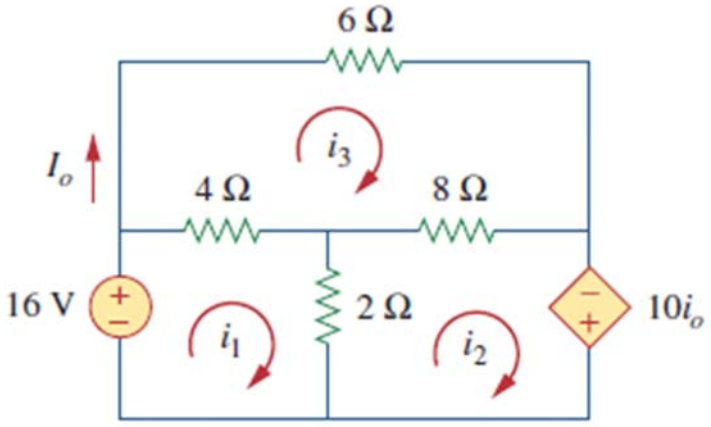
KVL@3

$$6i3 + 4(i3 - i1) + 8(i3 - i2) = 0$$

$$6i3 + 4i3 - 4i1 + 8i3 - 8i2 = 0$$

$$-4i1 - 8i2 + 18i3 = 0 \quad /2$$

$$-2i1 - 4i2 + 9i3 = 0 \rightarrow 3$$



Crammer

$$\begin{pmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{pmatrix}\begin{pmatrix} i1 \\ i2 \\ i3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta = 3(45-36) + (-9-8) - 2(9+10) = 27 - 17 - 38 = -27$$

$$\Delta 1 = \begin{pmatrix} 4 & -1 & -2 \\ 0 & 5 & -9 \\ 0 & -4 & 9 \end{pmatrix} = 4(45-36) = 36$$

$$i1 = \frac{\Delta 1}{\Delta} = -\frac{27}{36} = -0.75\ Amp$$

$$\Delta 2 = \begin{pmatrix} 3 & 4 & -2 \\ -1 & 0 & -9 \\ -2 & 0 & 9 \end{pmatrix} = -4(-9-18) = 108$$

$$i2 = \frac{108}{-27} = -4\ Amp = i0$$

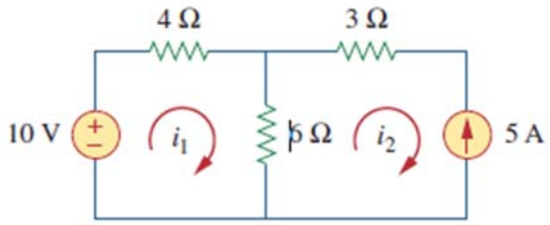
$$\Delta 3 = \begin{pmatrix} 3 & -1 & 4 \\ -1 & 5 & 0 \\ -2 & -4 & 0 \end{pmatrix} = 4(4+10) = 56$$

$$i3 = \frac{56}{-27} = -2.07\ Amp$$

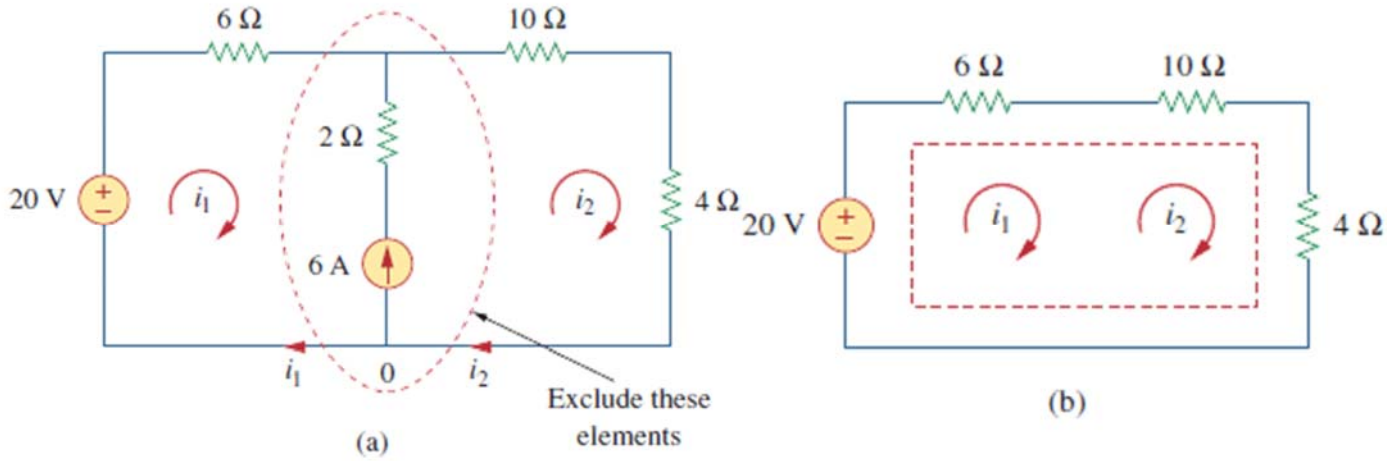
Mesh Analysis with Current Sources

- Case 1: When a current source exists only in one mesh, mesh current = source current

$$i_2 = -5 \text{ A}$$



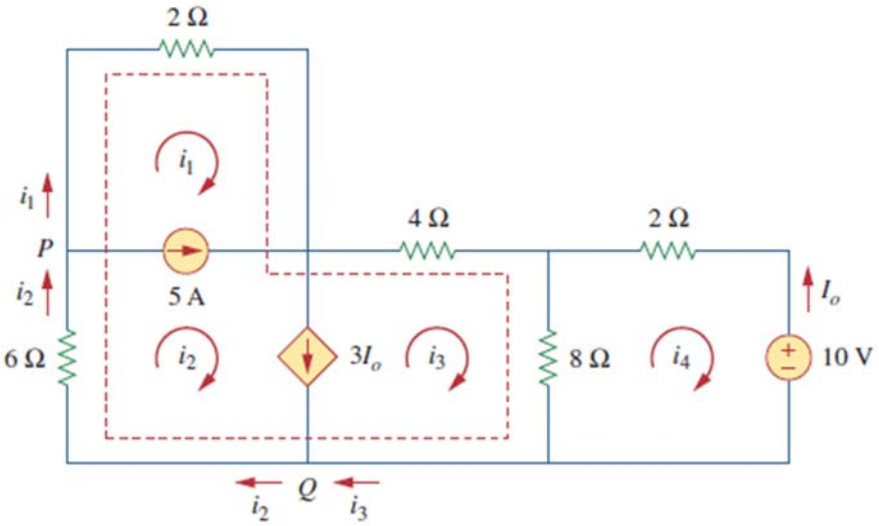
- Case 2: When a current source exists between two meshes; We create a *supermesh* by excluding the current source and any elements connected in series with it,



- Apply KCL : $-20 + 6i_1 + 10i_2 + 4i_2 = 0$

Example 05

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.



$i_1 = -7.5$ A, $i_2 = -2.5$ A, $i_3 = 3.93$ A, $i_4 = 2.143$ A

KVL@1,2,3 super mesh

$$2i1 + 6i2 + 4i3 + 8(i3 - i4) = 0$$

$$2i1 + 6i2 + 4i3 + 8i3 - 8i4 = 0$$

$$2i1 + 6i2 + 12i3 - 8i4 = 0 \ /2$$

$$i1 + 3i2 + 6i3 - 4i4 = 0 \rightarrow 1$$

KCL@ super 1,2

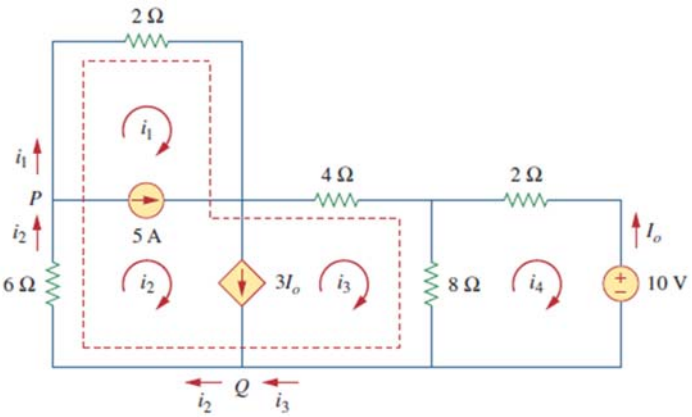
$$i2 = 5 + i1 \rightarrow 2$$

KCL@ super 2,3

$$i2 - i3 = 3i0$$

But $i0 = -i4$

$$i2 - i3 + 3i4 = 0 \rightarrow 3$$



KVL@4

$8(i4 - i3) + 2i4 + 10 = 0$

$8i4 - 8i3 + 2i4 = -10$

$-8i3 + 10i4 = -10 \ /2$

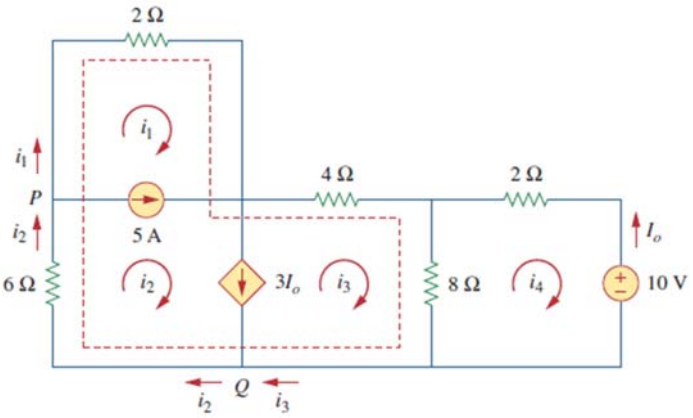
$-4i3 + 5i4 = -5 \rightarrow 4$

Sub 2 in 1,3

$i1 + 3(i1 + 5) + 6i3 - 4i4 = 0$

$4i1 + 6i3 - 4i4 = -15 \rightarrow 5$

$i1 - i3 + 3i4 = -5 \rightarrow 6$



Crammer

$$\begin{pmatrix} 4 & 6 & -4 \\ 1 & -1 & 3 \\ 0 & -4 & 5 \end{pmatrix} \begin{pmatrix} i1 \\ i2 \\ i3 \end{pmatrix} = \begin{pmatrix} -15 \\ -5 \\ -5 \end{pmatrix}$$

$\Delta = 4(-5 + 12) - (30 - 16) = 28 - 14 = 14$

$$\Delta 1 = \begin{pmatrix} -15 & 6 & -4 \\ -5 & -1 & 3 \\ -5 & -4 & 5 \end{pmatrix}$$

$= -15(-5 + 12) + 5(30 - 16) - 5(18 - 4)$

$= -105 + 70 - 70 = -105$

$$i1 = \frac{\Delta 1}{\Delta} = -\frac{105}{14} = -7.5 \ Amp$$

From 2

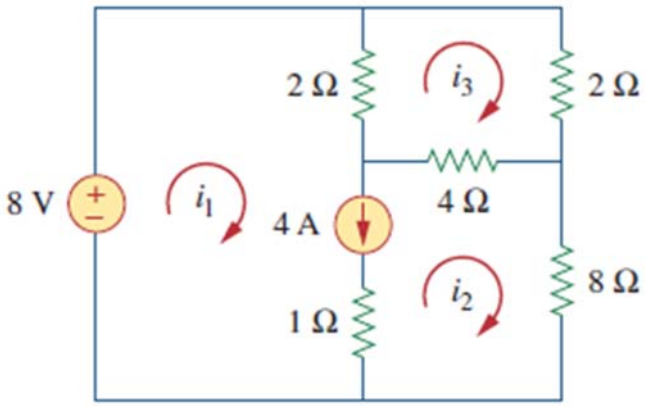
$i2 = 5 - 7.5 = -2.5 \ Amp$

$$\Delta 3 = \begin{pmatrix} 4 & -15 & -4 \\ 1 & -5 & 3 \\ 0 & -5 & 5 \end{pmatrix} = 4(-25+15) - (-75-20)$$
$$= -40 + 95 = 55$$
$$i3 = \frac{\Delta 3}{\Delta} = \frac{55}{14} = 3.93\ Amp$$

$$\Delta 4 = \begin{pmatrix} 4 & 6 & -15 \\ 1 & -1 & -5 \\ 0 & -4 & -5 \end{pmatrix} = 4(5-20) - (-30+60) = 60 - 30$$
$$= 30$$
$$i4 = \frac{\Delta 4}{\Delta} = \frac{30}{14} = 2.14\ Amp$$

Example 06

Use mesh analysis to determine i_1, i_2, and i_3 in Fig. 3.25.



$i_1 = 4.632$ A, $i_2 = 631.6$ mA, $i_3 = 1.4736$ A.

KVL@1,2 super mesh

$$-8 + 2(i1 - i3) + 4(i2 - i3) + 8i2 = 0$$

$$2i1 - 3i2 + 4i2 - 4i3 + 8i2 = 8$$

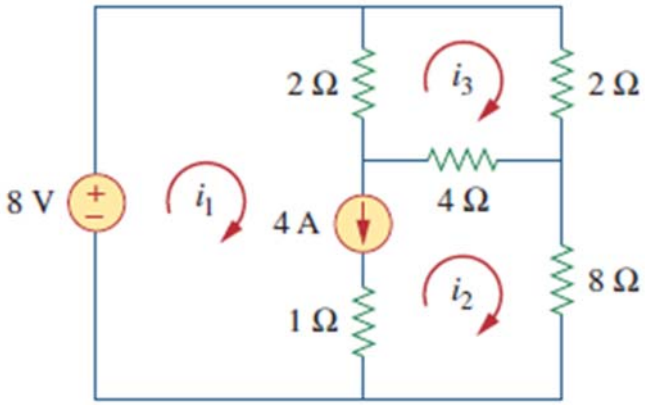
$$2i1 + 12i2 - 6i3 = 8 \ /2$$

$$i1 + 6i2 - 3i3 = 4 \rightarrow 1$$

KCL @ super mesh 1,2

$$i1 - i2 = 4$$

$$i1 = 4 + i2 \rightarrow 2$$



KVL@3

$$2(i3 - i1) + 2i3 + 4(i3 - i2) = 0$$

$$2i3 - 2i1 + 2i3 + 4i3 + 4i3 - 4i2 = 0$$

$$-2i1 - 4i2 + 8i3 = 0 \ /2$$

$$-i1 - 2i2 + 4i3 = 0 \rightarrow 2$$

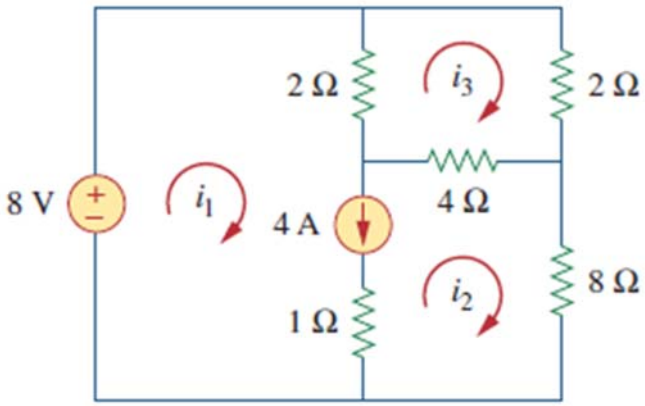
Sub 2 in 1,2

$$4 + i2 + 6i2 - 3i3 = 4$$

$$7i2 - 3i3 = 0 \rightarrow 4$$

$$-4 - i2 - 2i2 + 4i3 = 0$$

$$-3i2 + 4i3 = 4 \rightarrow 3$$



Mult 4 x 3 and 3 x 4

$$28i2 - 12i3 = 0$$
$$-9i2 + 12i3 = 12$$

$$19i2 = 12$$
$$i2 = \frac{12}{19} = 0.632\ Amp$$

Sub in 2

$$i1 = 4 + 0.632 = 4.632\ Amp$$

Sub in 1

$$i3 = \frac{i1 + 6i2 - 4}{3} = \frac{4.632 + 6(0.632) - 4}{3} = 1.47\ Amp$$