

Lecture (02) Basic laws – P1

By:

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Agenda

- Ohm's Law
- Nodes, Branches, and Loops
- Kirchhoff's Laws

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Ohm's Law

- Materials in general have a characteristic behavior of resisting the flow of electric charge, ability to resist current, is known as *resistance* and is represented by the symbol R .
- in mathematical form

$$R = \rho \frac{\ell}{A}$$

- ρ resistivity
- A area
- L length
- Good conductors, such as copper and aluminum, have low resistivity, while insulators, such as mica and paper, have high

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- Resistivity of common materials

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^2	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

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- Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor. "Georg Simon Ohm (1787–1854), a German physicist"

$$v \propto i$$

$$v = iR$$

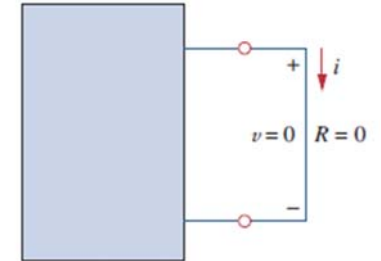
- R in Eq. is measured in the unit of ohms, designated Ω .
- The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).
- The direction of current i and the polarity of voltage v must conform with the passive sign convention

- This implies that current flows from a higher potential to a lower potential in order for

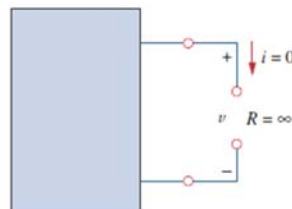
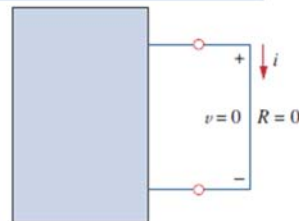
$$v = iR.$$

- If current flows from a lower potential to a higher potential

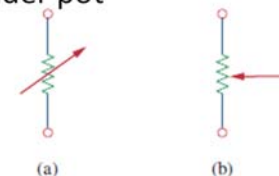
$$v = -iR$$



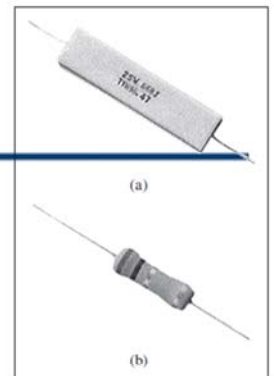
- the value of R can range from zero to infinity
- An element with $R=0$ is called a *short circuit*, $V=0$, $I=?$
- A short circuit is a circuit element with resistance approaching zero
- an element with $R = \infty$ is known as an *open circuit*
- indicating that the current is zero though the voltage could be anything
- An open circuit is a circuit element with



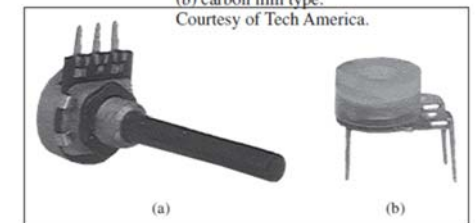
- Resistor types
 - Fixed
 - wirewound
 - composition
 - Variable (*potentiometer* or *pot*)
 - Composition
 - slider *pot*



Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

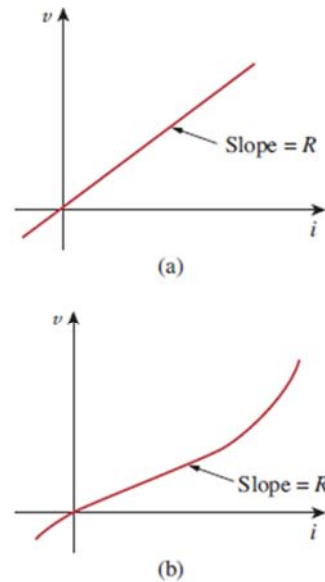


Fixed resistors: (a) wirewound type, (b) carbon film type. Courtesy of Tech America.



Variable resistors: (a) composition type, (b) slider *pot*. Courtesy of Tech America.

- Device mounting classification
 - surface mounted or
 - Integrated
- Linear resistor has a constant resistance and thus its current-voltage is straight has a R slope
- A *nonlinear* resistor does not obey Ohm's law.



The i - v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

- The conductance is a measure of how well an element will conduct electric current, unit of conductance is the *mho* σ or siemens (S),

$$G = \frac{1}{R} = \frac{i}{v}$$

$$1 \text{ S} = 1 \sigma = 1 \text{ A/V}$$

$$i = Gv$$

$$p = vi = i^2R = \frac{v^2}{R}$$

$$p = vi = v^2G = \frac{i^2}{G}$$

- Dissipated power properties
 - The power dissipated in a resistor is a nonlinear function of either current or voltage.
 - Since R and G are positive quantities, the power dissipated in a resistor is always positive.
 - Thus, a resistor always absorbs power from the circuit.
 - This confirms the idea that a resistor is a passive element, incapable of generating energy.

Example 1

- An electric iron draws 2 A at 120 V. Find its resistance.

Example 02

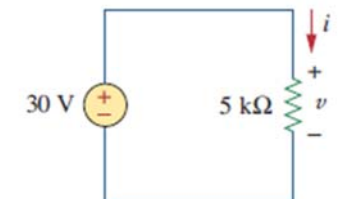
The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance $15\ \Omega$ at $110\ \text{V}$?

Solution

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Example 03

In the circuit shown in Fig. , calculate the current i , the conductance G , and the power p .



An electric iron draws $2\ \text{A}$ at $120\ \text{V}$. Find its resistance.

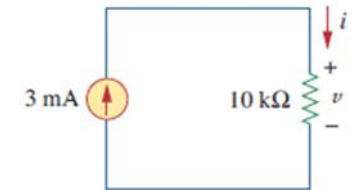
The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance $15\ \Omega$ at $110\ \text{V}$?

Answer

- $I = V/R = 110/15 = 7.33\ \text{ohm}$

Example 04

For the circuit shown in Fig. . calculate the voltage v , the conductance G , and the power p .



Answer

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

$$p = i^2 R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

$$p = v^2 G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

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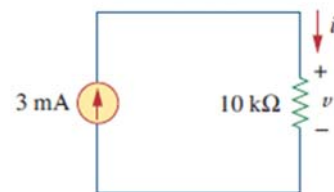
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Example 05

For the circuit shown in Fig. . calculate the voltage v , the conductance G , and the power p .

Answer

- $V = I \cdot R = 3 \times 10^{-3} \times 10 \times 10^3 = 30 \text{ volt}$
- $G = 1/R = 1 / 10 \times 10^3 = 100 \text{ u siemens}$
- $P = i^2 \times R = (3 \times 10^{-3})^2 \times 10 \times 10^3 = 0.09 \text{ w}$



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A voltage source of $20 \sin \pi t$ V is connected across a $5\text{-k}\Omega$ resistor. Find the current through the resistor and the power dissipated.

Answer

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

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example 06

A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R .

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A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R .

Answer

$$I = P/V = 30 \cos^2(t) / 15 \cos(t) = 2 \cos(t) \text{ Amp}$$

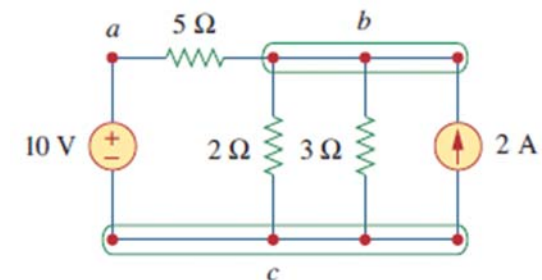
$$R = V^2/P = 225 \cos^2(t)/30 \cos^2(t) = 7.5 \text{ ohm}$$

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Nodes, Branches, and Loops

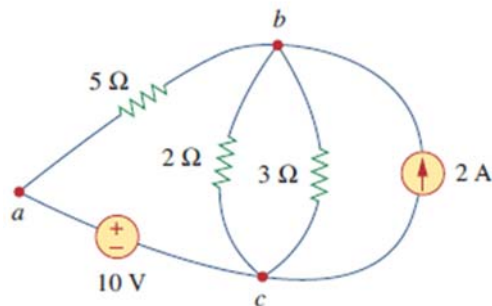
- A branch represents a single element such as a voltage source or a resistor.
- In other words, a branch represents any two-terminal element



- five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

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- A node is the point of connection between two or more branches (indicated by a dot in a circuit)
- If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.
- Below circuit has three nodes a, b, and c.



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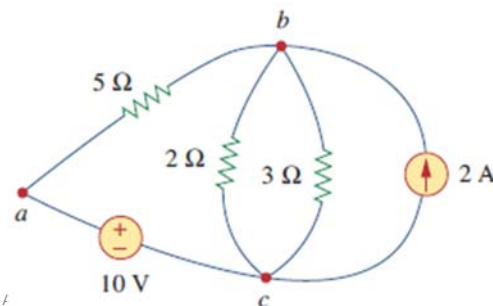
- A loop is any closed path in a circuit.
- A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- Independent loop; contains at least one branch which is not a part of any other independent loop
- Independent loops or paths result in independent sets of equations

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- fundamental theorem of network topology
- A network with b branches, n nodes, and l independent loops will satisfy that

$$L = b - n + 1$$

$b=5$
 $n=3$
 $l = 3$
 Independent loops:
 Abca == $5\ \Omega - 2\ \Omega - 10\text{V}$
 Abca == $5\ \Omega - 3\ \Omega - 10\text{V}$
 Abca == $5\ \Omega - 2\text{A} - 10\text{V}$

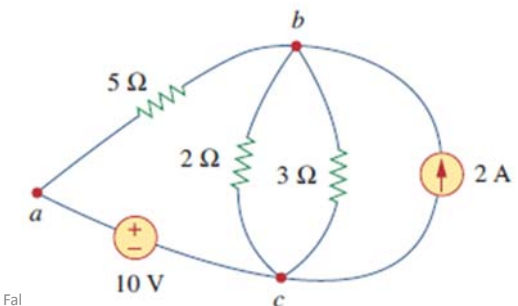


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- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them

Connection type	
In series	$5\ \Omega - 10\text{V}$
In parallel	$2\ \Omega - 3\ \Omega - 2\text{A}$
In parallel	$(5\ \Omega - 10\text{V}) - 2\ \Omega - 3\ \Omega - 2\text{A}$

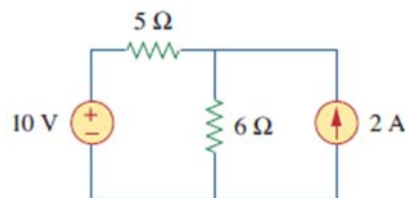


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Example 07

Determine the number of branches and nodes in the circuit shown in Fig. Identify which elements are in series and which are in parallel.

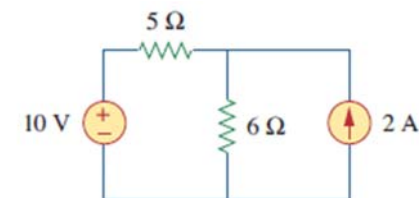


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Determine the number of branches and nodes in the circuit shown in Fig. Identify which elements are in series and which are in parallel.

- Solution
- $b = 4$
- $n = 2$
- $l = b - n + 1 = 3$
- Independent loops : $10V - 5\Omega - 6\Omega$, $10V - 5\Omega - 2A$, $6\Omega - 2A$



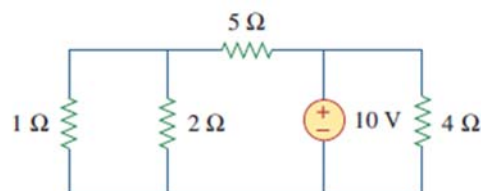
Connection type	
In series	$10V - 5\Omega$
In parallel	$6\Omega - 2A$
In parallel	$(10V - 5\Omega) - 6\Omega - 2A$

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Example 8

How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.

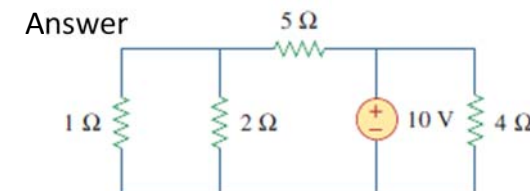


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How many branches and nodes does the circuit in Fig. have? Identify the elements that are in series and in parallel.

- $b = 5$
- $n = 3$
- $l = 5 - 3 + 1 = 3$
- Independent loops : $1\Omega - 5\Omega - 10V$, $1\Omega - 5\Omega - 4\Omega$, $4\Omega - 5\Omega - 2\Omega$



Connection type	
In parallel	$1\Omega - 2\Omega$
In parallel	$4\Omega - 10V$

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Kirchhoff's Laws

- Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.
- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

$$\sum_{n=1}^N i_n = 0$$

- where N is the number of branches connected to the node and i_n is the n^{th} current entering (or leaving) the node.

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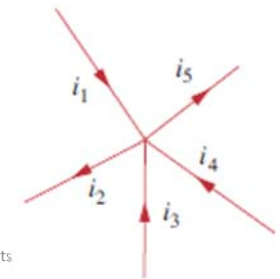
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- By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

- The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$



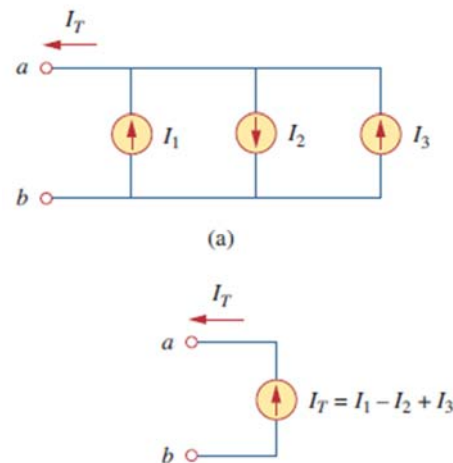
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- The combined or equivalent current source can be found by applying KCL to node a .

$$I_T + I_2 = I_1 + I_3$$

$$I_T = I_1 - I_2 + I_3$$



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- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^M V_m = 0$$

- M is the number of voltages in the loop (or the number of branches in the loop) and V_m is the m^{th} voltage.

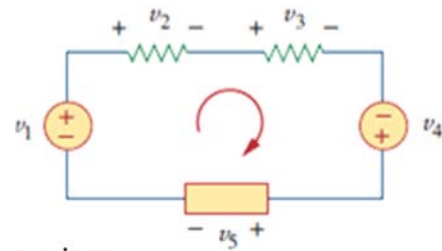
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- The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- We can start with any branch and go around the loop either clockwise or counterclockwise.
- 1st clockwise:

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$



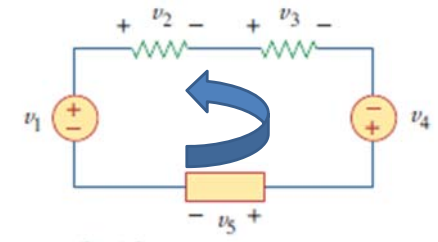
- Sum of voltage drops = Sum of voltage rises

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- 2nd counter clockwise:

$$v_1 - v_2 - v_3 + v_4 = 0$$



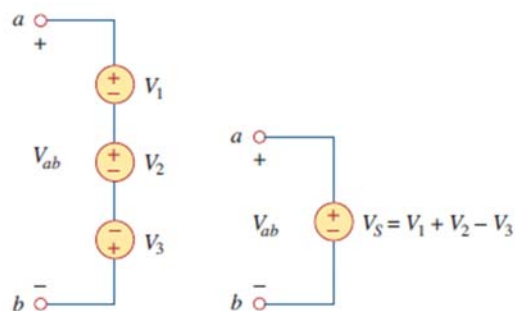
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- When voltage sources are connected in series

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

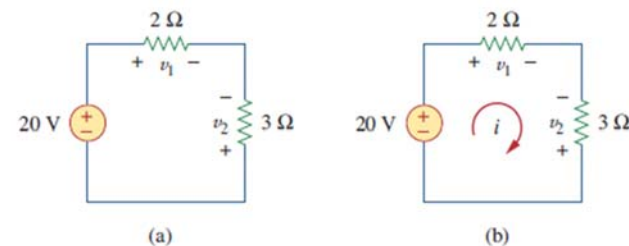


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Example 9

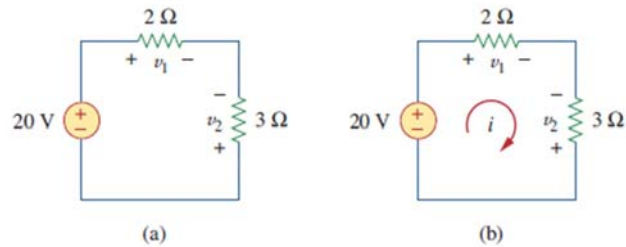
For the circuit in Fig. , find voltages v_1 and v_2 .



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For the circuit in Fig. , find voltages v_1 and v_2 .



Answer

- Ohm's law $v_1 = 2i, \quad v_2 = -3i$
- KVL $-20 + v_1 - v_2 = 0$
- substitute $-20 + 2i + 3i = 0$

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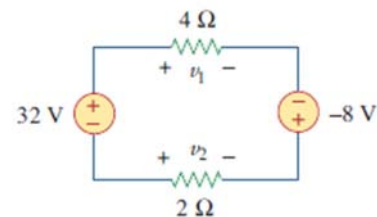
- $5i = 20 \Rightarrow i = 4 \text{ A}$
- $v_1 = 8 \text{ V}, \quad v_2 = -12 \text{ V}$

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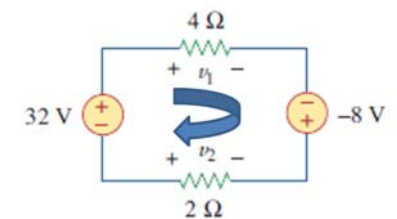
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Example 10

Find v_1 and v_2 in the circuit of Fig.



Find v_1 and v_2 in the circuit of Fig.



Answer

- Ohm's law : $V_1=4i, V_2 = -2i$
- KVL : $-32+ V_1+8 - V_2 = 0$
 $-32+4i+8 + 2i = 0$
 $6i = 24$
 $i = 4 \text{ Amp}$
- $V_1 = 16\text{volt}$
- $V_2 = -8 \text{ Volt}$

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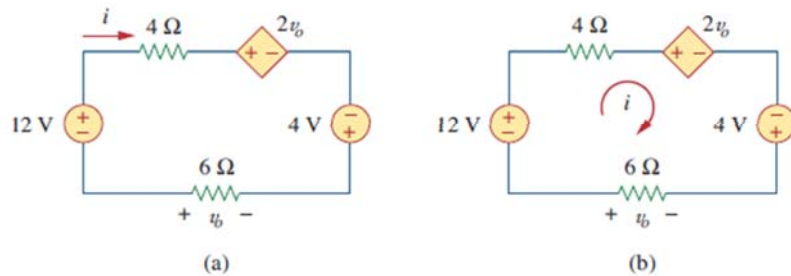
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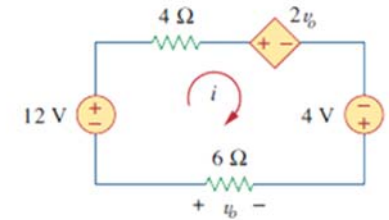
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Example 11

- Determine v_o and i in the circuit shown in Fig



- Determine v_o and i in the circuit shown in Fig



(a)

- Ohm's law : $V_0 = -6i$
- KVL : $-12 + V_{4\Omega} + 2V_0 - 4 - V_{6\Omega} = 0$
 $-12 + 4i + 2V_0 - 4 + 6i = 0$
 $-12 + 4i - 12i - 4 + 6i = 0$
 $-16 = 2i$
 $i = -8 \text{ Amp}$
 $V_0 = 48 \text{ volt}$

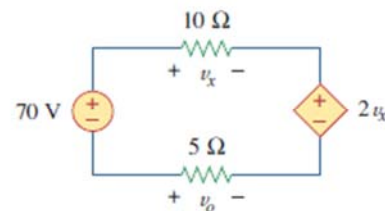
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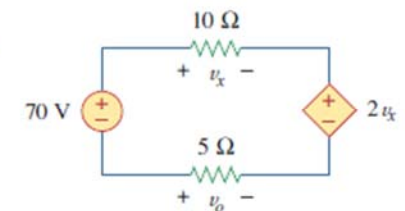
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Example 12

Find v_x and v_o in the circuit of Fig.



Find v_x and v_o in the circuit of Fig.



Answer

- Ohms law: $V_x = 10i$
 $V_0 = -5i$
- KVL: $-70 + V_x + 2V_x - V_0 = 0$
 $-70 + 10i + 2V_x + 5i = 0$
 $-70 + 10i + 20i + 5i = 0$
 $70 = 35i$
 $i = 2 \text{ Amp}$
 $V_x = 20 \text{ V}$
 $V_0 = -10 \text{ V}$

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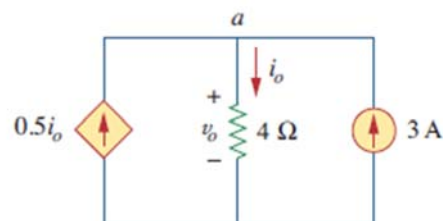
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Example 13

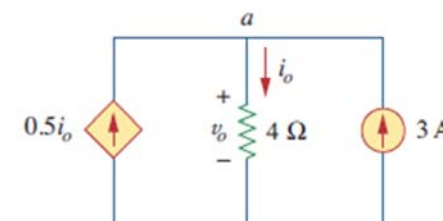
Find current i_o and voltage v_o in the circuit shown in Fig.



Find current i_o and voltage v_o in the circuit shown in Fig.

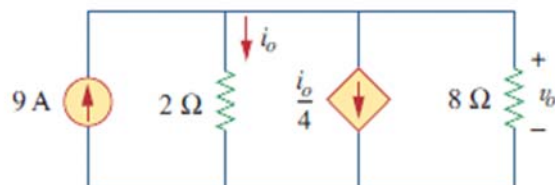
Answer

- KCL @ a : $i_o = 0.5i_o + 3$
 $0.5i_o = 3$
 $i_o = 6 \text{ Amp}$
- Ohm: $V_o = 4 \times i_o = 24 \text{ Volt}$



Example 14

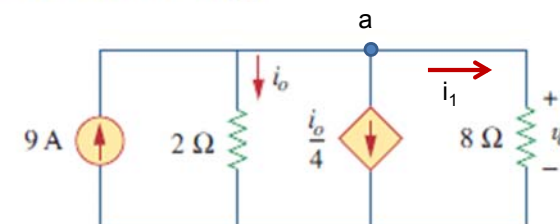
Find v_o and i_o in the circuit of Fig.



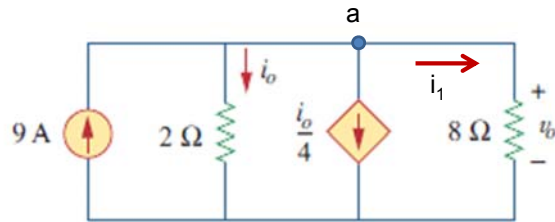
Find v_o and i_o in the circuit of Fig.

Solution:

- KCL @ a : $9 - i_o - (i_o / 4) - i_1 = 0$
- KVL @ (2 Ω - 8 Ω) : $V_{2\Omega} = V_{8\Omega}$
 $2i_o = 8i_1$
 $i_1 = i_o/4$



Find v_o and i_o in the circuit of Fig.



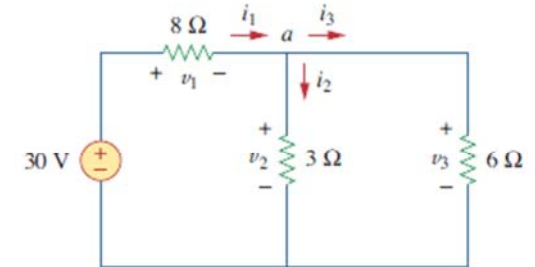
$$9 - i_o - i_o/4 - i_o/4 = 0$$

$$9 = 1.5 i_o \quad \rightarrow \quad i_o = 6 \text{ Amp}$$

$$i_1 = 1.5 \text{ Amp} \quad \rightarrow \quad V_o = 1.5 \times 8 = 12 \text{ Volt}$$

Example 15

Find currents and voltages in the circuit shown in Fig



(a)

Find currents and voltages in the circuit shown in Fig

Answer

Ohm: $V_1 = 8 i_1$

$V_2 = 3 i_2$

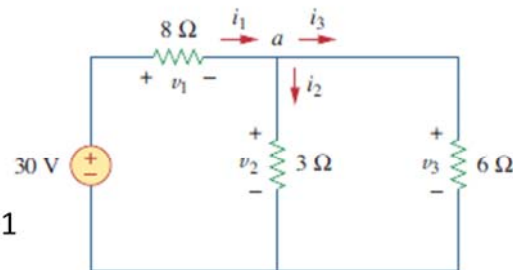
$V_3 = 3 i_3$

KCL @a: $i_1 - i_2 - i_3 = 0 \rightarrow 1$

KVL @ left: $-30 + V_1 + V_2 =$

$30 - 8 i_1 - 3 i_2 = 0$

$i_2 = (30 - 8 i_1)/3 \rightarrow 2$



(a)

Find currents and voltages in the circuit shown in Fig

Answer

KVL @ right: $V_2 = V_3$

$3 i_2 = 6 i_3$

$i_3 = 0.5 i_2 \rightarrow 3$

Substitute 2,3 in 1

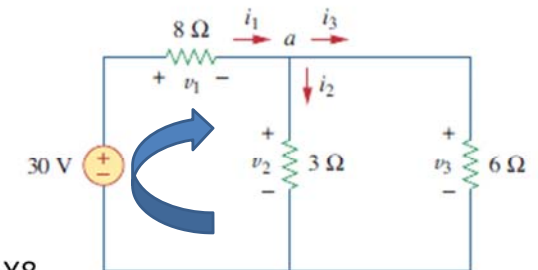
$(30 - 8 i_2)/8 - i_2 - i_2/2 = 0 \quad \times 8$

$30 - 8 i_2 - 8 i_2 - 4 i_2 = 0$

$i_2 = 2 \text{ Amp}$

$i_3 = 1 \text{ Amp}$

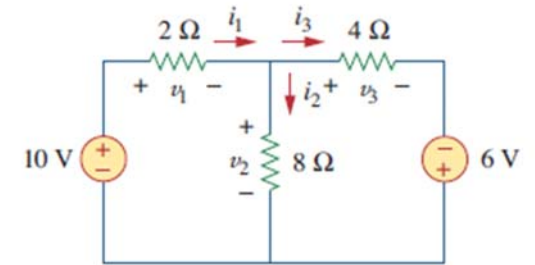
$i_1 = 3 \text{ Amp}$



(a)

Example 16

Find the currents and voltages in the circuit shown in Fig



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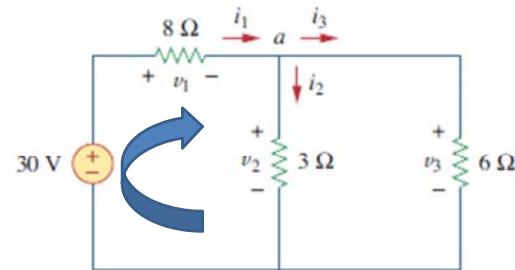
Find currents and voltages in the circuit shown in Fig

Answer

$$V1 = 24 \text{ Volt}$$

$$V2 = 6 \text{ Volt}$$

$$V3 = 6 \text{ Volt}$$



(a)

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Find the currents and voltages in the circuit shown in Fig

Answer

$$\text{Ohm: } V1 = 2 i_1$$

$$V2 = 8 i_2$$

$$V3 = 4 i_3$$

$$\text{KCL @ node: } i_1 - i_2 - i_3 = 0 \rightarrow 1$$

$$\text{KVL @ Left: } -10 + V1 + V2 = 0$$

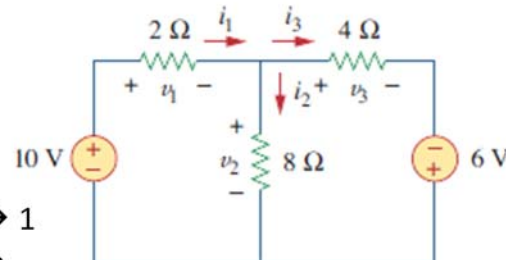
$$-10 + 2 i_1 + 8 i_2 = 0$$

$$i_1 = 5 - 4 i_2 \rightarrow 2$$

$$\text{KVL @ right: } -6 - V2 + V3 = 0$$

$$-6 - 8 i_2 + 4 i_3 = 0$$

$$i_3 = (6 + 8 i_2)/4 \rightarrow 3$$



Find the currents and voltages in the circuit shown in Fig

Answer

Substitute 2,3 in 1

$$5 - 4 i_2 - i_2 - (6 - 8 i_2)/4 = 0$$

$$20 - 16 i_2 - 4 i_2 - 6 - 8 i_2 = 0$$

$$14 = 28 i_2$$

$$i_2 = 0.5 \text{ Amp}$$

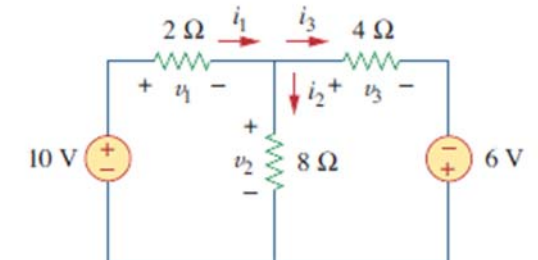
$$i_1 = 3 \text{ Amp}$$


$$i_3 = 2.5 \text{ Amp}$$

$$V1 = 6 \text{ V}$$

$$V2 = 4 \text{ V}$$

$$V3 = 10 \text{ V}$$





Thanks,..
See you next week (ISA),...

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